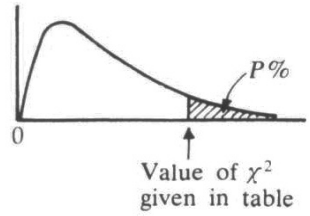
City & Guilds

# 9210-100
# Reference booklet

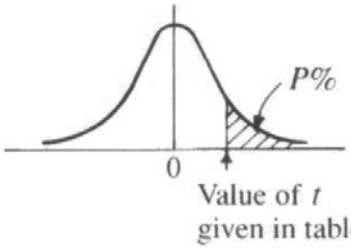
## Sample

## PERCENTAGE POINTS OF THE $\chi^2$-DISTRIBUTION




| Degrees of Freedom $\nu$ \ P | 99.5 | 99 | 97.5 | 95 | 90 | 10 | 5 | 2.5 | 1 | 0.5 | 0.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 | 10.83 |
| 2 | 0.01 | 0.02 | 0.05 | 0.10 | 0.21 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 | 13.82 |
| 3 | 0.07 | 0.11 | 0.22 | 0.35 | 0.58 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 | 16.27 |
| 4 | 0.21 | 0.30 | 0.48 | 0.71 | 1.06 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 | 18.47 |
| 5 | 0.41 | 0.55 | 0.83 | 1.15 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 | 20.52 |
| 6 | 0.68 | 0.87 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 | 22.46 |
| 7 | 0.99 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 | 24.32 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 | 26.12 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 | 27.88 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 | 29.59 |

## PERCENTAGE POINTS OF THE *t*-DISTRIBUTION




| ν \ P (2P) | 10 (20) | 5 (10) | 2.5 (5) | 1 (2) | 0.5 (1) | 0.1 (0.2) | 0.05 (0.1) |
|---|---|---|---|---|---|---|---|
| 1 | 3.08 | 6.31 | 12.7 | 31.8 | 63.7 | 318.3 | 636.6 |
| 2 | 1.89 | 2.92 | 4.30 | 6.96 | 9.92 | 22.3 | 31.60 |
| 3 | 1.64 | 2.35 | 3.18 | 4.54 | 5.84 | 10.2 | 12.92 |
| 4 | 1.53 | 2.13 | 2.78 | 3.75 | 4.60 | 7.17 | 8.61 |
| 5 | 1.48 | 2.02 | 2.57 | 3.36 | 4.03 | 5.89 | 6.87 |
| 6 | 1.44 | 1.94 | 2.45 | 3.14 | 3.71 | 5.21 | 5.96 |
| 7 | 1.41 | 1.89 | 2.36 | 3.00 | 3.50 | 4.79 | 5.41 |
| 8 | 1.40 | 1.86 | 2.31 | 2.90 | 3.36 | 4.50 | 5.04 |
| 9 | 1.38 | 1.83 | 2.26 | 2.82 | 3.25 | 4.30 | 4.78 |
| 10 | 1.37 | 1.81 | 2.23 | 2.76 | 3.17 | 4.14 | 4.59 |
| 11 | 1.36 | 1.80 | 2.20 | 2.72 | 3.11 | 4.03 | 4.44 |
| 12 | 1.36 | 1.78 | 2.18 | 2.68 | 3.05 | 3.93 | 4.32 |
| 13 | 1.35 | 1.77 | 2.16 | 2.65 | 3.01 | 3.85 | 4.22 |
| 14 | 1.35 | 1.76 | 2.14 | 2.62 | 2.98 | 3.79 | 4.14 |
| 15 | 1.34 | 1.75 | 2.13 | 2.60 | 2.95 | 3.73 | 4.07 |
| 16 | 1.34 | 1.75 | 2.12 | 2.58 | 2.92 | 3.69 | 4.01 |
| 17 | 1.33 | 1.74 | 2.11 | 2.57 | 2.90 | 3.65 | 3.96 |
| 18 | 1.33 | 1.73 | 2.10 | 2.55 | 2.88 | 3.61 | 3.92 |
| 19 | 1.33 | 1.73 | 2.09 | 2.54 | 2.86 | 3.58 | 3.88 |
| 20 | 1.33 | 1.72 | 2.09 | 2.53 | 2.85 | 3.55 | 3.85 |

