City & Guilds

UNIT 429 PRINCIPALS OF MECHANICAL ENGINEERINGS

Lesson 1: Construction of equilibrium diagrams **Suggested Teaching Time:** 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Explain the general conditions of static equilibrium and construct free body diagrams (AC 1.1, 1.2 and 1.3)	Principle of moments. The basic principles should be demonstrated using simple equipment, and learners should be given the opportunity to test the principles practically. Whole-class teaching should reinforce the practical work to embed the concept that a moment of a force is the product of the magnitude of that force and the **perpendicular** distance between the turning point and the **line** of action of the force. Tutors must stress that it is **not** the actual distance between the **point** at which the force acts and the turning point that matters and, as above, this should be demonstrated practically. Learners should then be introduced to the 'principle of moments' – that clockwise moments and anti-clockwise moments are equal for co-planar systems in equilibrium ($\Sigma M = 0$). This too should be demonstrated practically and/or checked practically by learners in small groups. Total load = total reaction. A tutor-led discussion should be used to extend the concept of static equilibrium to the realisation that, for systems in equilibrium, not only does $\Sigma M = 0$, but ΣV and ΣH also equal zero. That $\Sigma V = 0$ implies that the algebraic sum of all vertical forces equals zero, and that total loads equal total reactions in all cases. Tutor to demonstrate the concept practically and then show the students how to construct free body diagrams of components in equilibrium Learners will need to practise drawing loading diagrams from provided information. Tutors should provide exemplars from which they can derive such diagrams, for both 2D and 3D objects. Split class into smaller groups and issue the required information for the construction of the diagrams. Tutor to circulate and correct as required	**Books:** Hulse, R., Cain, J., *Structural Mechanics*, Palgrave Macmillan, ISBN: 0333804570 Hulse, R., Cain, J., *Structural Mechanics (Worked Examples)*, Palgrave Macmillan, ISBN: 0230579817 **Practical equipment:** Beams, rules, hanging weights, pulleys, string, 2D objects to balance, 3D objects to balance **Software:** Goya Siemens PLM RISA Technology **Website:** www.istructe.org

Lesson 2: Construction of equilibrium diagrams **Suggested Teaching Time:** 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Determination of beam reactions (AC 1.3)	Whole-class discussion covering concepts learned in lesson 1 and leading to an understanding of how this concept, together with the principle of moments, can be used to determine the value of reactions for loaded beams. Learners will need to practise drawing loading diagrams from provided information. Tutors should provide exemplars from which they can derive such diagrams, for both point and uniformly distributed loads. Beam reactions for point loading on simply supported beams (with and without overhangs) and cantilevers. Whole-class teaching should be used to introduce the learners to how the principles learned earlier can be used to determine beam reactions for point loading only. The tutor should work through typical examples of such calculations and the learners should then work through other examples of such calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained. There are software packages that can do all this, and it would be useful for the learners to check their answers against those obtained from the software programmes, but this should not be considered as the preferred method of determining beam reactions at this stage. Beam reactions for uniformly distributed loads (UDLs) as above. The tutor should demonstrate how a UDL can be considered as a point load by considering the load as concentrated at the centre of the loading. For instance, a UDL of 30kN/m, extending over 4m, can be considered as a point load of 120kN acting 2m from each end. Beam reactions for combination of point loads and UDLs. As above, with the tutor working through typical examples, the learners attempting to solve similar problems and the tutor offering regular feedback until the learners can consistently solve such problems.	**Books:** Hulse, R., Cain, J., *Structural Mechanics*, Palgrave Macmillan, ISBN: 0333804570 Hulse, R., Cain, J., *Structural Mechanics (Worked Examples)*, Palgrave Macmillan, ISBN: 0230579817 **Practical equipment:** Proprietary rigs for testing shear force and bending moments **Software:** Goya Siemens PLM ANSYS **Website:** www.istructe.org http://www.eng.auburn.edu/~marghitu/MECH2110/C_5.pdf

Lesson 3: Calculate shear force and bending moment values. **Suggested Teaching Time:** 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Determination of shear force and bending moment values (AC 1.5 and 1.8) Determination of deflections at mid-span (AC 1.6)	Whole-class teaching should be used to define the terms 'shear force' (SF) and 'bending moment' (BM) and to show how each can be determined at various points. This should be developed into the conversion of the values into 'SF diagrams' and 'BM diagrams'. Small-group work should follow with each group being given different loading diagrams for a variety of loading conditions (point loads or UDLs or both, simply supported beams or cantilevers). The tutor should circulate around the groups, correcting any mistakes along the way. A whole-class, tutor-led discussion should follow, with the tutor leading the class towards noting the: • Coincidence of the bending moment maximum and shear force zero • Importance of the point of contraflexure, at which positive bending becomes negative bending (or vice-versa) • Use to which the SF zero, BM max and position of the point of contraflexure are put. Whole-class teaching should be used to demonstrate the importance of limiting the amount by which a beam deflects under load, and predicting the amount by which the beam will deflect under a given load, to see if this is in within acceptable limits. Tutor to cover maximum stress, maximum load, beam dimensions and radius of curvature equations. Tutors should use worked examples to illustrate the point and learners should have the chance to practise similar calculations, with tutors providing ongoing feedback. A sheet of the formulae used to determine deflection under a range of loadings should be made available. There is no requirement for learners to commit these formulae to memory.	**Books:** Hulse, R., Cain, J., *Structural Mechanics*, Palgrave Macmillan, ISBN: 0333804570 Hulse, R., Cain, J., *Structural Mechanics (Worked Examples)*, Palgrave Macmillan, ISBN: 0230579817 **Practical equipment:** Proprietary rigs for testing shear force and bending moments **Software:** Goya Siemens PLM ANSYS **Website:** www.istructe.org

Lesson 4: Determination of magnitude and type of forces in frameworks **Suggested Teaching Time:** 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Determination of magnitude and type of forces in frameworks (AC 1.4 and 1.5)	Bow's notation. The tutor should develop the system used to annotate frames and the learners should then have the opportunity to annotate a series of different frames. This should be checked by the tutor. Graphical method of solving frames. This can be done manually or electronically. Whichever method is used, the tutor must stress the importance of accuracy in the drawing of both angles and lines. A discussion should follow in which the learners learn to differentiate between struts and ties from the direction of the forces in the individual force polygons joint. Method of resolution. The tutor should demonstrate solving frames using horizontal and vertical static equilibrium at each joint. Learners should then practise on different frames with different loadings. • Loadings to include: vertical, horizontal, inclined, point, uniformly distributed, combination of point and uniformly distributed • Frames to include simply supported and cantilever Method of sections. The tutor should demonstrate the procedures to use. Once again, there are software applications that can be used to solve frames and these can be used to check the learners' answers.	Hulse, R., Cain, J., *Structural Mechanics*, Palgrave Macmillan, ISBN: 0333804570 Durka, Frank, Al Nageim, Hassan, Morgan, W., Williams, D., *Structural Mechanics*, 7th Edn, Prentice Hall, ISBN-10: 0132239647, ISBN-13: 978-0132239646; ASBN: 0132239647 **Practical equipment:** Proprietary equipment for testing frames **Software:** Goya RISA Technology **Website:** www.istructe.org

Lesson 5: Moments of area

Suggested Teaching Time: 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Design of simple beams (AC 1.7)	General theory of bending. Learners should be able to use the formula to design simple beams. Tutors may derive the formula from first principles but learners are not required to do so. A simple hand-out will suffice. What is important is that learners understand the importance of the variables M, I, f and y, and of using consistent units. First and second moments of area. Tutors must emphasise the importance of sectional shape in beam sizing. Learners must be aware of the various formulae required to determine the second moment of area (I) – also known as the 'moment of inertia' – practice calculations should include universal beam sections for steel (Rectangular, I simple and complex, T, circular section). Learners must be given the opportunity to determine moments of inertia by using the formulae and by extraction of the values from tables, once the section modulus (z) has been determined. The class should be divided into several small groups and each should be given similar data to allow them to determine the required size of a beam. Comparing the answers will show that there are several beam sizes that satisfy the requirements for a given loading condition. Tutor-led discussion about the effect of differences in breadth, depth and sectional shape should lead to an agreed conclusion concerning the most practical size of beam to be used, and why this is so.	**Books:** Hulse, R., Cain, J., *Structural Mechanics*, Palgrave Macmillan, ISBN: 0333804570 **Manuals** *Steel Designers' Manual* (SC1) *Code of Practice for Structural Use of Concrete* 2013 *BS 5268-2:2002 Structural Use of Timber: Part 2* **Software:** Goya Siemens PLM **Website:** www.istructe.org

