9210-125
Level 6 Graduate Diploma in Engineering
Signals and systems

Sample Paper

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, ruler

Additional data is attached
- Fourier transform tables

General instructions
- This examination paper is of **three** hours duration.
- This examination paper contains **nine** questions.
- Answer any **five** questions only.
- All questions carry equal marks. The maximum marks for each section within a question are given against that section.
- An electronic, non-programmable calculator may be used, but the candidate **must** show clearly the steps prior to obtaining final numerical values.
- Drawings should be clear, in good proportion and in pencil. Do **not** use red ink.

1 a) Figure 1a shows two signals $x_1(t)$ and $x_2[n]$.

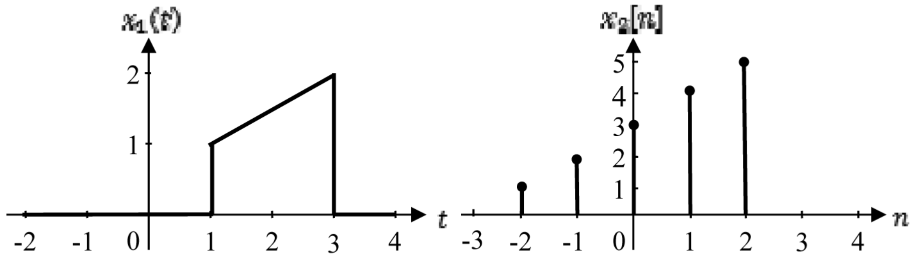

Figure 1a

Sketch the following signals.

i) $x_1(-t)$. (1 mark)

ii) $x_2(-t+2)$. (1 mark)

iii) $x_2[2n]$. (1 mark)

iv) $x_2[-n-1]$. (1 mark)

b) Determine if each of the following signals is periodic. If periodic, find the fundamental period.

i) $x_1(t) = \cos(2t + \pi/2)$. (2 marks)

ii) $x_2(t) = e^{j(\frac{\pi}{2}t-1)} + e^{j(\frac{\pi}{2}t-2)}$. (2 marks)

iii) $x_3[n] = \cos\frac{1}{2}n$. (1 mark)

c) Determine whether the following signals are energy signals, power signals, or neither. Here, $u(t)$ is the continuous-time unit step function, and $u[n]$ is the discrete-time unit step sequence.

i) $x_1(t) = u(t) - u(t-1)$. (1 mark)

ii) $x_2(t) = 2\cos\pi t$. (2 mark)

iii) $x_3[n] = (-0.5)^n u[n]$. (2 mark)

d) Sketch the following signals:

i) $x_1[n] = \sum_{k=0}^{\infty} k\delta[n-k]$. $\delta[n]$ is the discrete-time unit impulse (sample). (2 marks)

ii) $x_2(t) = e^{-0.5t}\cos(\pi t - \pi/2)$. (2 marks)

iii) $x_3[n] = (0.5)^n$. (2 marks)

2 a) i) State the expression for the exponential Fourier series representation of a periodic signal. (2 marks)

ii) Define the Fourier coefficients in i) above. (2 marks)

b) Consider the signal shown in Figure 2b.

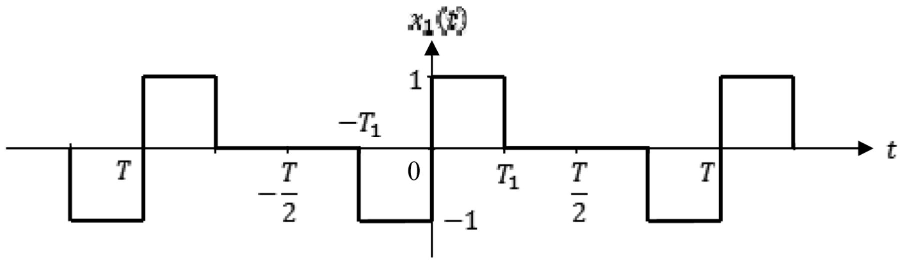

Figure 2b

i) Find the exponential Fourier series coefficients. (5 marks)

ii) Show that when $T_1 = \frac{T}{2}$,

$$a_k = \begin{cases} \frac{2}{jk\pi} & , \quad k \text{ odd}, \\ 0 & , \quad k \text{ even}. \end{cases}$$