Degrees of Freedom



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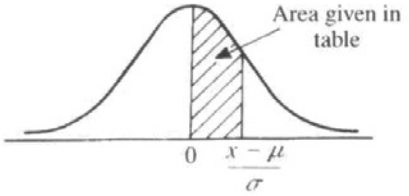
## THE NORMAL PROBABILITY INTEGRAL




| $\frac{x-\mu}{\sigma}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0000 | 0040 | 0080 | 0120 | 0160 | 0199 | 0239 | 0279 | 0319 | 0359 |
| .1 | 0398 | 0438 | 0478 | 0517 | 0557 | 0596 | 0636 | 0675 | 0714 | 0753 |
| .2 | 0793 | 0832 | 0871 | 0909 | 0948 | 0987 | 1026 | 1064 | 1103 | 1141 |
| .3 | 1179 | 1217 | 1255 | 1293 | 1331 | 1368 | 1406 | 1443 | 1480 | 1517 |
| .4 | 1555 | 1591 | 1628 | 1664 | 1700 | 1736 | 1772 | 1808 | 1844 | 1879 |
| .5 | 1915 | 1950 | 1985 | 2019 | 2054 | 2088 | 2123 | 2157 | 2190 | 2224 |
| .6 | 2257 | 2291 | 2324 | 2357 | 2389 | 2422 | 2454 | 2486 | 2517 | 2549 |
| .7 | 2580 | 2611 | 2642 | 2673 | 2703 | 2734 | 2764 | 2794 | 2822 | 2852 |
| .8 | 2881 | 2910 | 2939 | 2967 | 2995 | 3023 | 3051 | 3078 | 3106 | 3133 |
| .9 | 3159 | 3186 | 3212 | 3238 | 3264 | 3289 | 3315 | 3340 | 3365 | 3389 |
| 1.0 | 3413 | 3438 | 3461 | 3485 | 3508 | 3531 | 3554 | 3577 | 3599 | 3621 |
| 1.1 | 3643 | 3665 | 3686 | 3708 | 3729 | 3749 | 3770 | 3790 | 3810 | 3830 |
| 1.2 | 3849 | 3869 | 3888 | 3907 | 3925 | 3944 | 3962 | 3980 | 3997 | 4015 |
| 1.3 | 4032 | 4049 | 4066 | 4082 | 4099 | 4115 | 4131 | 4147 | 4162 | 4177 |
| 1.4 | 4192 | 4207 | 4222 | 4236 | 4251 | 4265 | 4279 | 4292 | 4306 | 4319 |
| 1.5 | 4332 | 4345 | 4357 | 4370 | 4382 | 4394 | 4406 | 4418 | 4429 | 4441 |
| 1.6 | 4452 | 4463 | 4474 | 4484 | 4495 | 4505 | 4515 | 4525 | 4535 | 4545 |
| 1.7 | 4554 | 4564 | 4573 | 4582 | 4591 | 4599 | 4608 | 4616 | 4625 | 4633 |
| 1.8 | 4641 | 4649 | 4656 | 4664 | 4671 | 4678 | 4686 | 4693 | 4699 | 4706 |
| 1.9 | 4713 | 4719 | 4726 | 4732 | 4738 | 4744 | 4750 | 4756 | 4761 | 4767 |
| 2.0 | 4772 | 4778 | 4783 | 4788 | 4793 | 4798 | 4803 | 4808 | 4812 | 4817 |
| 2.1 | 4821 | 4826 | 4830 | 4834 | 4838 | 4842 | 4846 | 4850 | 4854 | 4857 |
| 2.2 | 4861 | 4864 | 4868 | 4871 | 4875 | 4878 | 4881 | 4884 | 4887 | 4890 |
| 2.3 | 4893 | 4896 | 4898 | 4901 | 4904 | 4906 | 4909 | 4911 | 4913 | 4916 |
| 2.4 | 4918 | 4920 | 4922 | 4925 | 4927 | 4929 | 4931 | 4932 | 4934 | 4936 |