Lesson 6: Understanding the application of static theory to structures **Suggested Teaching Time:** 2 hours

Learning Outcome 1: Understand the application of static theory to structures

Topic	Suggested Teaching	Suggested Resources
Learning Outcome 1: revision	Tutor-led discussion to summarise the lessons learned, to include: • Explaining the general conditions of static equilibrium • The construction of free body diagrams of components in equilibrium • Evaluating the forces required to keep a 2D and 3D body in equilibrium • How to use bow's notation to determine the forces in simply supported and cantilever pin jointed frameworks subjected to: vertical, horizontal, inclined, point, uniformly distributed, combination of point and uniformly distributed loads • Calculating using the method of sections the forces in selected members of a simply supported or cantilever framework • Determining the shear force and bending moment loading at various points on a simply supported and/or cantilever beam • Calculating the second moment of area for beam cross sections (rectangular, i simple and complex, t, circular) • Using bending theory to find solutions to problems relating to beams (maximum stress, maximum load, beam dimensions, radius of curvature)	

Lesson 7: Stress and strain

Suggested Teaching Time: 2 hours

Learning Outcome 2: Understand the effects of loading components under various loads and conditions

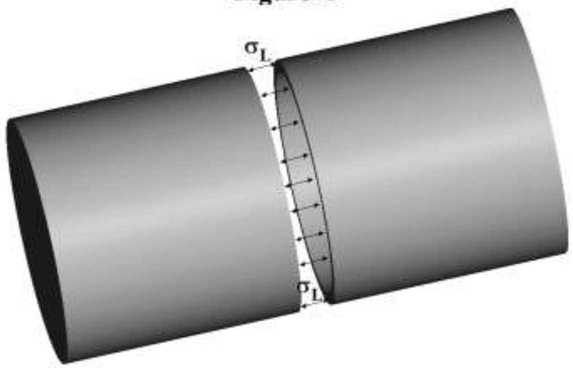
Topic	Suggested Teaching	Suggested Resources

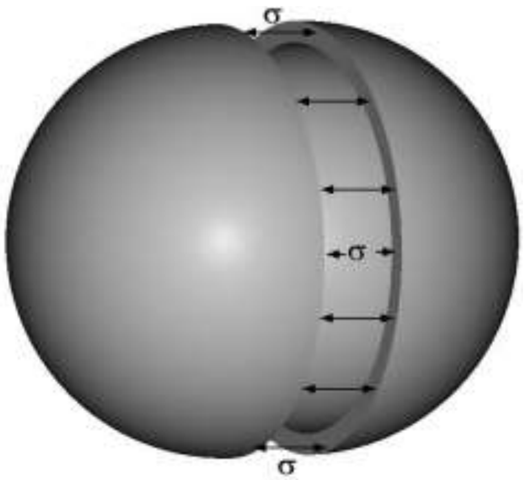
Basic theory revision
Calculate stress and strain in components under various conditions
(AC 2.1)
	Despite the prerequisite knowledge required for entering this course, take time at the beginning of the course to reinforce the basic equations.
	Whole-class teaching: involve the whole class in learner research and activity to cover the following principles and the meaning of the following terms:
	• Direct stress $\sigma = \frac{F}{A}$
	• Direct strain $\varepsilon = \frac{\Delta L}{L}$
	• Young's modulus {E}; $E = \frac{\sigma\left(stress\ \sigma=\frac{F}{A}\right)}{\varepsilon\left(Strain\ \varepsilon=\frac{\Delta L}{L}\right)}$
	• Shear modulus {G}; $G = \frac{E}{2(l+v)}$
	• Tensile or compressive stress may be caused by restricting thermal expansion
	• Thermal stress can be calculated as *σ = E ε = E α dt where*
	    ◦ *σ = stress due to temperature expansion (N/m^2, Pa)*
	    ◦ *E =Youngs Modulus (N/m^2)*
	    ◦ *ε = strain*
	    ◦ *α = temperature expansion coefficient (m/m^{o}C)*
	    ◦ *dt = temperature differance (^{o}C)*
	Demonstrate the solution of the different types of equations and then get the students to solve example questions, sample questions to include: different diameters, different materials, compound bars, and thermal strain. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required.
		Books:
		http://www-mdp.eng.cam.ac.uk/web/library/enginfo/cueddatabooks/materials.pdf
		Website:
		http://www.freestudy.co.uk/d209/t8.pdf
		Practical equipment:
		Laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of elastic theory
		Software:
		Computer-based finite element analysis software

Lesson 8: Pressure vessels

Suggested Teaching Time: 4 hours

Learning Outcome 2: Understand the effects of loading components under various loads and conditions

Topic	Suggested Teaching	Suggested Resources
Calculate stresses in thin walled cylindrical, pressure vessels (A.C. 2.2) calculate stresses in spherical pressure vessels (A.C. 2.2)	Whole-class discussion to cover how a cylinder is regarded as thin walled when the wall thickness t is less than 1/20 of the diameter D. Whole-class teaching to look at cylindrical pressure vessels. Tutor to show how when a cylinder of mean diameter D, wall thickness t and length L. has a pressure inside it larger than the pressure outside by an amount p, the cylinder will tend to split in the longitudinal direction (σ_L) and in the circumferential direction (σ_C). Stress in the circumferential direction is also called hoop stress or tangential stress. (Fig 1, Longitudinal forces; Fig 2, Circumferential forces.)  Figure 1  Figure 2	**Books:** http://www-mdp.eng.cam.ac.uk/web/library/enginfo/cueddatabooks/materials.pdf **Website:** http://www.engineersedge.com/pressure_vessels_menu.shtml http://www.freestudy.co.uk/d209/t8.pdf **Practical equipment:** Laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of theory **Software:** Computer-based finite element analysis software

	Pressure forces: F= pA = $p\frac{\pi D^2}{4}$ Stress F = σ_L multiplied by the area of the metal = $\sigma_L \pi D t$ $\sigma_L = \frac{pD}{4t}$ Pressure forces: F= pA=pLD Stress F = σ_C multiplied by the area of the metal = $\sigma_C 2Lt$ $\sigma_C = \frac{pD}{2t}$ Tutor to explain that for a given pressure the circumferential stress is twice the longitudinal stress. Whole-class teaching to look at spherical pressure vessels (Figure 3). Tutor to demonstrate that In a spherical vessel the stress produced in the material is equivalent to the longitudinal stress in the cylinder $\sigma_C = \frac{pD}{4t}$  Figure 3	

