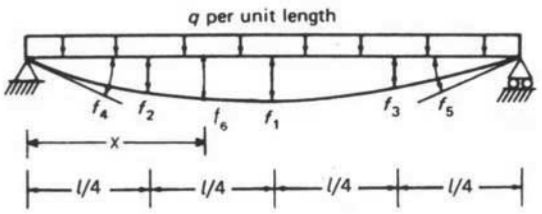
9210-111
Reference booklet

Sample

APPENDIX B Displacements of prismatic members

The following table gives the displacements in beams of constant flexural rigidity EI and constant torsional rigidity GJ, subjected to the loading shown on each beam. The positive directions of the displacements are downward for translation, clockwise for rotation. The deformations due to shearing forces are neglected.

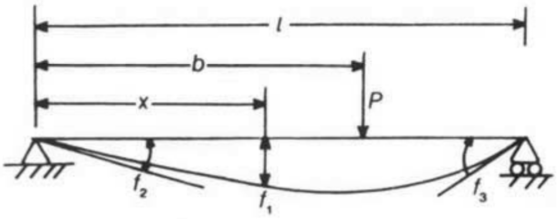


$$f_1 = \frac{5}{384}\frac{ql^4}{EI}$$

$$f_2 = f_3 = \frac{19}{2048}\frac{ql^4}{EI}$$

$$f_4 = -f_5 = \frac{ql^3}{24EI}$$

$$f_6 = \frac{qx}{24EI}(l^3 - 2lx^2 + x^3)$$

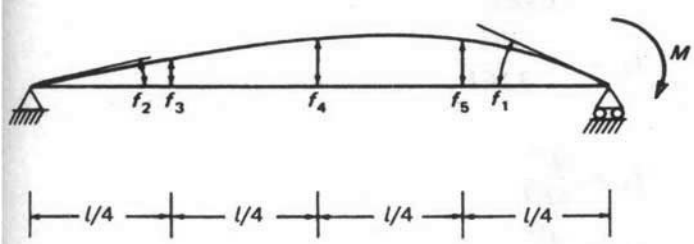


$$f_1 = \frac{P(l-b)x}{6lEI}(2lb - b^2 - x^2) \qquad \text{when } x \leqslant b$$

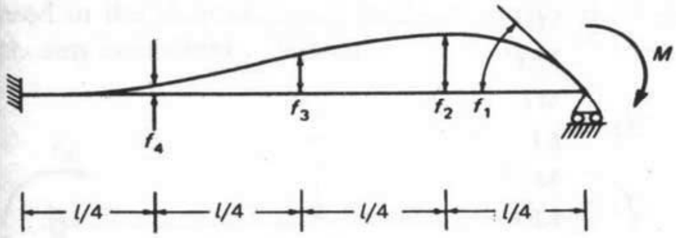
$$f_1 = \frac{Pb(l-x)}{6lEI}(2lx - x^2 - b^2) \qquad \text{when } x \geqslant b$$

$$f_2 = \frac{Pb(l-b)}{6lEI}(2l - b) \qquad f_3 = -\frac{Pb}{6lEI}(l^2 - b^2)$$

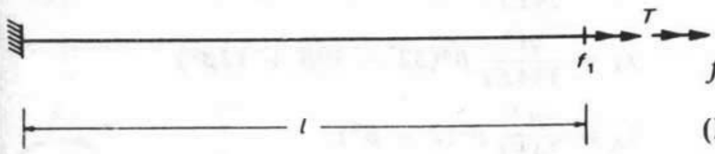
When $b = l/2$, $f_2 = -f_3 = Pl^2/(16EI)$, and $f_1 = Pl^3/48EI$ at $x = l/2$.



$$f_1 = \frac{Ml}{3EI}$$
$$f_2 = -\frac{Ml}{6EI}$$
$$f_3 = -\frac{15Ml^2}{384EI}$$
$$f_4 = -\frac{Ml^2}{16EI}$$
$$f_5 = -\frac{21Ml^2}{384EI}$$