(3 marks)

c) From the results of b) deduce the Fourier series representation of the waveforms shown in Figure 2c. (8 marks)

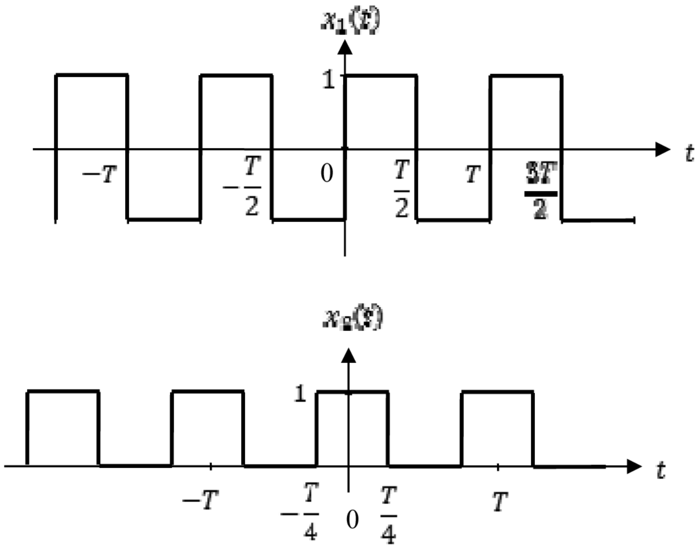

Figure 2c

3 a) Write the expression for the Fourier transform of an aperiodic continuous-time signal $x(t)$ and the inverse Fourier transform. (2 marks)

b) Find the Fourier transform of

i) $$x(t) = \begin{cases} +1 & , -\tau < t < 0 \\ -1 & , 0 < t < \tau, \\ 0 & \text{otherwise.} \end{cases}$$ (4 marks)

ii) $$h(t) = \begin{cases} 1 & , -\tau/2 < t < \tau/2, \\ 0 & , \text{otherwise.} \end{cases}$$ (4 marks)

c) Use the convolution property to sketch the inverse Fourier transform of

$Y(j\omega) = \dfrac{2}{j\omega}(\cos\omega - 1)\text{sinc}(\dfrac{\omega}{2}\pi)$. (5 marks)

d) i) Show that

$$F\left[\frac{dx(t)}{dt}\right] = j\omega X(j\omega)$$

where $X(j\omega)$ is the Fourier transform $x(t)$. (2 marks)

ii) Find the Fourier transform of the **derivative** of the signum function

$$sgn(t) = \begin{cases} -1 & , t < 0, \\ +1 & , t > 0. \end{cases}$$ (2 marks)

iii) Deduce the Fourier transform of the signum function. (1 mark)

4 a) i) Use Euler's formula to show that the Fourier series coefficients of the discrete time signal

$$x[n] = \cos\left(\frac{2\pi}{N}n\right)$$

are $a_1 = \frac{1}{2}$ and $a_{-1} = \frac{1}{2}$. Here, N is the period. (3 marks)

ii) Find the Fourier series coefficients of

$$x_1[n] = 2 + \cos\left(\frac{2\pi}{N}n\right) + \sin\left(\frac{2\pi}{N}n\right) + 4\sin\left(\frac{4\pi}{N}n + \frac{\pi}{2}\right).$$ (4 marks)

iii) Sketch the magnitude spectrum in ii). (2 marks)

b) Consider the discrete-time periodic square wave shown in Figure 4b.