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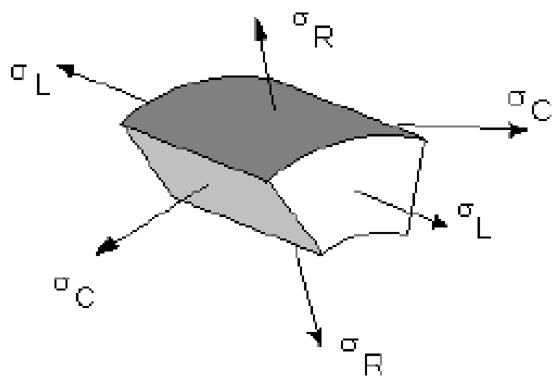
Topic	Suggested Teaching	Suggested Resources
Calculate stresses in thick-walled cylindrical, pressure vessels (AC 2.3) Calculating changes in volume of thin-walled vessels	Whole-class teaching to show that by using the equations used in the previous lesson, tutor to demonstrate that longitudinal strain is: • $\varepsilon_L = \frac{1}{E}(\sigma_L - v\sigma_c)$ show that by substituting σ_L & σ_C into this you get: • $\varepsilon_L = \frac{\Delta L}{L} = \frac{1}{E}\left(\frac{pD}{4t} - v\frac{pD}{2t}\right) = \frac{pD}{4tE}(1-2v)$ Then show that circumferential strain can be expressed as: • $\varepsilon_C = \frac{change\ in\ circumference}{Circumference} \therefore \varepsilon_C = \frac{\pi(D+\Delta D)-\pi D}{\pi D} = \frac{\Delta D}{D}$ Tutor then to show that the circumferential strain is therefore the same as the diametric strain Whole-class teaching to show that by using equations used in previous lesson, circumferential strain can be calculated using the formula $\varepsilon_C = \frac{1}{E}(\sigma_C - v\sigma_L)$ and by substituting σ_L & σ_C into this you get: $\varepsilon_C = \varepsilon_D = \frac{\Delta D}{D} = \frac{1}{E}\left(\frac{pD}{2t} - v\frac{pD}{4t}\right) = \frac{pD}{4tE}(2-v)$ Tutor to demonstrate how to calculate changes in area and volume for both cylinders and spheres and then get the students to solve example questions, sample questions to include: different diameters, different materials, compound bars, and thermal strain. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required.	**Books:** http://www-mdp.eng.cam.ac.uk/web/library/enginfo/cueddatabooks/materials.pdf **Website**: http://www.freestudy.co.uk/d209/t8.pdf http://www.engineersedge.com/pressure_vessels_menu.shtml **Practical equipment:** Laboratory equipment to evaluate stress and deflection in simple components and structures when subjected to complex loading to enable the learner to verify the predictions of theory **Software:** Computer-based finite element analysis software

Topic	Suggested Teaching	Suggested Resources
Lame's theory	**4.1 <u>LAME'S THEORY</u>** Consider a small section of the wall.  σ_L = Longitudinal stress σ_R = Radial stress σ_C = Circumferential stress Whole-class teaching, tutor to demonstrate Lame's theory, showing the formulae for the following: • $\varepsilon_L = \frac{1}{E}(\sigma_L - (\sigma_R + \sigma_C))$ Longitudinal • $\varepsilon_C = \frac{1}{E}(\sigma_C - (\sigma_L + \sigma_R))$ Circumferential • $\varepsilon_R = \frac{1}{E}(\sigma_R - (\sigma_C + \sigma_L))$ Radial Stress • $\sigma_R = a - \frac{b}{r^2}$ • $\sigma_C = a + \frac{b}{r^2}$	**Books:** Collins, Jack A., *Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention*, John Wiley and Sons, ISBN 0471558915, 9780471558910 **Website:** http://www.engineersedge.com/pressure_vessels_menu.shtml

Topic	Suggested Teaching	Suggested Resources
Lame's theory, continued	Tutor to demonstrate through a worked example of using Lame's theory e.g. A hydraulic cylinder is 100mm internal diameter and 140mm external diameter. It is pressurised internally to 100MPa gauge. Determine the radial and circumferential stress at the inner and outer surfaces. Take E = 205GPa and v = 0.25 **Solution:** The boundary conditions are: Inner surface r = 50 mm σR = - 100 MPa (compressive) Outer surface r = 70 mm σR = 0 MPa (compressive) Substituting into Lame's equation we have σR = - 100 x 10^6 = $a-\frac{b}{r^2}=a-\frac{b}{0.05^2}$ σR = 0 = $a-\frac{b}{r^2}=\ a-\frac{b}{0.07^2}$ Solving simultaneous equations b = 510 kN a = 104 MPa Now solve the circumferential stress. $\sigma_c=a-\frac{b}{r^2}$ Putting r = 0.05 σ_c = 308 MPa Putting r = 0.07 σ_c = 208 MPa Tutor should then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required	**Books:** Collins, Jack A., *Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention*, John Wiley and Sons, ISBN 0471558915, 9780471558910 **Website:** http://www.engineersedge.com/pressure_vessels_menu.shtml

Topic	Suggested Teaching	Suggested Resources
Factors affecting the thickness of the walls of pressure vessels (AC 2.3)	Whole-class teaching to explain the effect of different factors on the thickness of materials required for pressure vessel design. These factors to include: • Joint efficiency • Factor of safety • Type of fluids and gases Tutor should then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible include practical elements. Tutor to circulate and correct as required.	**Books:** Collins, Jack A., *Failure of Materials in Mechanical Design: Analysis, Prediction, Prevention*, John Wiley and Sons, ISBN 0471558915, 9780471558910 **Website:** http://www.engineersedge.com/pressure_vessels_menu.shtml

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Lesson 9: Strain energy **Suggested Teaching Time:** 4 hours

Learning Outcome 2: Understand the effects of loading components under various loads and conditions

Topic	Suggested Teaching	Suggested Resources
Strain energy Tensile strain energy Shear strain energy Torque strain energy	Whole-class discussion to introduce the concept of strain energy. *'When an elastic body is deformed, work is done. The energy used up is stored in the body as strain energy and it may be regained by allowing the body to relax. The best example of this is a clockwork device which stores strain energy and then gives it up'* Whole-class teaching on strain energy due to direct stress. Tutor to conduct a practical experiment to show that if a bar of length L and cross sectional area A has a tensile force applied, it stretches. Tutor to get the students to plot the graph of force v extension to demonstrate that it is usually a straight line. Tutor to explain how, when the force reaches a value of F and corresponding extension x, the work done (W) is the area under the graph. Tutor to demonstrate that work done = W = Fx/2. (The same as the average force x extension.) Tutor to demonstrate that since the work done is the energy used up, this is now stored in the material as strain energy hence giving the formula U = Fx/2. Using standard stress and strain formula, the tutor should demonstrate the following reasoning: • The stress in the bar is $\sigma = F/A$ hence $F = \sigma A$ • The strain in the bar is $\varepsilon = x/L$ hence $x = \varepsilon L$ For an elastic material up to the limit of proportionality, $\sigma / \varepsilon = E$ (The modulus of elasticity) hence $\varepsilon = \sigma / E$. Substituting we find : $U = \sigma A \varepsilon L/2 = \sigma^2 AL/2E$.	**Books:** Smallman, R. E., Bishop, Ray J., *Modern Physical Metallurgy and Materials Engineering: Science, Process, Applications*, Butterworth-Heinemann,, ISBN: 0750645644, 9780750645645 Gere, James, Goodno, Barry J., *Mechanics of Materials*, Cengage Learning, ISBN: 1285225783, 9781285225784 **Website:** http://www.me.mtu.edu/~mavable/MEEM4405/Energy_slides.pdf

	since the volume of the bar is AL we can write $U = \frac{\sigma^2}{2E} \times$ Tutor to demonstrate in a similar fashion how the formula for strain energy due to pure shear stress is: $U = \frac{\tau^2}{2E} \times v$ Tutors and classes should note that pure shear does not often occur in structures and the numerical values are very small compared to that due to other forms of loading so it is often (but not always) ignored. Tutor to demonstrate practically the relationship between torque T and angle of twist θ is normally a straight line and that the work done is the area under the torque-angle graph. For a given pair of values W = Tθ/2 The strain energy stored is equal to the work done hence U = Tθ/2 From the theory of torsion θ = TL/GJ where G is the modulus of rigidity and J is the polar second moment of area. $J = \pi R^4/2$ for a solid circle. Substitute θ = TL/GJ and we get $U = T^2L/2GJ$ also from torsion theory T = τJ/R where τ is maximum shear stress on the surface. Substituting for T we get the following: $U = (\tau J/R)^2/2GJ = \tau^2 JL/2GR^2$ Substitute $J = \pi R^4/2$ $U = \tau^2 \pi R^4 L/4GR^2 = \tau^2 \pi R^2 L/4G$ The volume of the bar is $AL = \pi R^2 L$ so it follows that: $U = (\tau^2/4G)$ x v. (τ is the maximum shear stress on the surface.)	