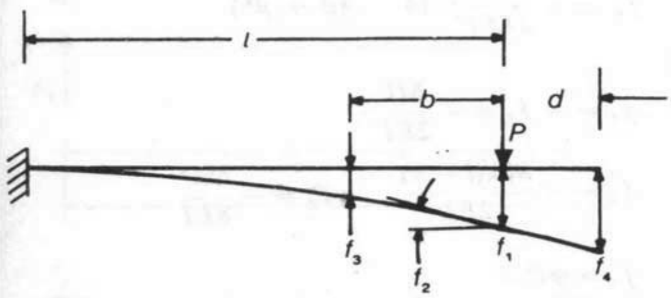


$$f_1 = \frac{Ml}{4EI}$$
$$f_2 = -\frac{9Ml^2}{256EI}$$
$$f_3 = -\frac{Ml^2}{32EI}$$
$$f_4 = -\frac{3Ml^2}{256EI}$$



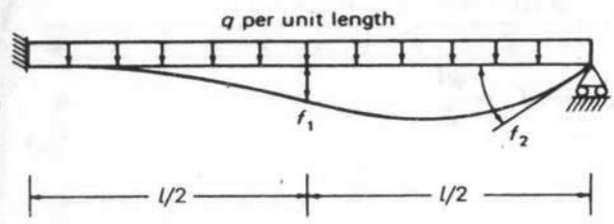
$$f_1 = \frac{Tl}{GJ}$$

(Effect of warping ignored

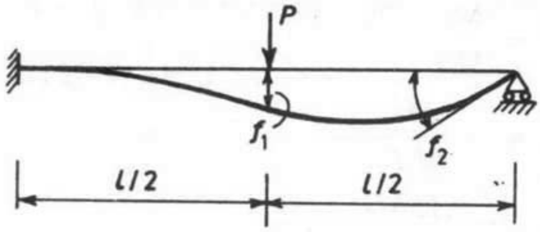


$$f_1 = \frac{Pl^3}{3EI}$$
$$f_2 = Pl^2/2EI$$
$$f_4 = f_1 + df_2$$
$$f_3 = \frac{Pl^3}{3EI}\left(1 - \frac{3b}{2l} + \frac{b^3}{2l^3}\right)$$

