9210-112
Graduate Diploma in Engineering
Circuits and waves

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, ruler

No additional data is attached

General instructions
- This examination paper is of **three hours** duration.
- This examination paper contains **five** questions over Section A and B.
- The candidates must answer **five** questions in total by selecting at least one question from each section.
- All questions carry equal marks. The maximum marks for each section within a question are given against that section.
- An electronic, non-programmable calculator may be used but candidates **must** show clearly the steps prior to obtaining final numerical values.
- Drawings should be clear, in good proportion and in pencil. Do **not** use red ink.

Section A

1 a) Use the superposition theorem to find the voltage V_0 in the circuit shown in Figure 1a. (5 marks)

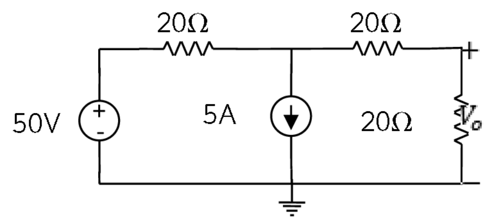

Figure 1a

b) Use nodal analysis to find the voltage V_3 in the circuit shown in Figure 1b. (5 marks)

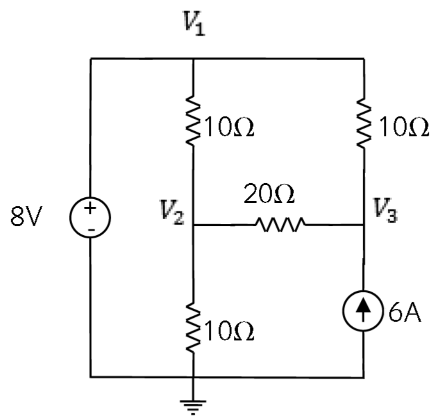

Figure 1b

c) Figure 1c shows a circuit with a current-controlled voltage source. Determine the output voltage V_0. (5 marks)

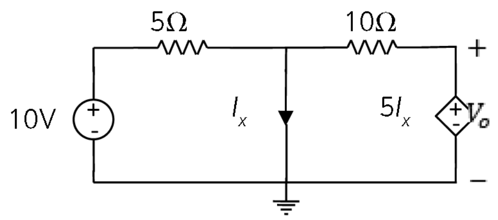

Figure 1c

d) Find the voltage in V_0 Figure 1d using mesh analysis. (5 marks)

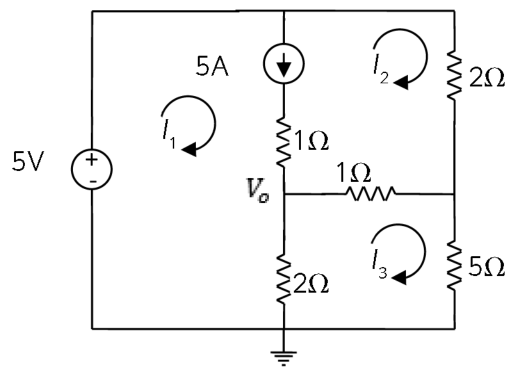

Figure 1d

2 a) Figure 2a shows a capacitor. The voltage across the capacitor at $t = 0$ is V_0.

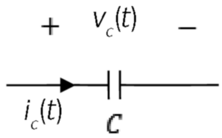

Figure 2a

i) Write an expression for the voltage across the capacitor $v_c(t)$ if the current through the capacitor is $i_c(t)$. (2 marks)

ii) The capacitor is suddenly connected across a resistor R. Show that

$v_c(t) = V_0 e^{-\frac{t}{RC}}$ (3 marks)

b) A capacitor of 10 µF charged to 24 V is suddenly connected across a resistor of 1 kΩ.

i) Compute the capacitor voltage after 20 ms. (1 mark)

ii) Compute the current after 20 ms. (1 mark)

iii) Compute the amount of energy transferred from the capacitor to the resistor by 20 ms. (3 marks)

c) Figure 2c shows a series *LRC* circuit with an initially charged capacitor.
The form of the solution for the current is

$i(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}$

Where p_1 and p_2 are characteristic roots.