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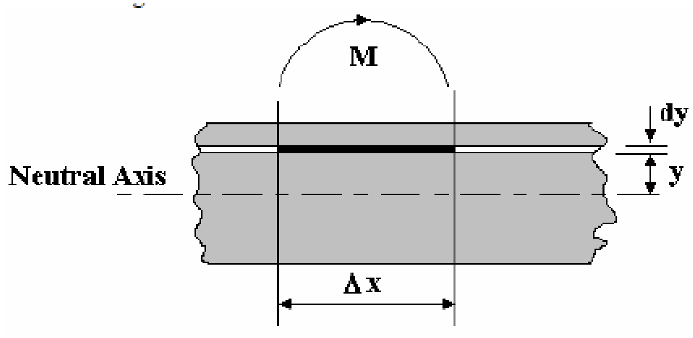
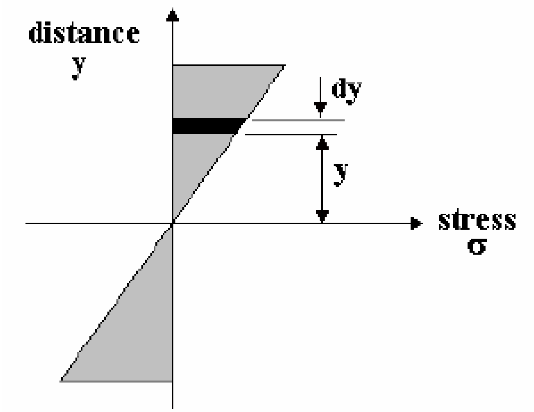
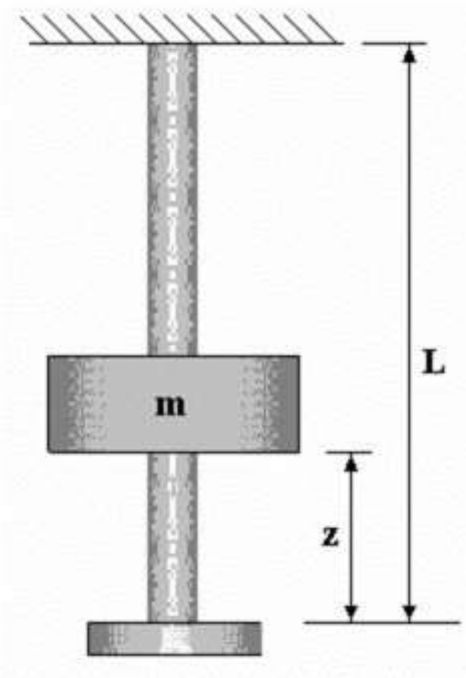
Topic	Suggested Teaching	Suggested Resources
Bending strain energy	**Strain energy due to bending** Whole-class discussion to cover how the strain energy produced by bending is usually large in comparison to the other forms. It should cover the fact that when a beam bends, layers on one side of the neutral axis are stretched and on the other side they are compressed. In both cases, this represents stored strain energy. Tutor to consider using an example such as the following:  Consider a point on a beam where the bending moment is M. Now consider an elementary layer within the material of length Δx and thickness dy at distance y from the neutral axis.  The cross-sectional area of the strip is dA. The bending stress is zero on the neutral axis and increases with distance y. This is tensile on one side and compressive on the other.	**Books:** Smallman, R. E., Bishop, Ray J., *Modern Physical Metallurgy and Materials Engineering: Science, Process, Applications*, Butterworth-Heinemann,, ISBN: 0750645644, 9780750645645 Gere, James, Goodno, Barry J., *Mechanics of Materials*, Cengage Learning, ISBN: 1285225783, 9781285225784 **Website:** http://www.me.mtu.edu/~mavable/MEEM4405/Energy_slides.pdf

	Each elementary layer has a direct stress (σ) on it and the strain energy stored has been shown to be U = (σ2/2E) x v The volume of the strip is Δx dA The strain energy in the strip is part of the total so du = (σ2/2E)Δx dA From bending theory we have σ = My/I where I is the second moment of area. Substituting for σ we get $du = \frac{(My/I)^2}{2E}\Delta x\ dA$ and in the limit as Δx → dx $du = \frac{(My/I)^2}{2E}\Delta x\ dA = \left(\frac{M^2}{2EI^2} dx\ y^2 dA\right)\ du = \left\{\frac{(My/I)^2}{2E}\right\}\Delta x\ dA$ The strain energy stored in an element of length dx is then $u = \frac{M^2}{2EI^2} dx \int y^2\ dA$ and by definition $I = \int y^2\ dA$ so this simplifies to $u = \frac{M^2}{2EI} dx$ In order to solve the strain energy stored in a finite length, we must integrate with respect to x. For a length of beam the total strain energy is $u = \frac{1}{2EI}\int M^2\ dx$ The problem however, is that M varies with x and M as a function of x has to be substituted. Tutor should then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required.	

Lesson 10: Impact loads

Suggested Teaching Time: 3 hours

Learning Outcome 2: Understand the effects of loading components under various loads and conditions

Topic	Suggested Teaching	Suggested Resources
Application of strain Energy to impact loads Simplified solution Exact Solution	Tutor to explain how when a load is suddenly applied to a structure (e.g. by dropping a weight on it), the stress and deflection resulting is larger than when a static load is applied. Whole-class teaching to demonstrate impact loading using a mass falling onto a collar at the end of a bar as shown. The bar has a length L and a cross sectional area A. The mass drops a distance z. At the moment the bar is stretched to its maximum:  • The force in the bar is F and the extension is x • The corresponding stress is $\sigma = F/A$ • The strain is $\varepsilon = x/L$ • The relationship between stress and strain is $E = \sigma / \varepsilon$ hence $x = \sigma L/E$ • The strain energy in the bar is $U = \sigma 2AL/2E$ • The potential energy given up by the falling mass is P.E. = $mg(z + x)$	**Books:** Smallman, R. E., Bishop, Ray J., *Modern Physical Metallurgy and Materials Engineering: Science, Process, Applications*, Butterworth-Heinemann,, ISBN: 0750645644, 9780750645645 Gere, James, Goodno, Barry J., *Mechanics of Materials*, Cengage Learning, ISBN: 1285225783, 9781285225784 **Website:** http://www.me.mtu.edu/~mavable/MEEM4405/Energy_slides.pdf

	Tutor to demonstrate that these problems can be solved in two ways. **Simplified solution** If the extension x is small compared to the distance z then we may say P.E. = mgz Equating the energy lost to the strain energy gained we have mgz = σ2AL/2E Hence $\sigma = \sqrt{\frac{2mgzE}{AL}}$ **Exact solution** Tutor to show that by equating P.E and strain energy we have mg{z + x) = σ2(AL/2E) And by substituting x = σL/E into the equation we get mg{z + (σL/E)} = σ2(AL/2E) ≈ mgz + mgσL/E= σ2(AL/2E) Tutor then to show by rearranging this into a quadratic equation σ2(AL/2E) - (mgL/E)σ - mgz =0 We can then solve the quadratic equation to find $\sigma = \frac{\frac{mgL}{E} \pm \sqrt{\left(\frac{mgL}{E}\right)^2 + \left(\frac{4ALmgz}{2E}\right)}}{\frac{2AL}{2E}} \rightarrow \sigma = \left(\frac{mg}{A}\right) \pm \sqrt{\left(\frac{mg}{A}\right)^2 + \left(\frac{2mgzE}{AL}\right)}$ Tutor should then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required

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Topic	Suggested Teaching	Suggested Resources
	Suddenly applied loads Whole-class teaching, tutor to explain how a suddenly applied load occurs when z = 0 and that this is not the same as a static load. Putting z = 0 yields the result $x = 2\,x_s$ They should then explain that it also follows that the instantaneous stress is double the static stress. Tutor-led discussion to discuss how this theory also applies to loads dropped on beams where the appropriate solution for the static deflection must be used. Tutor should then get the students to solve example questions. Tutor to assist individual students, correcting errors as required. Where possible, include practical elements. Tutor to circulate and correct as required.	