## Tables of Transforms

**Laplace and z Transforms**

| Definition | $f(t)$ from $t = 0-$ | $F(s) = L[f(t)] = \int_{0-}^{\infty} f(t) \cdot \exp(-st) \cdot dt$ | $F(z) = L[f^*(t)]_{z=\exp(Ts)}$ |
|---|---|---|---|
| Sum | $af_1(t) + bf_2(t)$ | $aF_1(s) + bF_2(s)$ | $aF_1(z) + bF_2(z)$ |
| First derivative | $(d/dt)f(t)$ | $sF(s) - f(0-)$ | |
| $n$th derivative | $(d^n/dt^n)f(t)$ | $s^nF(s) - s^{n-1}f(0-) - s^{n-2}f^{(1)}(0-) \ldots . - f^{(n-1)}(0-)$ | |
| Definite integral | $\int_{0-}^{t} f(\tau) \cdot d\tau$ | $\frac{1}{s}F(s)$ | |
| Shift in $t$ | $f(t - kT) \cdot H(t - kT)$ | $\exp(-skT)F(s)$ | $z^{-k}F(z)$ |
| Exponential multiplier | $\exp(-\alpha t) \cdot f(t)$ | $F(s + \alpha)$ | $F[z \exp(\alpha T)]$ |
| Periodic function (period $T$) | $f(t)$ | $\frac{1}{1 - \exp(-sT)} \int_{0-}^{T} \exp(-st)f(t) \cdot dt$ | |
| Initial value | $f(t), t \to 0+$ | $sF(s), s \to \infty$ | |
| Final value | $f(t), t \to \infty$ | $sF(s), s \to 0$ | |

| Description | $f(t)$ | $F(s)$ | $F(z)$ |
|---|---|---|---|
| 1. Unit impulse: at $t = 0$ | $\delta(t)$ | $1$ | $1$ or $z^{-0}$ |
| at $t = kT$ | $\delta(t - kT)$ | $\exp(-skT)$ | $z^{-k}$ |
| 2. Unit step | $H(t)$ | $\frac{1}{s}$ | $\frac{z}{z-1}$ |
| 3. Delayed step | | $\frac{\exp(-skT)}{s}$ | $\frac{z^{-k+1}}{z-1}$ |
| 4. Rectangular pulse (duration $kT$) | $H(t) - H(t - kT)$ | $\frac{1 - \exp(-skT)}{s}$ | $\frac{z - z^{-k+1}}{z-1}$ |
| 5. Unit ramp | $t$ | $\frac{1}{s^2}$ | $\frac{Tz}{(z-1)^2}$ |
| 6. Delayed ramp | $(t - kT)H(t - kT)$ | $\frac{\exp(-skT)}{s^2}$ | $\frac{Tz^{-k+1}}{(z-1)^2}$ |
| 7. Acceleration function | $\frac{1}{2}t^2$ | $\frac{1}{s^3}$ | $\frac{1}{2}\frac{T^2z(z+1)}{(z-1)^3}$ |
| 8. $n$th order ramp | $t^n$ | $\frac{n!}{s^{n+1}}$ | $\lim_{\alpha \to 0} (-1)^n \frac{\partial^n}{\partial \alpha^n}\left(\frac{z}{z - \exp(-\alpha T)}\right)$ |
| 9. Exponential decay | $\exp(-\alpha t)$ | $\frac{1}{s + \alpha}$ | $\frac{z}{z - \exp(-\alpha T)}$ |