for $0 \leqslant b \leqslant l$

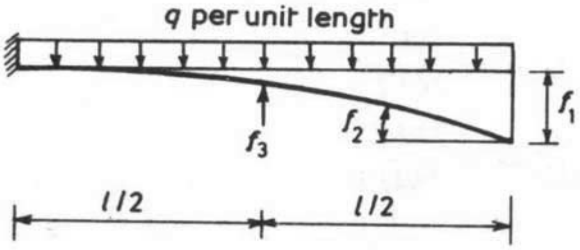


$$f_1 = \frac{ql^4}{192EI}$$
$$f_2 = -\frac{ql^3}{48EI}$$



$$f_1 = \frac{7Pl^3}{768EI}$$

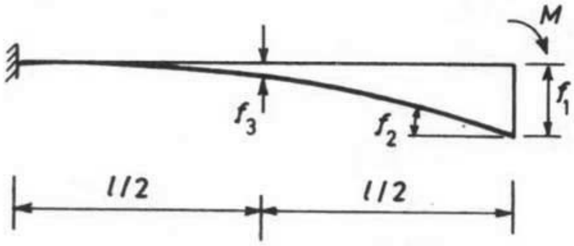
$$f_2 = -\frac{Pl^2}{32EI}$$



$$f_1 = \frac{ql^4}{8EI}$$

$$f_2 = \frac{ql^3}{6EI}$$

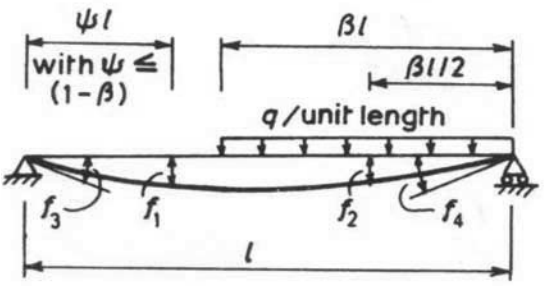
$$f_3 = \frac{17ql^4}{384EI}$$



$$f_1 = \frac{Ml^2}{2EI}$$

$$f_2 = \frac{Ml}{EI}$$

$$f_3 = \frac{Ml^2}{8EI}$$

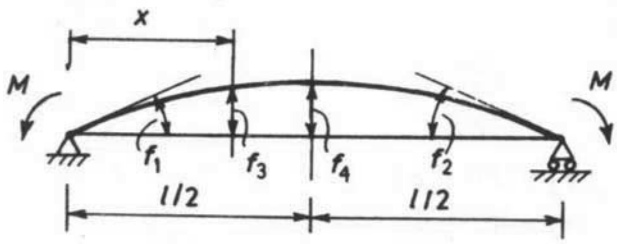


$$f_1 = \frac{ql^4}{24EI}\beta^2\psi(2 - \beta^2 - 2\psi^2)$$

$$f_2 = \frac{ql^4}{384EI}\beta^3(32 - 39\beta + 12\beta^2)$$

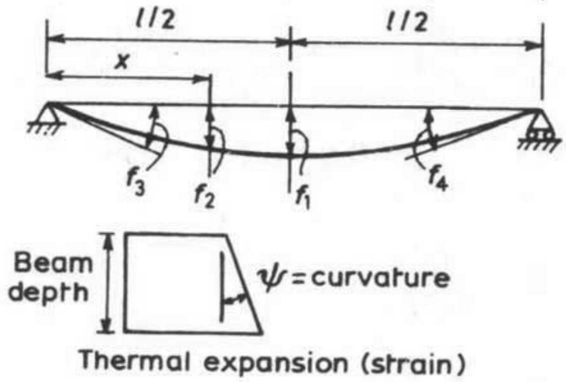
$$f_3 = \frac{ql^3}{24EI}\beta^2(2 - \beta^2)$$

$$f_4 = -\frac{ql^3}{24EI}\beta^2(4 - 4\beta + \beta^2)$$



$$f_1 = -f_2 = -\frac{Ml}{2EI}$$

$$f_3 = -\frac{Mx(l-x)}{2EI} \qquad f_4 = -\frac{Ml^2}{8EI}$$



$$f_1 = \psi l^2/8$$

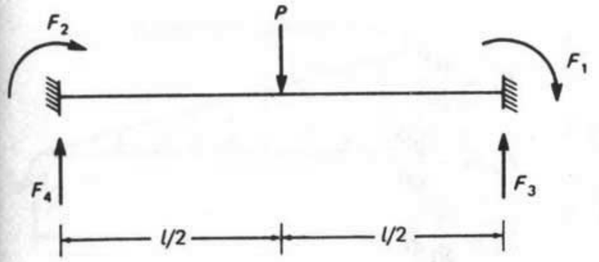
$$f_2 = \frac{\psi x(l-x)}{2}$$

$$f_3 = -f_4 = \frac{\psi l}{2}$$

APPENDIX C Fixed-end forces of prismatic members

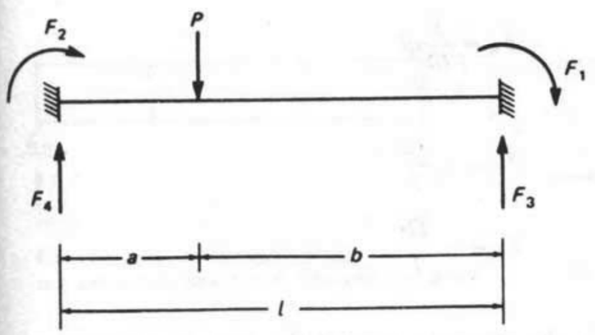
The following table gives the fixed-end forces in beams of constant flexural rigidity and constant torsional rigidity due to applied loads. The forces are considered positive if upward or in the clockwise direction. A twisting couple is positive if it acts in the direction of rotation of a right-hand screw progressing to the right. When the end-forces are used in the displacement method, appropriate signs have to be assigned according to the chosen coordinate system.

