City & Guilds

9210-135
Graduate Diploma in Engineering
Mechanics of solids

Sample Paper

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, eraser, ruler

A work book is attached

General instructions
- This examination paper is of **three hours** duration.
- This examination paper contains **nine** questions.
- Answer any **five** questions.
- All questions carry equal marks. The maximum marks for each section within a question are given against that section.
- An electronic, non-programmable calculator may be used, but the candidate must show clearly the steps prior to obtaining final numerical values.
- Drawings should be clear, in good proportion and in pencil. Do **not** use red ink.

1 a) For a stress element with only two perpendicular direct tensile stresses, construct the Mohr's circle diagram, identifying the following on the diagram.

i) Two axes. (2 marks)

ii) Points and magnitudes of principal stresses. (3 marks)

iii) Point and magnitude of maximum shear stress. (3 marks)

b) A thin steel plate is subjected to two 60 MPa compressive stresses, two 80 MPa tensile stresses and shear stresses of magnitude 40 MPa as shown in Figure Q1. For the given state of stress, determine the normal and shearing stresses after the element has been rotated through

i) 30° clockwise (6 marks)

ii) 20° counter clockwise. (6 marks)

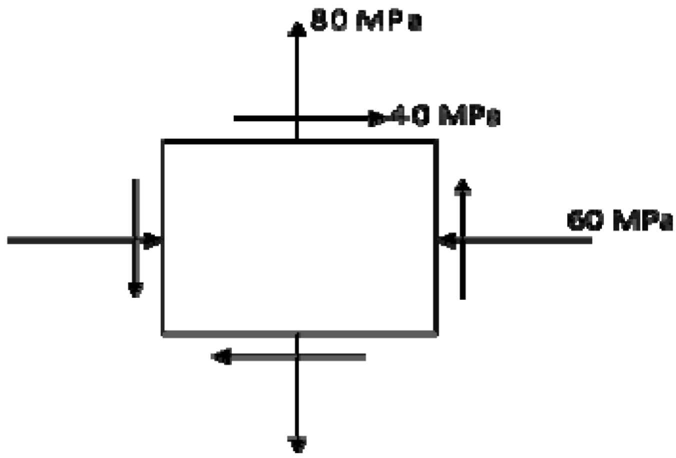

Figure Q1

2 a) Name different types of strain gauges. (4 marks)

b) Explain why electrical resistance strain gauges are very popular for strain measurements. (4 marks)

c) A rectangular rosette is used to obtain the state of plane stress at a point on the surface of a body. The strains measured along the three directions are

$_a = +300\,\mu$ $_b = 150\,\mu$ $_c = 0$.

The directions ***a*** and ***c*** are perpendicular to each other and the direction ***b*** is inclined at 45° to ***a*** and ***c***. The material of the body has a Young's modulus of 200 GPa and a Poisson's ratio of 0.3.

i) Considering the direction of ***a*** as that of the x-axis, determine the principal stresses and the principal direction at the point. (11 marks)

ii) If an equiangular rosette, with one direction the same as the x-axis had been used in the above case, what strain measurements would have been obtained? (1 mark)

3 a) Discuss the use of shear force and bending moment diagrams in engineering applications. (5 marks)

b) A beam AB simply supported at A and B is 8 m in length. Between A and C it carries a uniformly distributed load (UDL) with intensity of 6 kN/m and between D and B it carries a UDL with 4 kN/m. A concentrated load of 8 kN is applied at E. AC = CD = DE = 2 m and DB = 4 m. The above loading configuration is illustrated in Figure Q3.

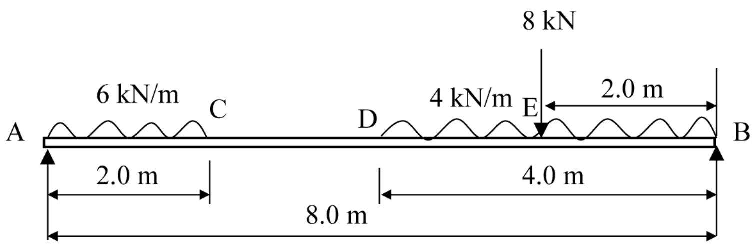

Figure Q3

i) Draw the shear force and bending moment diagram for the beam. (9 marks)

ii) Find the position and value of the maximum bending moment. (6 marks)

4 a) Define the term bending stress. (2 marks)

b) State the theory of simple bending explaining all the elements with relevant units. (3 marks)

c) State the assumptions made in the theory of simple bending. (3 marks)

d) A simply supported beam carries a UDL of intensity 3 kN/m over entire span of 6 m. The cross section of the beam is a T section having the dimensions as shown in Figure Q4. Calculate the maximum shear stress for the section of the beam. (12 marks)