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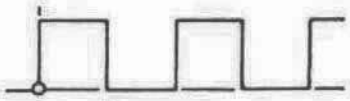
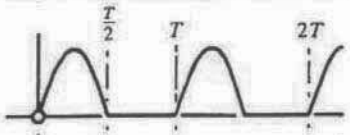
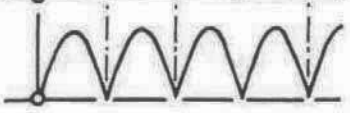
| Description | f(t) | F(s) | F(z) |
|---|---|---|---|
| 10. Exponential rise | $1 - \exp(-\alpha t)$ | $\frac{\alpha}{s(s+\alpha)}$ | $\frac{[1-\exp(-\alpha T)]z}{(z-1)[z-\exp(-\alpha T)]}$ |
| 11. Exponential × $t$ | $t\exp(-\alpha t)$ | $\frac{1}{(s+\alpha)^2}$ | $\frac{Tz\exp(-\alpha T)}{[z-\exp(-\alpha T)]^2}$ |
| 12. Exponential × $t^n$ | $t^n\exp(-\alpha t)$ | $\frac{n!}{(s+\alpha)^{n+1}}$ | $(-1)^n\frac{\partial^n}{\partial\alpha^n}\left[\frac{z}{z-\exp(-\alpha T)}\right]$ |
| 13. Difference of exponentials | $\exp(-\alpha t) - \exp(-\beta t)$ | $\frac{\beta-\alpha}{(s+\alpha)(s+\beta)}$ | $z\left[\frac{1}{z-\exp(-\alpha T)} - \frac{1}{z-\exp(-\beta T)}\right]$ |
| 14. Sine | $\sin \omega t$ | $\frac{\omega}{s^2+\omega^2}$ | $\frac{z\sin \omega T}{z^2 - 2z\cos \omega T + 1}$ |
| 15. Phase-advanced sine | $\sin(\omega t + \phi)$ | $\frac{\omega\cos\phi + s\sin\phi}{s^2+\omega^2}$ | $\frac{z^2\sin\phi + z\sin(\omega T - \phi)}{z^2 - 2z\cos\omega T + 1}$ |
| 16. Sine × $t$ | $t\sin \omega t$ | $\frac{2\omega s}{(s^2+\omega^2)^2}$ | |
| 17. Exponentially decaying sine | $\exp(-\alpha t)\sin \omega t$ | $\frac{\omega}{(s+\alpha)^2+\omega^2}$ | $\frac{z\exp(-\alpha T)\sin \omega T}{z^2 - 2z\exp(-\alpha T)\cos \omega T + \exp(-2\alpha T)}$ |
| 18. Cosine | $\cos \omega t$ | $\frac{s}{s^2+\omega^2}$ | $\frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$ |
| 19. Phase-advanced cosine | $\cos(\omega t + \phi)$ | $\frac{s\cos\phi - \omega\sin\phi}{s^2+\omega^2}$ | $\frac{z^2\cos\phi - z\cos(\omega T - \phi)}{z^2 - 2z\cos\omega T + 1}$ |
| 20. Offset cosine | $1 - \cos \omega t$ | $\frac{\omega^2}{s(s^2+\omega^2)}$ | $\frac{z}{z-1} - \frac{z(z-\cos \omega T)}{z^2 - 2z\cos \omega T + 1}$ |
| 21. Cosinc × $t$ | $t\cos \omega t$ | $\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$ | |
| 22. Exponentially decaying cosine | $\exp(-\alpha t)\cos \omega t$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$ | $\frac{z^2 - z\exp(-\alpha T)\cos \omega T}{z^2 - 2z\exp(-\alpha T)\cos \omega T + \exp(-2\alpha T)}$ |
| 23. Trigonometric function | $\sin \omega t - \omega t\cos \omega t$ | $\frac{2\omega^3}{(s^2+\omega^2)^2}$ | |
| 24. Exponentially decaying trig. fn. | $\exp(-\alpha t)$ × $[\sin \omega t - \omega t\cos \omega t]$ | $\frac{2\omega^3}{[(s+\alpha)^2+\omega^2]^2}$ | |
| 25. Hyperbolic sine | $\sinh \omega t$ | $\frac{\omega}{s^2-\omega^2}$ | $\frac{z\sinh \omega T}{z^2 - 2z\cosh \omega T + 1}$ |
| 26. Hyperbolic cosine | $\cosh \omega t$ | $\frac{s}{s^2-\omega^2}$ | $\frac{z(z-\cosh \omega T)}{z^2 - 2z\cosh \omega T + 1}$ |