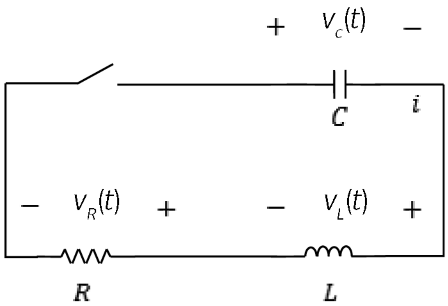

Figure 2c

i) Obtain the characteristic equation starting from KVL without using Laplace techniques. (3 marks)

ii) Obtain an expression for the characteristic roots. (2 marks)

d) In the circuit in Figure 2c, $L = 1$ H, $R = 7\ \Omega$, and $C = 0.1$ F. The capacitor is initially charged to 60 V. Show that the current is $i(t) = 20(e^{-5t} - e^{-2t})$. (5 marks)

See next page

3 a) Figure 3a shows a general two-port network. Express the voltages V_1 and V_2 in terms of z-parameters. (5 marks)

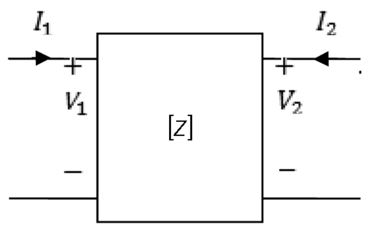

Figure 3a

b) The following measurements were made on an unknown two-port network.

With $I_2 = 0$	With $I_1 = 0$
$V_1 = 36$ V	$V_1 = 16$ V
$V_2 = 24$ V	$V_2 = 20$ V
$I_1 = 4$ A	$I_2 = 2$ A

Determine the z-parameters. (5 marks)

c) For the two-port network described by h-parameters

$V_1 = h_{11}I_1 + h_{12}V_2,$

$I_2 = h_{21}I_1 + h_{22}V_2,$

Find the h-parameters of the circuit shown in Figure 3c. (5 marks)

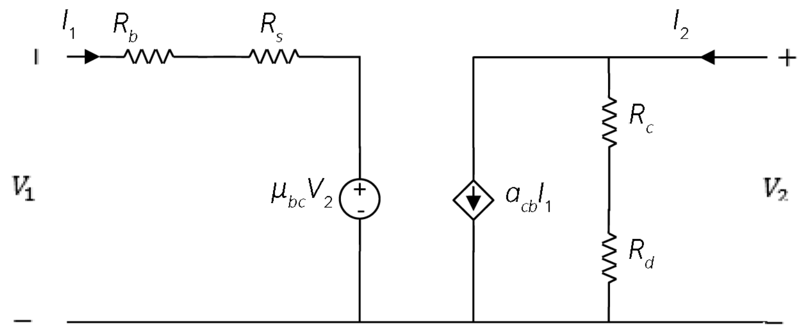

Figure 3c

d) Assume that, in the circuit in Figure 3c, a load $R_L = 30$ kΩ is connected at the output V_2. The circuit has the following parameters:

$R_b + R_e = 1.5$ kΩ, $a_{cb} = 50$, $\mu_{bc} = 3 \times 10^{-4}$, $1/(R_c + R_d) = 25$ μS

Find the input impedance. (5 marks)

4 a) Consider the following admittance function

$$Y(s) = \frac{24s^4 + 13s^2 + 1}{24s^3 + 7s}.$$

i) Realize this using the first Cauer form. (3 marks)

ii) Sketch the circuit. (2 marks)

b) Consider the following admittance function

$$Y(s) = \frac{(6s + 1)(s + 1)}{2(3s + 1)}.$$

i) Realize this using the second Foster form. (3 marks)

ii) Sketch the circuit. (2 marks)

c) Figure 4c shows an *RLC* circuit.

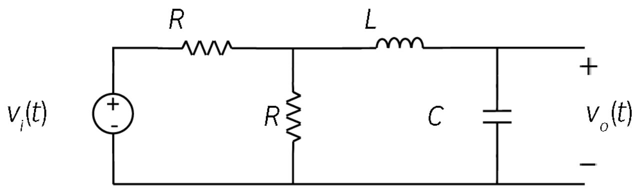

Figure 4c

Show that the transfer function is

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{2LC}}{s^2 + \frac{R}{2L}s + \frac{1}{LC}}.$$ (5 marks)

d) The parameters of the circuit shown in Figure 4c are $R = 2\ \Omega$, $L = 1$ H and $C = 0.4$ F.

i) Sketch the pole-zero diagram. (4 marks)

ii) Characterize the network with respect to damping. (1 mark)