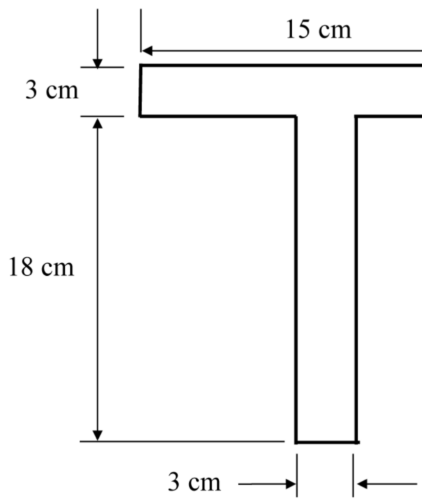

Figure Q4

5 a) Prove the relationship in the case of torsion in a circular shaft using

θ – Angle of twist in radians.
G – Modulus of Rigidity of shaft material.
l – Length of the shaft.
T – Shear stress at a radius R of the section. (5 marks)

b) Two shafts made of the same material, are connected in series. If each shaft transmits the same torque T, show that the total angle of twist θ, is given by,

$$\theta = \frac{T}{G}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2}\right]$$

Where T – Torque transmitted by each shaft.
l_1, l_2 – Respective length of the two shafts.
G – The modulus of the rigidity of the material.
J_1, J_2 – Respective of polar moment of inertia. (3 marks)

c) A solid shaft 50 mm diameter is to be replaced by a hollow shaft of the same material with internal diameter equal to half of the external diameter.
Find the diameter of the hollow shaft if the torque transmitted by both solid and hollow shafts is the same. (12 marks)

6 a) By giving suitable examples, describe how you would improve fatigue resistance in machine components. (6 marks)

b) A shaft is made of alloy steel for which $K_{IC} = 87\ MNm^{-3/2}$. Non-destructive testing by ultrasonic methods shows that the component contains a crack of up to $2a = 0.2$ mm in length. Laboratory tests show that the crack growth rate under cyclic loading is given by, $da/dN = A(\Delta K)^4$, where $A = 5 \times 10^{-13}\ (MNm^{-3/2})^{-4}m^{-1}$.
The component is subjected to an alternating stress of range $\Delta\sigma = 220\ MNm^{-2}$ about a mean stress of $\Delta\sigma/2$.
Given that $\Delta K = \Delta\sigma\sqrt{\pi a}$, calculate the number of cycles to failure. (14 marks)

7 a) Show that the deflection of a cantilever at its free end is given by the relation

$$y = \frac{wl^4}{8EI}$$

Where w = Uniformly distributed load per unit length of the cantilever
l = Span of the cantilever.
EI = Flexural rigidity of the cantilever. (8 marks)

b) A cantilever beam made of steel having a modulus of elasticity E = 200 GPa and a length of 0.5 m, is subjected to a point load of 5 kN at the free end of the beam. The beam is standard **I** – profile having a second moment of area $198 \times 10^{-8}\ m^4$ about the axis of bending.
Find
i) the slope of the support (8 marks)
ii) the maximum deflection of the beam. (4 marks)

8 a) Explain the meaning of the term discretization. (4 marks)

b) What are the types of boundary conditions used in Finite Element Analysis? (4 marks)

c) Explain the stiffness method. (4 marks)

d) Define shape function and explain the characteristics of shape function. (4 marks)

e) Explain types of elements, which can be used for finite element analysis. (4 marks)

9 The radial and tangential stresses in a thin rotating disk (under plane stress conditions) are respectively given by,

$$\sigma_{rr} = \frac{C_1}{2} + \frac{C_2}{r^2} - \frac{(3 + \nu)}{8} \rho \varpi^2 r^2 \ ;$$

$$\sigma_{\theta\theta} = \frac{C_1}{2} - \frac{C_2}{r^2} - \frac{(1 + 3\nu)}{8} \rho \varpi^2 r^2$$

Where '***r***' is the radial distance, 'ν' is the Poisson's ratio and 'ω' is the angular velocity. C_1 and C_2 are constants.
A thin circular disk of uniform thickness has an outside radius 0.8 m and a concentric hole of radius 0.2 m at the centre. Material properties of the disk are density (ρ) = 7850 kg/m^3 and Poisson's ratio (ν) = 0.25. If the disk is rotated at an angular velocity (ω) = 250 rad/s, determine the intensities of principal stresses when the disc is

a) a solid disk without the central hole

b) disk with the central hole. (20 marks)