APPENDIX B  Displacements of prismatic members

The following table gives the displacements in beams of constant flexural rigidity $EI$ and constant torsional rigidity $GJ$, subjected to the loading shown on each beam. The positive directions of the displacements are downward for translation, clockwise for rotation. The deformations due to shearing forces are neglected.

$$f_1 = \frac{5}{384} \frac{ql^4}{EI}$$

$$f_2 = f_3 = \frac{19}{2048} \frac{ql^4}{EI}$$

$$f_4 = -f_5 = \frac{ql^3}{24EI}$$

$$f_6 = \frac{q}{24EI} \left(l^3 - 2lx^2 + x^3\right)$$

$$f_1 = \frac{P(l-b)x}{6EI} \left(2lb - b^2 - x^2\right) \quad \text{when } x \leq b$$

$$f_2 = \frac{Pb(l-x)}{6EI} \left(2lx - x^2 - b^2\right) \quad \text{when } x \geq b$$

$$f_3 = -\frac{Pb}{6EI} (l^2 - b^2)$$

When $b = l/2$, $f_3 = -f_5 = Pl^2/(16EI)$, and $f_1 = Pl^3/48EI$ at $x = l/2$. 
\[ f_1 = \frac{Ml}{3EI} \]
\[ f_2 = -\frac{Ml}{6EI} \]
\[ f_3 = -\frac{15Ml^2}{384EI} \]
\[ f_4 = -\frac{Ml^2}{16EI} \]
\[ f_5 = -\frac{21Ml^2}{384EI} \]
\[ f_6 = \frac{Ml}{4EI} \]
\[ f_7 = -\frac{9Ml^2}{256EI} \]
\[ f_8 = -\frac{Ml^2}{32EI} \]
\[ f_9 = -\frac{3Ml^2}{256EI} \]

\[ f_1 = \frac{Tl}{GJ} \]

(Effect of warping ignored)

\[ f_1 = \frac{Pl^3}{3EI} \]
\[ f_2 = \frac{Pl^3}{2EI} \]
\[ f_4 = f_1 + df_2 \]
\[ f_3 = \frac{Pl^3}{3EI} \left( 1 - \frac{3b}{2l} + \frac{b^3}{2l^3} \right) \]
for \( 0 \leq b \leq l \)

\[ f_1 = \frac{ql^4}{192EI} \]
\[ f_2 = -\frac{ql^3}{48EI} \]
\[ f_1 = \frac{7Pl^3}{768EI} \]
\[ f_2 = -\frac{Pl^2}{32EI} \]

\[ f_1 = \frac{ql^4}{8EI} \]
\[ f_2 = \frac{ql^3}{6EI} \]
\[ f_3 = \frac{17ql^4}{384EI} \]

\[ f_1 = \frac{Ml^2}{2EI} \]
\[ f_2 = \frac{Ml}{EI} \]
\[ f_3 = \frac{Ml^2}{8EI} \]

\[ f_1 = \frac{ql^4}{24EI} \beta^2 \psi(2 - \beta^2 - 2\psi^2) \]
\[ f_2 = \frac{ql^4}{384EI} \beta^3(32 - 39\beta + 12\beta^2) \]
\[ f_3 = \frac{ql^3}{24EI} \beta^2(2 - \beta^2) \]
\[ f_4 = -\frac{ql^3}{24EI} \beta^2(4 - 4\beta + \beta^2) \]

\[ f_1 = -f_2 = -\frac{Ml}{2EI} \]
\[ f_3 = -\frac{M(l - x)}{2EI} \]
\[ f_4 = -\frac{Ml^2}{8EI} \]

\[ f_1 = \psi l^2 / 8 \]
\[ f_2 = \psi x(l - x) / 2 \]
\[ f_3 = -f_4 = \psi l / 2 \]
APPENDIX C  Fixed-end forces of prismatic members

The following table gives the fixed-end forces in beams of constant flexural rigidity and constant torsional rigidity due to applied loads. The forces are considered positive if upward or in the clockwise direction. A twisting couple is positive if it acts in the direction of rotation of a right-hand screw progressing to the right. When the end-forces are used in the displacement method, appropriate signs have to be assigned according to the chosen coordinate system.