Fixed-End Force



$$F_1 = -F_2 = \frac{Pl}{8}$$

$$F_3 = F_4 = \frac{P}{2}$$

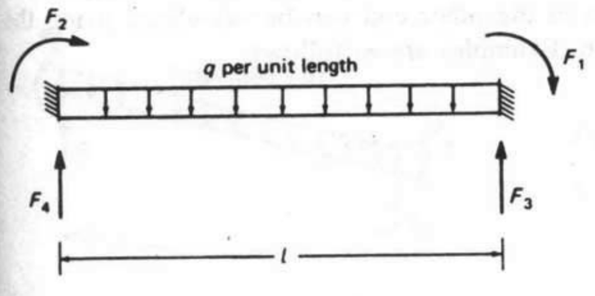


$$F_1 = \frac{Pa^2b}{l^2}$$

$$F_2 = -\frac{Pab^2}{l^2}$$

$$F_3 = P\left(\frac{a}{l} + \frac{a^2b}{l^3} - \frac{ab^2}{l^3}\right)$$

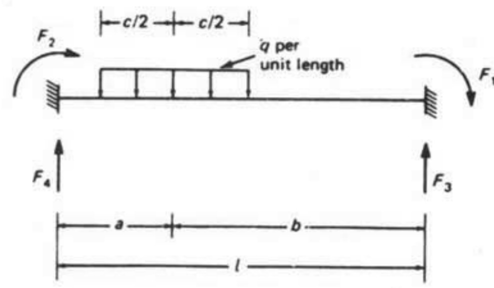
$$F_4 = P\left(\frac{b}{l} - \frac{a^2b}{l^3} + \frac{ab^2}{l^3}\right)$$



$$F_1 = -F_2 = \frac{ql^2}{12}$$

$$F_3 = F_4 = \frac{ql}{2}$$

Fixed-End Force

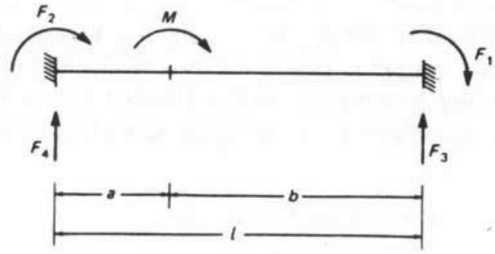


$$F_1 = \frac{qc}{12l^2}\left[12a^2b + c^2(l-3a)\right]$$

$$F_2 = -\frac{qc}{12l^2}\left[12ab^2 + c^2(l-3b)\right]$$

$$F_3 = \frac{qca}{l} + \frac{(F_1+F_2)}{l}$$

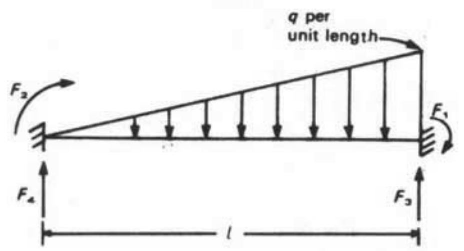
$$F_4 = \frac{qcb}{l} - \frac{(F_2+F_1)}{l}$$