## Periodic Functions

| | f(t) to base t | F(s) |
|---|---|---|
| 1. Rectangular wave (period $T$) |  | $\frac{1 + \tanh(\frac{1}{4}sT)}{2s}$ |
| 2. Half-wave rectified sine ($T = 2\pi/\omega$) |  | $\frac{\omega \exp(\frac{1}{4}sT)\,\mathrm{cosech}(\frac{1}{4}sT)}{2(s^2 + \omega^2)}$ |
| 3. Full-wave rectified sine ($T = 2\pi/\omega$) $\lvert \sin \omega t \rvert$ |  | $\frac{\omega \coth(\frac{1}{4}sT)}{s^2 + \omega^2}$ |

## Fourier Transforms

$$X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft)\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) \exp(j2\pi ft)\, df$$

$$\int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$

## Transform theorems

| Name | Signal | Fourier transform |
|---|---|---|
| Superposition | $ax_1(t) + bx_2(t)$ | $aX_1(f) + bX_2(f)$ |
| Time delay | $x(t - t_0)$ | $X(f) \exp(-j2\pi ft_0)$ |
| Scale change | $x(at)$ | $\lvert a \rvert^{-1} X(f/a)$ |
| Frequency translation | $x(t) \exp(j2\pi f_0 t)$ | $X(f - f_0)$ |
| Modulation | $x(t) \cos 2\pi f_0 t$ | $\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$ |
| Differentiation | $\frac{d^n x(t)}{dt^n}$ | $(j2\pi f)^n X(f)$ |
| Integration | $\int_{-\infty}^{t} x(t')\, dt'$ | $(j2\pi f)^{-1} X(f) + \frac{1}{2}X(0)\delta(f)$ |
| Convolution | $\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\, dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\, dt'$ | $X_1(f)X_2(f)$ |
| Multiplication | $x_1(t)x_2(t)$ | $\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\, df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\, df'$ |



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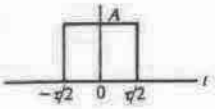
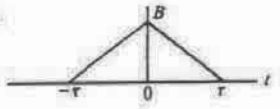
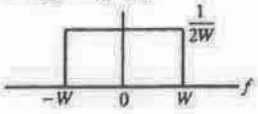
Fourier transform pairs

| Signal $x(t)$ | Transform $X(f)$ |
|---|---|
| 1.   | $A\tau\dfrac{\sin \pi f\tau}{\pi f\tau} \triangleq A\tau \operatorname{sinc} f\tau$ |
| 2.   | $B\tau\dfrac{\sin^2 \pi f\tau}{(\pi f\tau)^2} \triangleq B\tau \operatorname{sinc}^2 f\tau$ |
| 3. $\exp(-\alpha t)u(t)$ | $\dfrac{1}{\alpha + j2\pi f}$ |
| 4. $\exp(-\lvert t\rvert/\tau)$ | $\dfrac{2\tau}{1 + (2\pi f\tau)^2}$ |
| 5. $\exp[-\pi(t/\tau)^2]$ | $\tau \exp[-\pi(f\tau)^2]$ |
| 6. $\dfrac{\sin 2\pi Wt}{2\pi Wt} \triangleq \operatorname{sinc} 2Wt$ |   |
| 7. $\exp[j(2\pi f_c t + \phi)]$ | $\exp(j\phi)\delta(f - f_c)$ |
| 8. $\cos(2\pi f_c t + \phi)$ | $\frac{1}{2}\delta(f - f_c)\exp(j\phi) + \frac{1}{2}\delta(f + f_c)\exp(-j\phi)$ |
| 9. $\delta(t - t_0)$ | $\exp(-j2\pi f t_0)$ |
| 10. $\sum_{m=-\infty}^{\infty} \delta(t - mT_3)$ | $\dfrac{1}{T_3}\sum_{n=-\infty}^{\infty} \delta\left(f - \dfrac{n}{T_3}\right)$ |
| 11. $\operatorname{sgn} t = \begin{cases} +1, & t > 0 \\ -1, & t < 0 \end{cases}$ | $-\dfrac{j}{\pi f}$ |
| 12. $u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$ | $\frac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$ |