**Fixed-End Force**

- \( F_1 = -F_2 = \frac{Pl}{8} \)
- \( F_3 = F_4 = \frac{P}{2} \)
- \( F_1 = \frac{Pa^2b}{l^2} \)
- \( F_2 = -\frac{Pab^2}{l^2} \)
- \( F_3 = P\left(\frac{a}{l} + \frac{a^2b}{l^3} - \frac{ab^2}{l^3}\right) \)
- \( F_4 = P\left(\frac{b}{l} - \frac{a^2b}{l^3} + \frac{ab^2}{l^3}\right) \)
- \( F_1 = -F_2 = \frac{ql^2}{12} \)
- \( F_3 = F_4 = \frac{ql}{2} \)
Fixed-End Force

\[ F_1 = \frac{qc}{12l^2} \left[ 12a^2 b + c^4 (l - 3a) \right] \]
\[ F_2 = -\frac{qc}{12l^2} \left[ 12ab^2 + c^4 (l - 3b) \right] \]
\[ F_3 = \frac{qca}{l} \left( \frac{F_1 + F_2}{l} \right) \]
\[ F_4 = \frac{qcb}{l} \left( \frac{F_2 + F_3}{l} \right) \]

\[ F_1 = \frac{Ma}{l} \left( 2 - \frac{3a}{l} \right) \]
\[ F_2 = \frac{Mb}{l} \left( 2 - \frac{3b}{l} \right) \]
\[ F_3 - F_4 = \frac{6Mab}{l^3} \]

\[ F_1 = \frac{ql^2}{20} \]
\[ F_2 = -\frac{ql^2}{30} \]
\[ F_3 = \frac{7}{20} ql \]
\[ F_4 = \frac{3}{20} ql \]

\[ F_1 = -\frac{Ta}{l} \]
\[ F_2 = -\frac{Tb}{l} \]

If the totally fixed support in any of the above cases, except the last, is changed to a hinge or a roller, the fixed-end moment at the other end can be calculated using the equations of this appendix and Eq. 11.46. Examples are as follows:
Appendi.

\[ F_1 = \frac{q l^2}{8} \]
\[ F_2 = \frac{3q l}{8} \]
\[ F_3 = \frac{5q l}{8} \]

\[ F_1 = \frac{P a b}{l^2} \left( a + \frac{b}{2} \right) \]
\[ F_2 = P \left[ \frac{b}{l} - \frac{a b}{l^3} \left( a + \frac{b}{2} \right) \right] \]
\[ F_3 = P \left[ \frac{a}{l} + \frac{a b}{l^3} \left( a + \frac{b}{2} \right) \right] \]

\[ F_1 = \frac{q l^2}{15} \]
\[ F_2 = \frac{q l}{10} \]
\[ F_3 = \frac{2q l}{5} \]

For the case when both ends are encastré, see Fig. 6.10g and Eq. 6.42

\[ F_1 = \frac{3E I \alpha}{2h} (T_{\text{bot}} - T_{\text{top}}) \]
\[ F_2 = -F_3 = \frac{3E I}{2hl} \alpha (T_{\text{bot}} - T_{\text{top}}) \]

\[ \alpha = \text{coefficient of thermal expansion} \]

\[ F_1 = F_2 = \frac{q l}{2} \]
\[ F_3 = -F_4 = -\frac{q l^2}{h^2} \]
APPENDIX D  End-forces caused by end-displacements of prismatic members

The following table gives the forces at the ends of beams due to a unit translation or unit rotation of one end. The positive directions for the forces are upward and clockwise. The effect of the deformation caused by the shearing forces is neglected; this topic is considered in Section 15.2. Moreover, the equations do not account for the bending moment due to axial forces; if a member is subjected to a large axial force, its effect may be included using Table 14.2 instead of this appendix. The beams have a constant flexural rigidity \( EI \) and a constant torsional rigidity \( GJ \).

\[
\begin{align*}
F_1 &= F_2 = \frac{6EI}{l^2} \\
F_3 &= -F_4 = \frac{12EI}{l^3}
\end{align*}
\]
Beam

Force

\[ F_1 = \frac{3EI}{l^2} \]

\[ F_2 = -F_3 = \frac{3EI}{l^3} \]

\[ F_1 = \frac{3EI}{l} \]

\[ F_2 = -F_3 = \frac{3EI}{l^3} \]

Angle of twist \( \theta = 1 \)

\( (\text{Effect of warping ignored}) \)

\[ F_1 = -F_2 = \frac{GJ}{l} \]

\[ F_1 = -F_2 = \frac{2EI}{l} \]