$$F_1 = \frac{Ma}{l}\left(2 - \frac{3a}{l}\right)$$

$$F_2 = \frac{Mb}{l}\left(2 - \frac{3b}{l}\right)$$

$$F_3 = -F_4 = \frac{6Mab}{l^3}$$

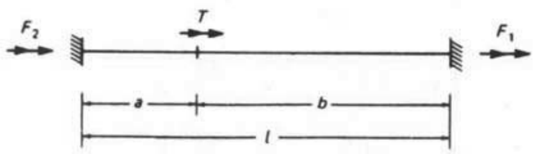


$$F_1 = \frac{ql^2}{20}$$

$$F_2 = -\frac{ql^2}{30}$$

$$F_3 = \frac{7}{20}ql$$

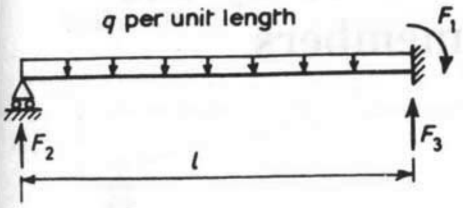
$$F_4 = \frac{3}{20}ql$$



$$F_1 = -\frac{Ta}{l}$$

$$F_2 = -\frac{Tb}{l}$$

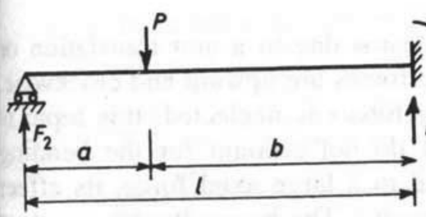
If the totally fixed support in any of the above cases, except the last, is changed to a hinge or a roller, the fixed-end moment at the other end can be calculated using the equations of this appendix and Eq. 11.46. Examples are as follows:



$$F_1 = \frac{ql^2}{8}$$

$$F_2 = \frac{3ql}{8}$$

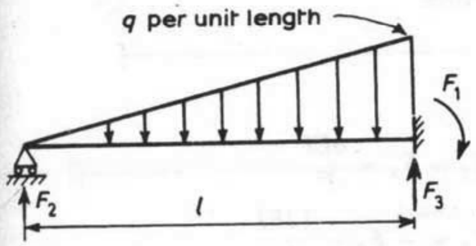
$$F_3 = \frac{5ql}{8}$$



$$F_1 = \frac{Pab}{l^2}\left(a + \frac{b}{2}\right)$$

$$F_2 = P\left[\frac{b}{l} - \frac{ab}{l^3}\left(a + \frac{b}{2}\right)\right]$$

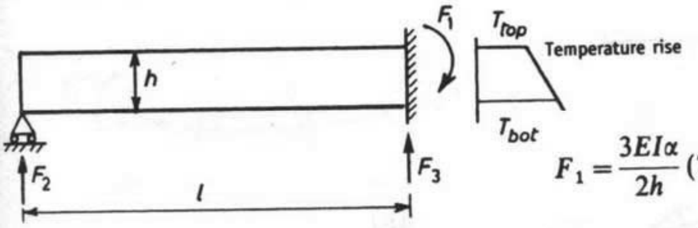
$$F_3 = P\left[\frac{a}{l} + \frac{ab}{l^3}\left(a + \frac{b}{2}\right)\right]$$