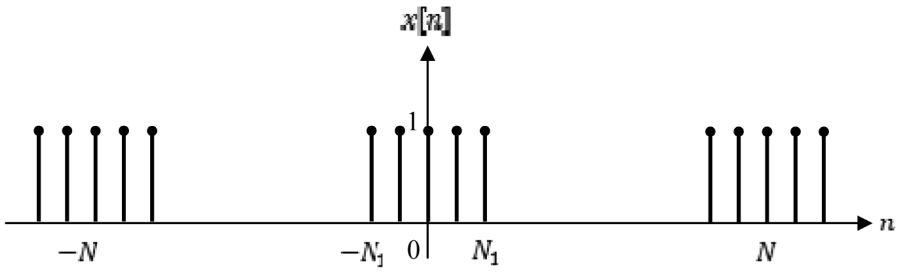

Figure 4b

i) Show that

$$a_k = \begin{cases} \frac{1}{N}\frac{\sin[2\pi k(N_1 + 1/2)/N]}{\sin(\pi k/N)} & , k \neq 0, \pm N, \pm 2N, \ldots \\ \frac{2N_1 + 1}{N} & , k = 0, \pm N, \pm 2N, \ldots \end{cases}$$ (4 marks)

ii) Sketch the magnitude spectrum for the case where $N_1 = 2$ and $N = 10$. (3 marks)

c) Find the discrete-time Fourier transform of

i) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$, (2 marks)

ii) $x_2[n] = \left(\frac{1}{2}\right)^{n-1} u[n-1]$. (2 marks)

5 a) i) Write the expression for the Laplace transform of $x[t]$. (2 marks)

ii) Obtain the Laplace transform and the region of convergence (ROC) of $x(t) = e^{-at}u(t)$. (2 marks)

iii) Deduce the Laplace transform of unit step function $u(t)$. (2 marks)

iv) Obtain the Laplace transform of

$$x(t) = \begin{cases} 1 & , 1 < t < 2, \\ 0 & , \text{otherwise.} \end{cases}$$ (2 marks)

b) i) Verify that if $x(t) \xrightarrow{L} X(s)$ with ROC = R then $-tx(t) \xrightarrow{L} \frac{dX(s)}{ds}$ with ROC = R. (2 marks)

ii) Obtain the Laplace transform of $te^{-at}u(t)$. (2 marks)

iii) Obtain the inverse Laplace transform of $X(s) = \frac{s}{(s+1)^2(s+2)}$, $\text{Re}\{s\} > -1$. (4 marks)

c) A system represented by $H(s) = \frac{1}{s+1}$, $\text{Re}\{s\} > -1$ is applied a step input. Obtain the expression for its output $y(t)$. (4 marks)

6 a) A continuous-time linear time-invariant system is described by the following equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} - 3y(t) = x(t)$$

i) Find the system function $H(s)$. (2 marks)

ii) Determine the impulse response $h_1(t)$ if the system is causal. (4 marks)

ii) Determine the impulse response $h_2(t)$ if the system is stable. (4 marks)

b) Consider the second-order RLC series network shownin Figure 6b.

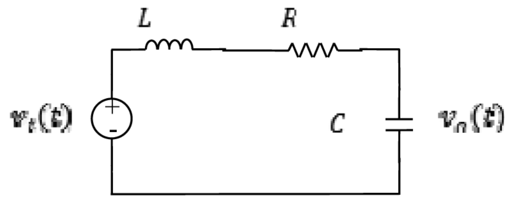

Figure 6b

i) Obtain an expression for the voltage-gain transfer function $G(s) = \dfrac{V_o(s)}{V_i(s)}$. (2 marks)

ii) Comparing $G(s)$ with the standard expression

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

show that

$$\omega_n^2 = \frac{1}{LC}, \text{ and } R = 2\zeta\sqrt{\frac{L}{C}}.$$ (2 marks)

iii) If $\omega_n = 1000$ rad/s and $L = 10$ mH compute the values of R to achieve damping ratios $\zeta = 0.25$ and $\zeta = 1.0$ respectively. (2 marks)

iv) Sketch the step response of the system for $\zeta = 0.25$ and $\zeta = 1.0$ respectively. (4 marks)

7 a) Write the expressions for

i) Convolution integral for continuous-time signals and systems, (2 marks)

ii) Convolution sum for the discrete-time signals and systems. (2 marks)

b) Consider the two signals

$x_1(t) = u(t) - u(t-1)$,

and

$x_2(t) = e^{-t}u(t)$.