5 a) A periodic waveform may be expressed using the Fourier series as

$f(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \ldots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots$

i) Write expressions for a_0, a_k, and b_k for all values of $k \neq 0$. (3 marks)

ii) State a case each when a_0, a_k, and b_k would individually become zero. (2 marks)

b) For the waveform $f(t)$ shown in Figure 5a

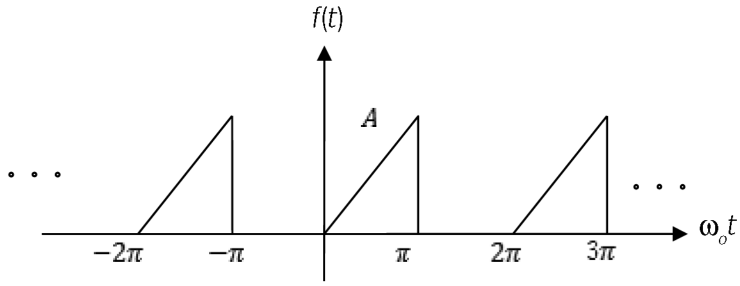

Figure 5a

i) Compute a_0. (2 marks)

ii) Compute a_k. (3 marks)

iii) Compute b_k. (3 marks)

iv) Write the Fourier series. (2 marks)

c) A current waveform,

$i(t) = 2 \sin \omega_0 t + 4 \sin 3\omega_0 t + \sin 5\omega_0 t$

is applied to a $10\ \Omega$ resistor.

i) Sketch the frequency spectrum showing the magnitudes of different frequency components. (3 marks)

ii) Compute the average power in the resistor. (2 marks)

Section B

6 a) Express Gauss' law

i) in integral form (2 marks)

ii) in words. (2 marks)

b) Figure 6b shows two long thin conductors, separated by a large distance d.
The linear charge density in one is $+\rho$ and the other is $-\rho$.

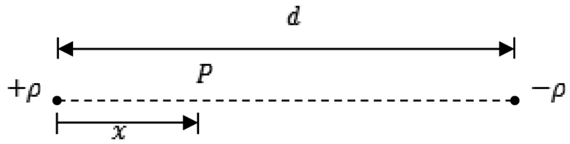

Figure 6b

i) Apply Gauss' law considering a cylindrical surface to obtain an expression for the electric field strength at a point P due to the charge $+\rho$ as shown in Figure 6bi. (2 marks)

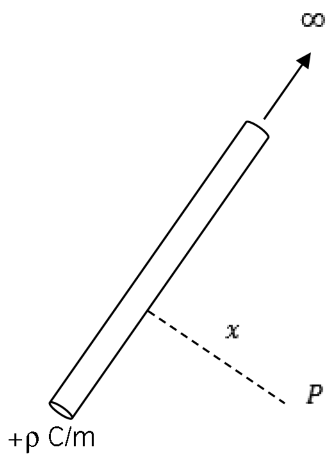

Figure 6bi: Thin Infinitely long conductor carrying a positive charge

ii) Obtain a similar expression as above due to a charge $-\rho$ as shown in Figure 6bii. (2 marks)

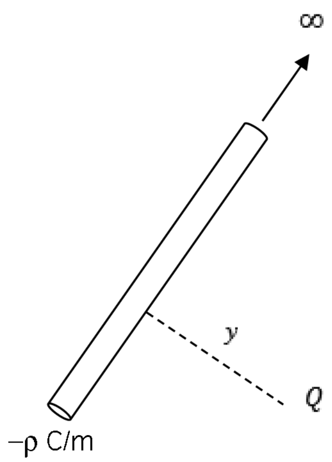

Figure 6bii: Thin Infinitely long conductor carrying a negative charge

iii) Obtain an expression for the resultant electric field strength at P of the twin transmission line as shown in Figure 6biii. (2 marks)

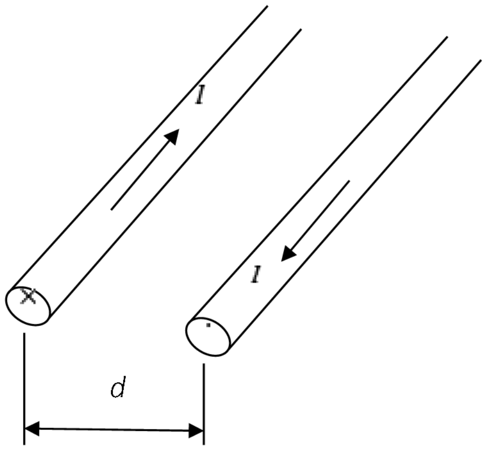

Figure 6biii: A twin transmission Line

iv) Plot the electric field strength for $x \in (0,d)$. (2 marks)

c) Two parallel metal plates, each of area A, are separated by a dielectric of permittivity $\in$ of uniform thickness d. The charge in one plate is Q and in the other is $-Q$.