Lesson 11: Calculate the polar moment of inertia of shafts

Suggested Teaching Time: 4 hours

Learning Outcome 2: Understand the effects of loading components under various loads and conditions

Topic	Suggested Teaching	Suggested Resources
The polar moment of inertia of shafts Polar moment of Inertia	Whole-class teaching, tutor to demonstrate practically how when a shaft is subjected to a torque or twisting, a shearing stress is produced in the shaft. The shear stress varies from zero in the axis to a maximum at the outside surface of the shaft. They should illustrate how the shear stress in a solid circular shaft in a given position can be expressed as:τ = T r / J (1) where • τ = Shear stress (MPa, psi) • T = Twisting moment (Nmm, in lb) • r = Distance from centre to stressed surface in the given position (mm, in) • J = Polar moment of inertia of an area (mm^4, in4) Tutor to explain how the polar moment of inertia of an area is a measure of a beam's ability to resist torsion. The polar moment of inertia is defined with respect to an axis perpendicular to the area considered. It is analogous to the area moment of inertia, which characterises a beam's ability to resist bending - required to predict deflection and stress in a beam Note: 'polar moment of inertia of an area' is also called 'polar moment of inertia', 'second moment of area', 'area moment of inertia', 'polar moment of area' or 'second area moment'.	**Book:** Bansal, R. K., *Mechanical Engineering (O.T.)*, Firewall Media, ISBN: 8170081920, 9788170081920 **Website:** http://hyperphysics.phy-astr.gsu.edu/hbase/icyl.html

Topic	Suggested Teaching	Suggested Resources
	Tutor to demonstrate how the polar moment of inertia of a circular solid shaft can be expressed as $J = \frac{\pi R^4}{2}$ $= \frac{\pi \left(\frac{D}{2}\right)^4}{2}$ $= \pi \frac{D^4}{32}$ Where D = shaft outside diameter And also show that the polar moment of inertia of a circular hollow shaft can be expressed as $\pi \frac{D^4 - d^4}{32}$ Where d = shaft inside diameter	

Topic	Suggested Teaching	Suggested Resources
Mechanical power transmission by a shaft	Tutor-led discussion to bring together formulae used earlier and formulae for work and power to show that: Mechanical power is defined as work done per second. Work done is defined as force times distance moved. Hence $P = Fx/t$ where P is the Power, F is the force, x is distance moved and t is the time taken. Since distance moved/time taken is the velocity of the force we may write $\mathbf{P = F\,v}$ where v is the velocity. When a force rotates at radius R it travels one circumference in the time of one revolution. Hence the distance moved in one revolution is $x = 2\pi R$ If the speed is N rev/second then the time of one revolution is 1/N seconds. The mechanical power is hence $P = F\,2\pi R/(1/N) = 2\pi NFR$ Since FR is the torque produced by the force this reduces to $\mathbf{P = 2\pi NT}$ Since $2\pi N$ is the angular velocity ω radians/s it further reduces to $\mathbf{P = \omega T}$ **Example** A shaft is made from tube. The ratio of the inside diameter to the outside diameter is 0.6. The material must not experience a shear stress greater than 500 kPa. The shaft must transmit 1.5 MW of mechanical power at 1500 rev/min. Calculate the shaft diameters.	**Book:** Bansal, R. K., *Mechanical Engineering (O.T.)*, Firewall Media, ISBN: 8170081920, 9788170081920 Sarkar, B. K., *Strength of Materials*, Tata McGraw-Hill Education, ISBN: 0070494843, 9780070494848 **Website:** http://www.engineeringtoolbox.com/torsion-shafts-d_947.html

Topic	Suggested Teaching	Suggested Resources
	Solution The important quantities are P = 1.5 x 10^6 Watts, $\tau = 500 \text{ x } 10^3$ Pa, N = 1500 rev/min and d = 0.6D. $N = 1500\, Rev/min = 1500/60 = 25\, rev/s\; P = 2\pi NT$ Hence $T = \frac{P}{2\pi N} = \frac{1.5 \times 10^6}{2\pi \times 25} = 9549.3Nm$ $J = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi\{D^4 - (0.6D)^4\}}{32} = \frac{\pi\{D^4 - 0.36D^4\}}{32} = 0.08545D^4$ $\frac{T}{J} = \frac{\tau}{R} = \frac{2\tau}{D}\; Hence\; \frac{9549.3}{0.08545D^4} = \frac{2 \times 500 \text{ x } 10^3}{D}$ $= \frac{9549.3}{0.08545D^4 \times 2 \times 500 \text{ x } 10^3} - \frac{D^4}{D} = D^3$ $D^3 = 0.11175\; d = \sqrt[3]{0.11175} = 0.4816M = 481.6MM\; d = 0.6D = 289mm$	

Lesson 12: Kinematics motions **Suggested Teaching Time:** 1 hour

Learning Outcome 1: Understand the principles of kinematics

Topic	Suggested Teaching	Suggested Resources
Revision of Basic Concepts	Taking time at the beginning of this section to reinforce the basic equations. Whole-class teaching: involve the whole class ion learner research and activity to cover the following principles and terms: • The difference between scalar and vector quantities ○ Any physical quantity that requires a direction to be stated in order to define it completely is known as a vector quantity ○ A scalar quantity, such as time, is adequately defined when the magnitude is given in the appropriate units • Force and motion ○ Force, measured in newtons, is a vector quantity because its effect depends upon its magnitude and direction • How to determine the resultant of two coplanar vectors by using a vector triangle • How to calculate the resultant of two perpendicular vectors Show video. Involve the whole class in learner research activity to cover the following principles and the different kinematic motions: • Translation • Rotation • General motion • Relative motion Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required.	**Books:** Johnson, K., (2006) *Physics for You*, Nelson Thorne Jason, Z., (2009), *Force and Motion*, Johns Hopkins University Press Oxlade, C., (2005), *Forces and Motion*, Hodder Wayland Doherty, J. J. J., (2008), *Kinematics and Dynamics*, Bibliolife Wilson, C. E., (2003), *Kinematics and Dynamics of Machinery*, Pearson **Website**: www.metacafe.com/tags/Kinematics/page-3 http://www.physicsclassroom.com/Shockwave-Physics-Studios www.revisionworld.co.uk?node/7814 **Practical equipment:** Laboratory equipment for evaluating forces, velocity and acceleration

Lesson 13: Kinematics motions (continued) **Suggested Teaching Time:** 1 hour

Learning Outcome 1: Understand the principles of kinematics

Topic	Suggested Teaching	Suggested Resources
Revision of Basic Concepts (Continued)	Whole-class teaching: Tutor to involve the whole class in learner research and activity to cover the following principles and the meaning of the following terms: • Displacement: the change of position of a body in a particular direction and is a vector quantity • Speed: ratio of distance to time taken by a moving body and is a scalar quantity • Velocity: the rate of motion in a given direction and is a vector quantity • Acceleration: the rate of change of velocity is a scalar quantity Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required.	**Books:** As per lesson 12 **Website**: www.scienceaid.co.uk/physics/forces/motion.html http://www.physicsclassroom.com/Shockwave-Physics-Studios http://www.bbc.co.uk/learningzone/clips/topics/secondary.shtml#engineering http://www.YourOtherTeacher.com **Practical equipment:** Laboratory equipment for evaluating forces, displacement, velocity and acceleration