Here, $u(t)$ is the continuous-time unit step function.

i) Use convolution to obtain an expression for $x(t) = x_1(t) * x_2(t)$. (6 marks)

ii) Sketch the result $x(t)$. (2 marks)

c) A system whose impulse response $h(n)$ shown in Figure 7c receives a step sequence, also shown in Figure 7c.

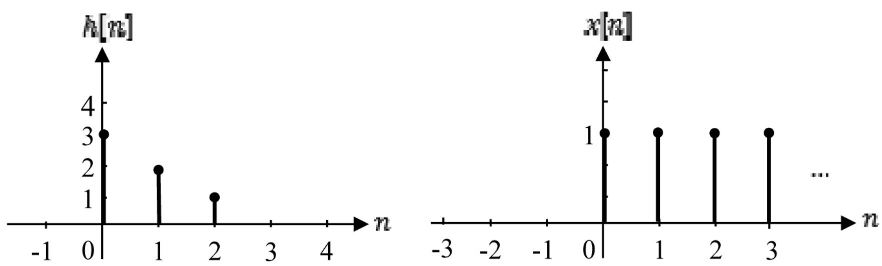

Figure 7c

i) Compute and sketch the system output. (4 marks)

ii) Is the system causal? Justify the answer. (2 marks)

iii) Is the system stable? Justify the answer. (2 marks)

8 a) Find the z-transform and the corresponding region of convergence of
   i) $x_1[n] = a^n u[n]$. Here, $u[n]$ is the discrete-time unit step sequence. (2 marks)
   ii) $x_1[n] = -a^n u[-n-1]$. (4 marks)

b) Consider the discrete-time signal

$$x[n] = \left(\frac{1}{3}\right)^2 u[n] + \left(\frac{1}{2}\right)^2 u[-n-1].$$

   i) Find the z-transform. (3 marks)
   ii) Sketch the pole-zero diagram and the region of convergence. (3 marks)

c) A discrete-time, linear, shift-invariant system is represented by

$$y[n+1] - \frac{5}{2} y[n] + y[n-1] = x[n]$$

where $x[n]$ is the input and $y[n]$ is the output. The system is stable.

   i) Show that the system function is $H(z) = \dfrac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}$. (2 marks)
   ii) Sketch the pole-zero diagram and the region of convergence. (2 marks)
   iii) Determine the unit sample output $h[n]$. (4 marks)

9 a) List four application areas where the concepts of signals and systems are used. (4 marks)

b) A radio transmission station may broadcast several channels using several carrier frequencies. Consider such an AM broadcasting station that uses two carrier frequencies $f_c = f_1 = 800$ kHz, $f_c = f_2 = 810$ kHz. For convenience, assume $s(t) = A\cos\omega_s t$. Assume that the AM signal model is
$x_c(t) = A_c[1 + s(t)] \cos(2\pi f_c t)$

   i) Obtain an expression for the spectrum of a signal $x(t) = A\cos\omega_0 t$. (2 marks)
   ii) Sketch the spectrum of the AM signal $x_c(t)$. (2 marks)
   iii) If the receiving antenna receives both the channels equally, sketch the spectrum of the received signal. (4 marks)
   iv) What must be the bandwidth of the filter to extract one channel? (2 marks)

c) A discrete-time feedback system is shown Figure 9c.

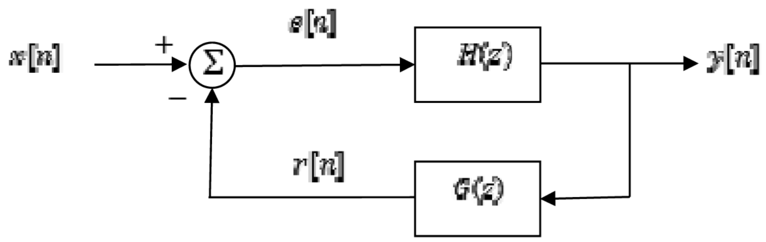

Figure 9c

Here, $H(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}}$, and $G(z) = 1 - bz^{-1}$. Find the range of real values of b for which the system is stable. (6 marks)