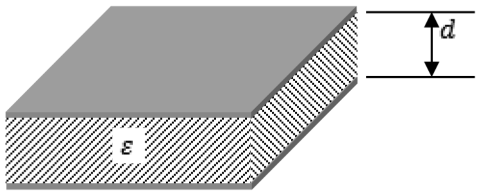

Figure 6c

i) Obtain an expression for the electric field strength between the plates. (3 marks)

ii) Find the capacitance of this parallel-plate capacitor. (2 marks)

iii) Show that the energy stored in the capacitor is $W = \frac{Q^2}{2C}$. (3 marks)

7 a) State Ampere's circuital law in integral form. (2 marks)

b) Consider a solid cylinder of radius a that carries a uniform current density of J_0 A/m^2 for $r < a$. The total current is I_0 A. Show that the magnetic flux density is

$$B = \begin{cases} \dfrac{\mu_0 r I_0}{2\pi a^2}, & \text{for } r < a, \\ \dfrac{\mu_0 I_0}{2\pi r}, & \text{for } r > a. \end{cases}$$

(4 marks)

c) Consider a hollow cylinder that carries a uniform current (total current is I_0) as shown in Figure 7c. Obtain an expression for the magnetic flux density in terms of r. (6 marks)

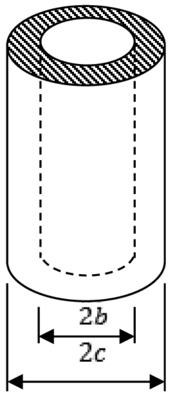

Figure 7c

d) Consider a coaxial cable, which has a solid cylindrical core of radius a, surrounded by a hollow cylinder of inner radius b and outer radius c.

i) Use results for parts b) and c) to obtain an expression for the magnetic flux density as a function of r. (6 marks)

ii) Show that the coaxial cable exhibits magnetic shielding. (2 marks)

8 a) i) Write Maxwell's equations in differential form. (2 marks)

ii) Derive the time-harmonic form of Maxwell's equations for source-free media. (4 marks)

b) i) Show that for free-space and time-harmonic fields, the wave equation is of the form $\nabla^2 E + \gamma^2 E = 0$. (4 marks)

ii) Write an expression for γ^2. (2 marks)

c) Consider the plane wave

$E = E_{ox}\cos(\omega t - \beta z)a_x + E_{oy}\cos(\omega t - \beta z + \theta)a_y$.

i) Show that the wave is plane-polarized if $\theta = 0$. (3 marks)

ii) Show that the wave is circularly polarized if $\theta = 90°$. (3 marks)

iii) List two applications of polarization. (2 marks)

9 a) List **two** advantages each of

i) metal waveguides (2 marks)

ii) dielectric waveguides. (2 marks)

b) The propagation constant for TE_{mn} and TM_{mn} modes for an air-filled rectangular metal waveguide, shown in Figure 9a of dimensions $a \times b$, is given by

$$\gamma_{mn} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

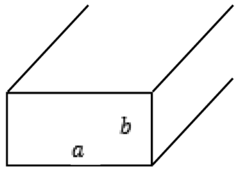

Figure 9a

The dimensions are a = 2 cm and b = 1 cm.

i) Show that the cut-off frequency for the TE_{mn} or TM_{mn} mode is

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}.$$ (6 marks)

ii) Compute the cut off frequency for the TE_{10} mode. (3 marks)

iii) Compute the cut off frequency for the TE_{20} mode (3 marks)

iv) What is the dominant TE mode for this waveguide? (4 marks)

10 a) List **five** fundamental parameters of antennas. (5 marks)

b) The radiation intensity of an antenna is given by

$$U(\theta, \phi) = \begin{cases} \sin\theta \sin\phi, & 0 \le \theta \le \pi, 0 \le \phi \le \pi, \\ 0, & \pi < \theta \le 2\pi, \pi < \phi \le 2\pi \end{cases}$$

i) Compute the total power radiated by the antenna, P_{rad}. (4 marks)

ii) Obtain an expression for the directivity. (4 marks)

iii) Find the maximum directivity of the antenna in dB. (2 marks)

c) A 10 GHz communication link consists of a transmitting antenna of gain 25 dB and a receiving antenna of gain 30 dB oriented to receive the maximum power. The distance between the two antennas is 20 km. Assuming there are no system losses, estimate the smallest transmit power if the received power is to be greater than –50 dBm? (5 marks)