Lesson 14: Kinematics motions

Suggested Teaching Time: 1 hour

Learning Outcome 1: Understand the principles of kinematics

Topic	Suggested Teaching	Suggested Resources
Kinematic modelling of simple mechanisms (A.C. 1.1)	Whole-class teaching to explain kinematic modelling of simple mechanisms. Tutor to involve the whole class in learner research and activity to cover the following principles and the meaning of the following terms: • Reference frames: the movement of components of a mechanical system is analysed by attaching a reference frame to each part and determining how the reference frames move relative to each other. If the structural strength of the parts is sufficient then their deformation can be neglected and rigid transformations used to define this relative movement. • Degrees of freedom: the degrees of freedom (DOF) of a rigid body is defined as the number of independent movements it has e.g. a rigid body on a plane has 3 DOF. The bar can be translated along the x axis, translated along the y axis, and rotated about its centroid. • Rigid body links: two or more rigid bodies in space are collectively called a rigid body system. We can hinder the motion of these independent rigid bodies with kinematic constraints. Kinematic constraints are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system	**Books:** Johnson, K., *Physics for You*, Nelson Thorne Jason, Z., (2009), Force and Motion, Johns Hopkins University Press Oxlade, C., (2005), Forces and Motion, Hodder Wayland Doherty, J. J. J., (2008), *Kinematics and Dynamics*, Bibliolife Wilson, C. E., (2003), *Kinematics and Dynamics of Machinery*, Pearson **Website**: www.metacafe.com/tags/Kinematics/page-3 http://www.physicsclassroom.com/Shockwave-Physics-Studios http://kmoddl.library.cornell.edu/model.php?m=reuleaux

Lesson 15: Evaluation of velocities in kinematic mechanisms by graphical analysis **Suggested Teaching Time:** 4 hours

Learning Outcome 3: Understand the principles of kinematics

Topic	Suggested Teaching	Suggested Resources
Evaluate velocities in kinematic mechanisms by graphical analysis (A.C. 1.2)	**Velocity diagrams** This involves the construction of diagrams which need to be done accurately and to a suitable scale. Students should use: a drawing board, ruler, compass, protractor, and triangles or a suitable CAD package with which the students are familiar. Tutor-led learning: learner research and activity on the concepts of: absolute and relative velocity, and the motion of a slider-crank. Tutor should demonstrate the drawing of the different types of diagrams and then get the students to solve example questions using the graphical method, tutor to assist individual students, correcting errors as required Diagram types to include: velocity diagrams for the relative velocities of two unconnected bodies and the following types of mechanisms: Four-bar linkage; and slider-crank; Where possible include practical examples, tutor to circulate and correct as required	**Software:** Basic CAD programme **Practical equipment:** Drawing board, ruler, compass, protractor, and triangles Examples of resolute and prismatic joins, kinematic chains, planar kinematic mechanisms, and spatial kinematic mechanisms, including: • Four-bar linkage • Crank and rocker • Drag link • Slider-crank • Scotch yoke • Quick-return **Website:** http://www.freestudy.co.uk/dynamics/velaccdiag.pdf https://www.youtube.com/watch?v=lzaZ38Rn9Tk

Lesson 16: The application of the conservation of momentum to collisions	**Suggested Teaching Time:** 4 hours

Learning Outcome 4: Understand dynamic principles of systems under the action of forces

Topic	Suggested Teaching	Suggested Resources
Explain the application of the conservation of momentum to collisions	Tutor-led discussion, covering the Momentum Conservation Principle. Tutor to lead the discussion to cover the fact that In a collision between two objects, each object is interacting with the other object. This interaction involves a force acting between the objects for some amount of time. This force and time constitutes an impulse and the impulse changes the momentum of each object. Such a collision is governed by Newton's laws of motion; and as such, the laws of motion can be applied to the analysis of the collision (or explosion) situation. Tutor to discuss the following: • In a collision between object 1 and object 2, the force exerted on object 1 (F_1) is equal in magnitude and opposite in direction to the force exerted on object 2 (F_2). In equation form: $F_1 = -F_2$ • In a collision between object 1 and object 2, the time during which the force acts upon object 1 (t_1) is equal to the time during which the force acts upon object 2 ($_{t2}$). In equation form: $t_1 = t_2$ Tutor to cover the concept that when we have two equations which relate the forces exerted upon individual objects involved in a collision and the times over which these forces occur. It is accepted mathematical logic to state the following: • If A = - B and C = D then A x C = - B x D The above logic is fundamental to mathematics and can be used here to analyse a collision. • If $F_1 = -F_2$ and $t_1 = t_2$ then $F_1 \times t_1 = -F_2 \times t_2$	**Book:** Giordano, Nicholas, *College Physics: Reasoning and Relationships*, Cengage Learning, ISBN 1285225341, 9781285225340 **Website:** http://www.physicsclassroom.com/class/momentum/u4l2b.cfm http://www.freestudy.co.uk/dynamics/impulse%20and%20momentum.pdf

Topic	Suggested Teaching	Suggested Resources
	The above equation states that in a collision between object 1 and object 2, the impulse experienced by object 1 (F_1 x t_1) is equal in magnitude and opposite in direction to the impulse experienced by object 2 (F_2 x t_2). Objects encountering impulses in collisions will experience a momentum change. The momentum change is equal to the impulse. Thus, if the impulse encountered by object 1 is equal in magnitude and opposite in direction to the impulse experienced by object 2, then the same can be said of the two objects' momentum changes. The momentum change experienced by object 1 (m1 x Delta v1) is equal in magnitude and opposite in direction to the momentum change experienced by object 2 (m_2 x Delta v_2). This statement could be written in equation form as $m_1 \times \Delta v_1 = - m_2 \times \Delta v_2$	

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Topic	Suggested Teaching	Suggested Resources
Explain the application of the conservation of momentum to collisions (continued)	This equation claims that in a collision, one object gains momentum and the other object loses momentum. The amount of momentum gained by one object is equal to the amount of momentum lost by the other object. The total amount of momentum possessed by the two objects does not change. Momentum is simply transferred from one object to the other object. The sum of the momentum of object 1 and the momentum of object 2 before the collision is equal to the sum of the momentum of object 1 and the momentum of object 2 after the collision. The following mathematical equation is often used to express the above principle. $m_1 \times v_1 + m_2 \times v_2 = m1 \times v_{1'} + m_2 \times v_{2'}$ (Note that a ‘ symbol is used to indicate after the collision.) **Direction matters** Momentum is a vector quantity; The direction of the momentum vector is always in the same direction as the velocity vector. Because momentum is a vector, the addition of two momentum vectors is conducted in the same manner by which any two vectors are added. For situations in which the two vectors are in opposite directions, one vector is considered negative and the other positive.	**Book:** Giordano, Nicholas, *College Physics: Reasoning and Relationships*, Cengage Learning, ISBN 1285225341, 9781285225340 **Website** http://www.physicsclassroom.com/class/momentum/u4l2b.cfm http://www.freestudy.co.uk/dynamics/impulse%20and%20momentum.pdf

Topic	Suggested Teaching	Suggested Resources
	Two-dimensional collision problems A two-dimensional collision is a collision in which the two objects are not originally moving along the same line of motion. They could be initially moving at right angles to one another or at least at some angle (other than 0 degrees and 180 degrees) relative to one another. In such cases, vector principles must be combined with momentum conservation principles in order to analyse the collision. The underlying principle of such collisions is that both the x and the y momentum are conserved in the collision. The analysis involves determining pre-collision momentum for both the x- and the y-directions. If inelastic, then the total amount of system momentum before the collision (and after) can be determined by using the Pythagorean theorem. Since the two colliding objects travel together in the same direction after the collision, the total momentum is simply the total mass of the objects multiplied by their velocity. Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required.	