$$F_1 = \frac{ql^2}{15}$$

$$F_2 = \frac{ql}{10}$$

$$F_3 = \frac{2ql}{5}$$

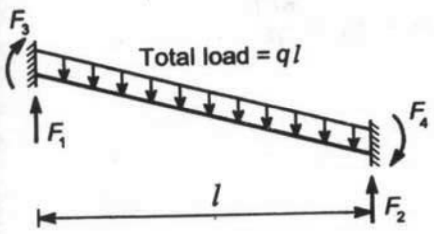


For the case when both ends are encastré, see Fig. 6.10g and Eq. 6.42

$$F_1 = \frac{3EI\alpha}{2h}(T_{bot} - T_{top})$$

$$F_2 = -F_3 = \frac{3EI}{2hl}\alpha(T_{bot} - T_{top})$$

α = coefficient of thermal expansion



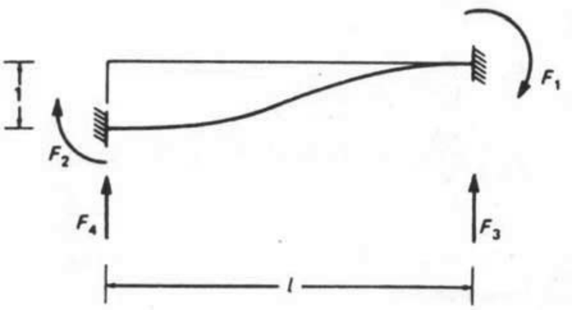
$$F_1 = F_2 = ql/2$$

$$F_3 = -F_4 = -ql^2/12$$

APPENDIX D End-forces caused by end-displacements of prismatic members

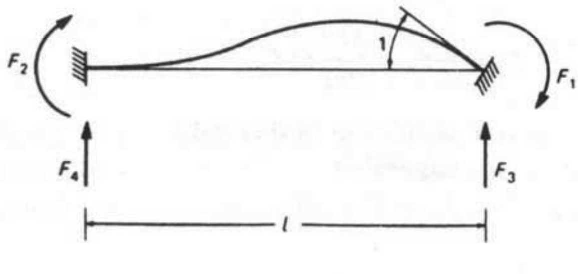
The following table gives the forces at the ends of beams due to a unit translation or unit rotation of one end. The positive directions for the forces are upward and clockwise. The effect of the deformation caused by the shearing forces is neglected; this topic is considered in Section 15.2. Moreover, the equations do not account for the bending moment due to axial forces; if a member is subjected to a large axial force, its effect may be included using Table 14.2 instead of this appendix. The beams have a constant flexural rigidity *EI* and a constant torsional rigidity *GJ*.

Beam **Force**



$$F_1 = F_2 = \frac{6EI}{l^2}$$

$$F_3 = -F_4 = \frac{12EI}{l^3}$$

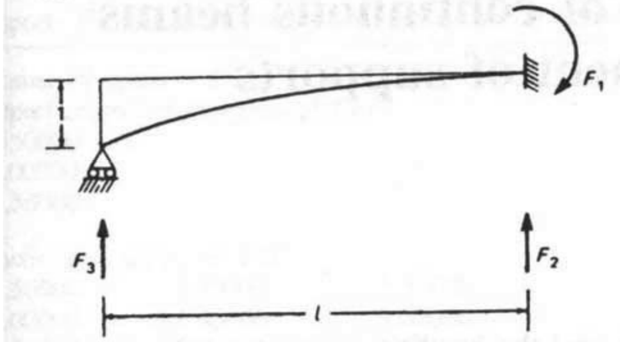


$$F_1 = \frac{4EI}{l}$$

$$F_2 = \frac{2EI}{l}$$

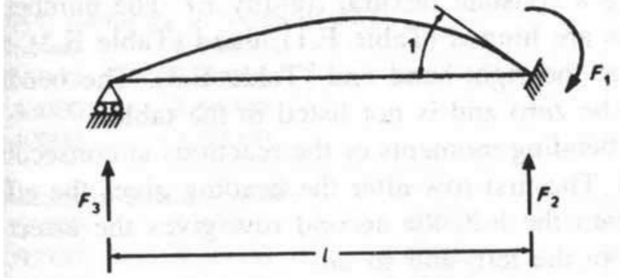
$$F_3 = -F_4 = \frac{6EI}{l^2}$$

Beam | **Force**



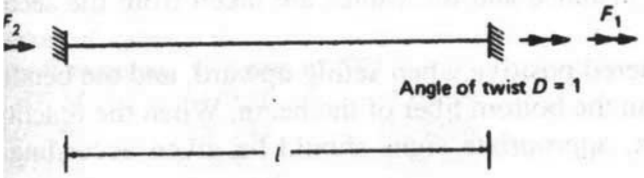
$$F_1 = \frac{3EI}{l^2}$$

$$F_2 = -F_3 = \frac{3EI}{l^3}$$



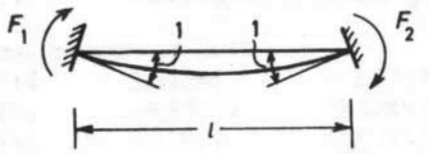
$$F_1 = \frac{3EI}{l}$$

$$F_2 = -F_3 = \frac{3EI}{l^2}$$



$$F_1 = -F_2 = \frac{GJ}{l}$$

(Effect of warping ignored)



$$F_1 = -F_2 = \frac{2EI}{l}$$