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Lesson 17: Solving problems of dynamics of simple systems **Suggested Teaching Time:** 2 hours

Learning Outcome 4: Understand dynamic principles of systems under the action of forces

Topic	Suggested Teaching	Suggested Resources
	Whole-class discussion to cover Newton's laws of motion Discussion to cover the concepts of: • Momentum, mass x velocity = m u kg m/s • Impulse Force x Time = Ft Tutor to relate these concepts top everyday equipment such as vehicles, and material-handling equipment. Tutor-led discussion to cover the concept of Pile Drivers - devices which drop large masses onto a pile in order to drive the pile into the ground. The pile has no initial momentum and the motion given to it is quickly decelerated by the frictional resistance as it moves into the ground. The velocity of the driver is obtained by gravitational acceleration and conversion of potential energy into kinetic energy. **Worked Example** A pile driver has a mass of 100 kg and falls 3m onto the pile which has a mass of 200 kg. The coefficient of restitution is 0.7. Calculate the velocity of the pile and the driver immediately after impact.	**Book:** Giordano, Nicholas, *College Physics: Reasoning and Relationships*, Cengage Learning, ISBN 1285225341, 9781285225340 **Website:** http://physics.tutorvista.com/momentum.html

Topic	Suggested Teaching	Suggested Resources
	Solution Initial potential energy of driver = mgz Kinetic energy at impact = $\frac{mu_1^2}{2}$ Equating $mgz = \frac{mu_1^2}{2}$ $u_1 = (2gz)^{1/2} = (2 \times 9.81 \times 3)^{1/2} = -7.672\ m/s$ Since this is down, by normal convention it is negative. $u_1 = -7.672\ m/s$ Initial velocity of the pile $u_2 = 0$ Initial momentum = -100 x 7.672 = -767.2 kg m/s Final momentum = $m_1 v_1 + m_2 v_2$ $100v_1 + 200v_2$ = -767.2 kg m/s $\Rightarrow$ $\mathrm{v1} = 1.023\ \mathrm{m/s}$ (upwards) Initial relative velocity = $u_1 - u_2$ = -7.672 m/s (coming together). Final relative velocity = $v_1 - v_2$ = -0.7(-7.672) = 5.37 m/s parting. $v_1 - v_2$ = 5.37 m/s s $\Rightarrow$ $\mathrm{v_2} = 4.35\ \mathrm{m/s}$ (downwards) Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required.	

Lesson 18: Solving problems of dynamics of simple systems **Suggested Teaching Time:** 8 hours

Learning Outcome 4: Understand dynamic principles of systems under the action of forces

Topic	Suggested Teaching	Suggested Resources
Evaluate the moment of inertia of a body about an axis of rotation	Whole-class discussion to ensure students are familiar with the following: • The laws relating angular displacement, velocity and acceleration. • The laws relating angular and linear motion. • The forms of mechanical energy. Whole-class teaching, Tutor to cover: **Angle θ** Angle may be measured in revolutions, degrees, or radians. In engineering we normally use radians. The links between them are 1 revolution = 360^{O} = 2π radian **Angular Velocity ω** Angular velocity is the rate of change of angle with time and may be expressed in calculus terms as the differential coefficient $\omega = \frac{d\theta}{dt}$ **Angular Acceleration α** Angular acceleration is the rate of change of angular velocity with time and in calculus terms may be expressed by the differential coefficient $\alpha = \frac{d\omega}{dt}$ or the second differential coefficient $\alpha = \frac{d^2\theta}{dt^2}$	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

Topic	Suggested Teaching	Suggested Resources
Evaluate the moment of inertia of a body about an axis of rotation (continued) Radius of Gyration *k*	**Link Between Linear and Angular Quantities**. Any angular quantity multiplied by the radius of the rotation is converted into the equivalent linear quantity as measured long the circular path. Hence Angle is converted into the length of an arc by $x = \theta R$ Angular velocity is converted into tangential velocity by $v = \omega R$ Angular acceleration is converted into tangential acceleration by $a = \alpha R$ **Torque** When we rotate a wheel, we must apply torque to overcome the inertia and speed it up or slow it down. You should know that torque is a moment of force. A force applied perpendicular to the axle of a wheel will not make it rotate, whereas a force applied at a radius will (figure 1B). The torque is F r (N m). Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required. Whole-class discussion to cover the principle that the moment of inertia is that property of a body which makes it reluctant to speed up or slow down in a rotational manner. Tutor to explain that clearly it is linked with mass (inertia) and in fact 'moment of inertia' is also known as the second moment of mass. It is not only the mass that governs this reluctance but also the location of the mass. You should appreciate that a wheel with all the mass near the axle is easier to speed up than one with an equal mass spread over a larger diameter.	

Topic	Suggested Teaching	Suggested Resources
	Introduce a case where all the mass is rotating at one radius. This might be a small ball or a rim (like a bicycle wheel) with mass δm at radius r. and show how the angular velocity is ω rad/s. go onto explain that: If we multiply the mass by the radius we get the first moment of mass r m If we multiply by the radius again we get the second moment of mass r^2 m Explain that this second moment is commonly called the moment of inertia and has a symbol I. Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements. Tutor to circulate and correct as required Tutor to discuss how unfortunately most rotating bodies do not have the mass concentrated at one radius and the moment of inertia is not calculated as easily as this. All rotating machinery such as pumps, engines and turbines have a moment of inertia. The radius of gyration is the radius at which we consider the mass to rotate such that the moment of inertia is given by $I = Mk^2$ where M is the total mass and k is the radius of gyration. Explain how the main problem with this approach is that the radius of gyration must be known and often this is deduced from tests on the machine	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

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Topic	Suggested Teaching	Suggested Resources
Evaluate the moment of inertia of a body about an axis of rotation (continued)	**Plain disc** Whole-class teaching; tutor to explain principles by going back to basics with a plain disc. Tutor to demonstrate the following. Consider a plain disc and suppose it to be made up from many concentric rings or cylinders. Each cylinder is so thin that it may be considered as being at one radius r and the radial thickness is a tiny part of the radius δr. These are called elementary rings or cylinders. If the mass of one ring is a small part of the total we denote it δm. The moment of inertia is a small part of the total and we denote it δI and this is given by δI = r2 δm. The total moment of inertia is the sum of all the separate small parts so we can write $\mathrm{I} = \sum \delta\mathrm{I} = \sum \mathrm{r}^2\, \delta\mathrm{m}$ Tutor to explain how if the disc is b metres deep, we can establish the formula for the mass of one ring. By using the following ideology. The elementary thin cylinder if cut and unrolled would form a flat sheet with • A length = circumference = $2\pi r$ and • A depth = b. • The thickness = δr The volume would therefore be calculated with the formula • Volume = length x depth x thickness = $2\pi r\ b\ \delta r$ From this we can calculate the mass $\delta\mathrm{m}$ by multiplying by the density of the material ρ. • $Mass = \delta m = \rho\ b\ 2\pi r\ \delta r$ If the mass is multiplied by the radius twice we get the moment of inertia $\delta\mathrm{I}$. • $\delta\, I = \rho\ b\ 2\pi r\ r2\ \delta r = \rho\ b\ 2\pi r3\ \delta r$	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

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Topic	Suggested Teaching	Suggested Resources
Evaluate the moment of inertia of a body about an axis of rotation (continued)	Tutor to demonstrate that as the radial thickness δr gets thinner and tends to zero, the equation becomes precise and we may replace the finite dimensions δ with the differential d. $d\,I \;=\; \rho\, b\, 2\pi r^3 dr$ The total moment of inertia is found by integration which is a way of summing all the rings that make up the disc. $I = \int_0^R 2\pi b\rho r^3 dr$ and taking the constants outside the integral the sign we have $I = 2\pi b\rho \int_0^R r^3 dr$ Completing the integration and substituting the limits of r = 0 (the middle) and r = R (the outer radius) we get the following. $I = 2\pi b\rho \int_0^R r^3 dr = \; 2\pi b\rho \frac{[r^4]_0^R}{4} = \; 2\pi b\rho \frac{[R^4 - 0^4]}{4}$ $I = \pi b\rho \frac{R^4}{2}$	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

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Topic	Suggested Teaching	Suggested Resources
	Now consider the volume and mass of the disc. The volume of the plain disc is the area of a circle radius R times the depth b. Volume = $\pi R^2 b$ Mass = volume x density = $\rho\pi R^2 b$ Examine the formula for I again. $I = \pi b\rho\frac{R^4}{2} = \pi b\rho R^2\frac{R^2}{2} = Mass \times \frac{R^2}{2}\ I = \frac{MR^2}{2}$ For a plain disc the moment of inertia is MR2/2 If we compare this to $\mathrm{I} = \mathrm{M\,k}^2$ we deduce that the radius of gyration for a plain disc is: $k = \sqrt{\frac{R^2}{2}} = \frac{R}{\sqrt{2}} = 0.707R$ The effective radius at which the mass rotates is clearly not at the midpoint between the middle and the outside but nearer the edge.	

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Lesson 19: Solving problems of dynamics of simple systems **Suggested Teaching Time:** 2 hours

Learning Outcome 4: Understand dynamic principles of systems under the action of forces

Topic	Suggested Teaching	Suggested Resources
Angular kinetic energy	Whole-class discussion to cover how linear kinetic energy is given by the formula K.E. = $\frac{mv^2}{2}$ Develop from this the fact that It requires energy to accelerate a wheel up to speed so rotating bodies also possess kinetic energy and the formula is K.E. =$\frac{m\omega^2}{2}$ Whole-class teaching: consider a disc and an elementary ring. If a point rotates about a centre with angular velocity ω rad/s, at radius r, the velocity of the point along the circle is v m/s and it is related to ω by $v = \omega r$. The mass of the ring is δm. The kinetic energy of the ring is $\frac{\delta m\, v^2}{2}$ If we convert v into ω the kinetic energy becomes $\frac{\delta m\, \omega^2}{2}$. Tutor show how the total kinetic energy for the disc is found by integration $$\text{K. E. of disc} = \int_0^R \frac{\delta m \omega^2 r^2}{2} = \frac{\omega^2}{2}\int_0^R \delta m r^2$$ By definition the term $\int_0^R \delta m r^2$ is the moment of Inertia so we may write K.E. $= \frac{I\omega^2}{2}$ Whole-class teaching, tutor to show how we can derive the formula for Friction Torque $T = \frac{FD}{2}$ Split class into smaller groups and issue a series of questions covering driving and frictional torque, angular momentum, rotational energy. Where possible include practical elements. Tutor to circulate and correct as required.	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

Lesson 20: Solving problems of dynamics of simple systems

Suggested Teaching Time: 6 hours

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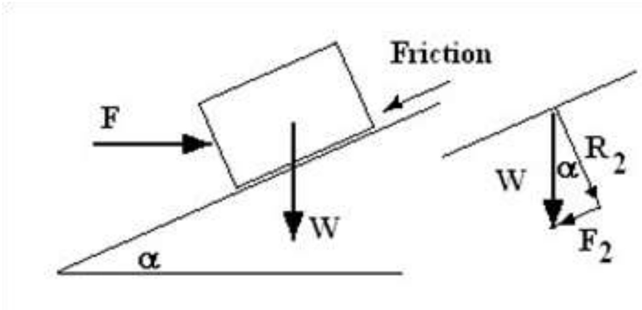
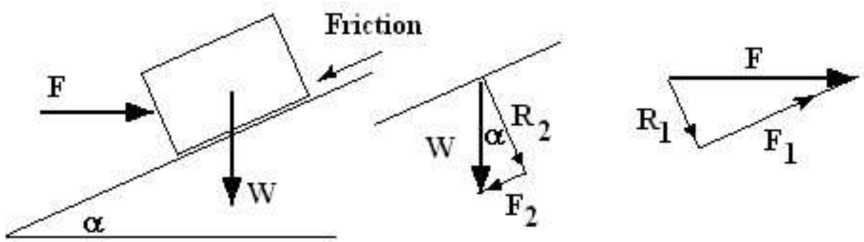
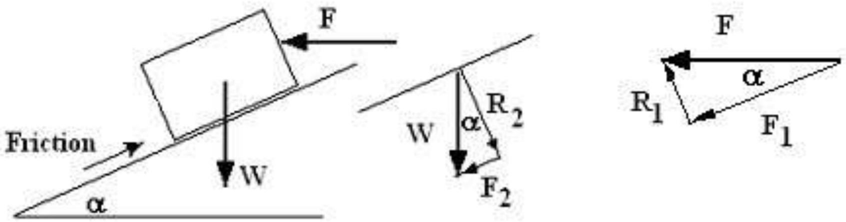
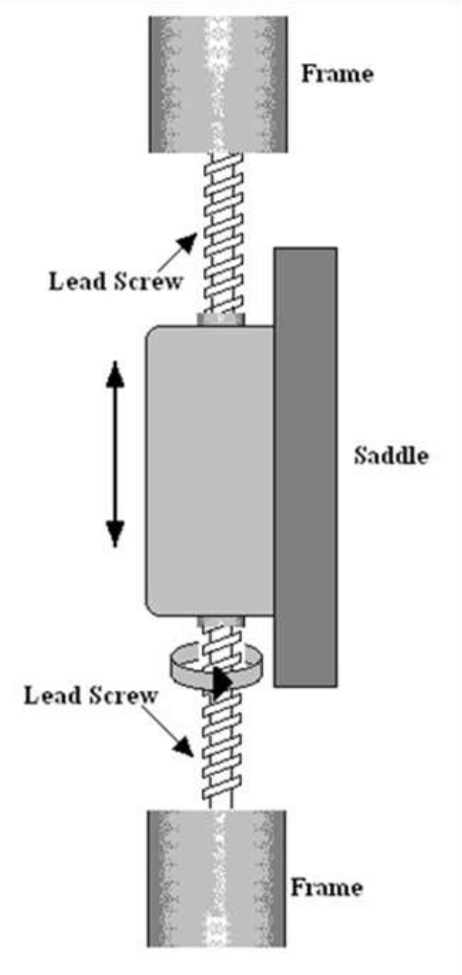
	Whole-class discussion to cover the principles of friction and to ensure students understand that the coefficient of friction is defined as μ = F/R where F is the force parallel to the surface and R is the force normal to the surface.  Tutor to demonstrate the following Consider a block on an inclined plane at angle ϕ to the horizontal. The weight acts vertically downwards. This must be resolved into two components parallel and perpendicular to the plane. Resolving R = W cos α and F_1 = W sin α If no other force is involved then the block will slide down the plane if F_1 is greater than the friction force. In this case $F1 > \mu R \; or \; F1 > \mu \, W \, cos \, \alpha$ The block will just slide when $F_1 = \mu W cos \, \alpha$ so it follows that $\mu = W \frac{sin \, \alpha}{W cos \, \alpha} = \tan \alpha$ and this is a way of finding μ and is called the friction angle	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

	 **Upwards** Tutor to demonstrate the following. Consider the case of a block sliding under the action of a horizontal force such that the block slides up the plane. We must resolve the weight and the force parallel and perpendicular to the plane as shown. The total force acting parallel to the plane is F1 – F2 and the total reaction is R = R1 + R2 The block will just slide up the plane if F1 – F2 = μ (R1 + R2)  **Downwards** In this case the force acts to make the body slide down the plane. The total force acting parallel to the plane is F1 + F2 and the total reaction is R = R2 - R1 The block will just slide up the plane if F1 + F2 = μ (R2 - R1)	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

Topic		Suggested Resources
	Application to screw thread The motion of two mating threads is the same as the previous problems. The vertical load is the thrust acting axially on the nut (e.g. the load on a screw jack). The angle of the plane is given by: $tan\ \alpha\ =\ pitch/ circumference\ =\ p/\pi D$ **Lead screw** Lead screw usually has a square thread. They are used to convert rotational motion into linear motion. The thread is rotated and the saddle moves in guides. This is used on many machines from lathes to linear electric actuators. The diagram shows a typical arrangement. The saddle carries a load and is moved up or down by rotation of the lead screw.  	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf

	Worked example A lead screw has square threads with a pitch of 3 mm and a mean diameter of 12 mm. The coefficient of friction is 0.2. Calculate the torque needed to turn it when the load is 4 kn. **Solution** The pitch is 3 mm and the circumference is 12mm so the angle of the plane is $\alpha = \tan^{-1}(3/12\pi) = 4.55^{\circ}$ The friction angle is $\beta = \tan^{-1} 0.2 = 11.31°$ The axial force is the force equivalent to the weight W. The torque T is the product of the force F and radius at which it acts which is the radius of the thread (6 mm). $F = W \tan(\beta + \alpha) = 4000 \tan(15.86°) = 1136\, N$ $T = F \times radius = 1136 \times 0.006 = 6.8\, Nm$	**Book:** *Engineering Physics*, Krishna Prakashan Media, ISBN: 8187224193, 9788187224198 **Website:** http://people.rit.edu/vwlsps/IntermediateMechanics2/Chapter8A.pdf