City & Guilds

9210-136
Level 6 Graduate Diploma in Engineering
Control systems

Sample Paper

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, ruler

No additional data is attached

General instructions
- This examination paper is of **three hours** duration.
- This examination paper contains **nine** questions.
- Answer **any five** questions.
- All questions carry equal marks. The maximum marks for each section within a question are given against that section.
- An electronic, non-programmable calculator may be used, but the candidate **must** show clearly the steps prior to obtaining final numerical values.
- Drawings should be clear, in good proportion and in pencil. Do **not** use red ink.

1 a) A liquid filling apparatus is shown in Figure 1, where the inflow, outflow, cross-section areas of the container and outlet, and the liquid height are indicated as $f_i(t)$, $f_o(t)$, A, a, and $h(t)$ respectively. Outflow is proportional to liquid height and outlet cross-section area with a constant of proportionality k.

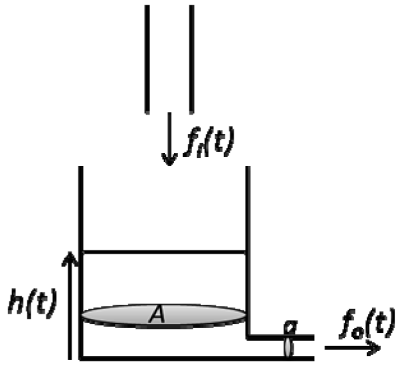

Figure 1

Derive the system transfer function for the liquid height $H(s)$ for a given inflow rate $F_i(s)$ and initial liquid level $h(0)$. (10 marks)

b) Calculate the stable liquid level h_s for $F = 100\ \text{cm}^3\text{s}^{-1}$, $a = 0.5\ \text{cm}^2$, and $k = 10\ \text{s}^{-1}$. (1 mark)

c) Sketch how the liquid level changes for the two cases (i) $h(o) > h_s$, and (ii) $h(o) < h_s$. (4 marks)

d) Estimate the time taken for the liquid level to settle. Assume $A = 25\ \text{cm}^2$ in addition to the given values earlier. (5 marks)

2 a) The transfer function of an open loop system is given by $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{2s + 1}{s^2 + 2s + 3}$.

Determine the ordinary differential equation of the open loop plant. (5 marks)

b) Calculate open loop poles and zeros and draw the pole-zero plot. (5 marks)

c) Draw the feedback control system for the plant with a single feedback gain K and derive the closed loop transfer function $G_c(s) = \dfrac{Y(s)}{R(s)}$, where $R(s)$ is the reference input to the plant. (5 marks)

d) Calculate K for critically damping the response. (5 marks)

3 a) Describe briefly the pole-zero cancellation in system approximation. (3 marks)

b) Approximate the system $G(s) \approx \dfrac{s^2 + 5s + 6}{(s + 2.1)(s^2 + 4s + 13)}$ to a second order system. (3 marks)

c) Determine the following attributes of the approximate plant.

i) Natural undamped frequency.
ii) Damping ratio.
iii) Rise time.
iv) Peak time.
v) 1% settling time.
vi) Peak overshoot. (12 marks)

d) Critically comment whether the system response is acceptable for an industrial temperature control system. (2 marks)

4 a) A system has a zero at –3, and poles at –2, –5, and $-1.5 \pm j\sqrt{15/2}$. Determine the transfer function of the system as a rational polynomial. (4 marks)

b) This plant is controlled by a feedback through a single gain K.
Determine the following.

i) Number of asymptotes and asymptote angles. (3 marks)

ii) Asymptote intersection point. (3 marks)

iii) Departure angle from the open loop pole $-1.5 + j\sqrt{15/2}$. (3 marks)

iv) Calculate the maximum stable feedback gain K_{max}. (3 marks)

c) Sketch the root locus of the controlled plant. (4 marks)

5 a) Explain briefly the following terms with regard to frequency response of systems.

i) Gain margin.

ii) Phase margin. (4 marks)

b) The frequency response of a controlled plant described by the transfer function

$G(s) = \dfrac{(s+3)(s+4)}{(s^3 + 10s^2 + 13s + 2)(s^2 + 13s + 2)}$ is shown below.

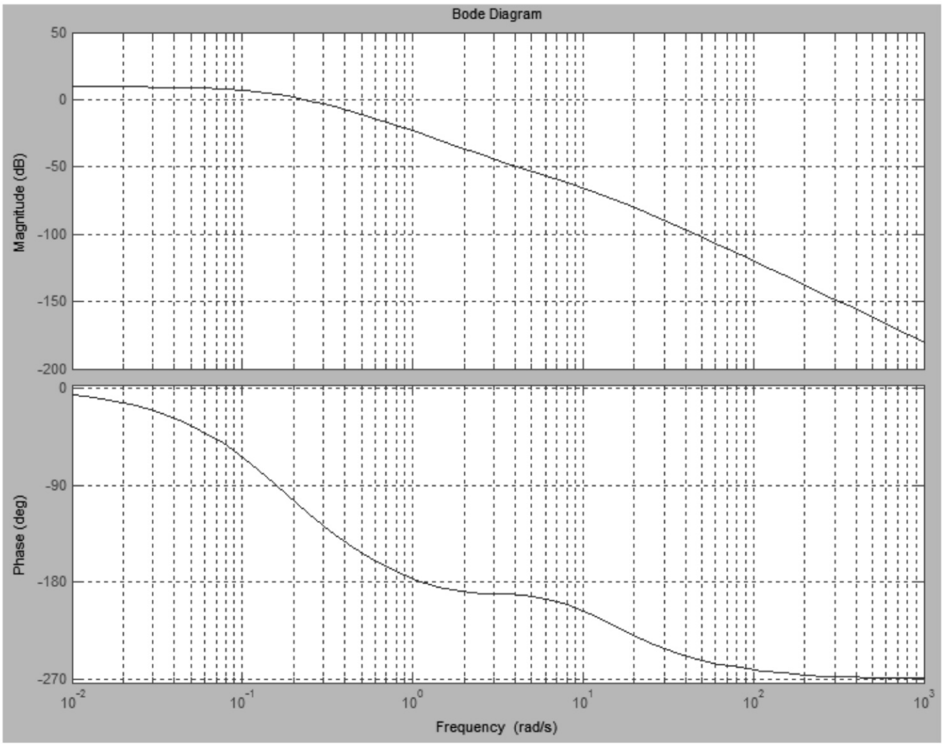

Figure 5

Following values for gain and phase can be read from the graphs.

Frequency	Gain	Phase
0.233 rad/s	1.0	–113.5°
1.135 rad/s	0.052	–180.0°

Table 5

Calculate the gain and phase margin of the system. (6 marks)

c) Comment whether the controlled system can track a reference signal, which contains frequencies up to 2 rad/s. (3 marks)

d) Calculate the forward gain to be added to increase the bandwidth to 3 rad/s. (4 marks)

e) Explain whether the phase margin will change after the bandwidth improvement in 4d) above. (3 marks)

6 a) Describe the three-term PID controller emphasizing the behaviour of the three terms separately. (6 marks)

b) A plant step response is shown in Figure 6a, where GM_{co} and f_{co} are gain margin and phase crossover frequency.

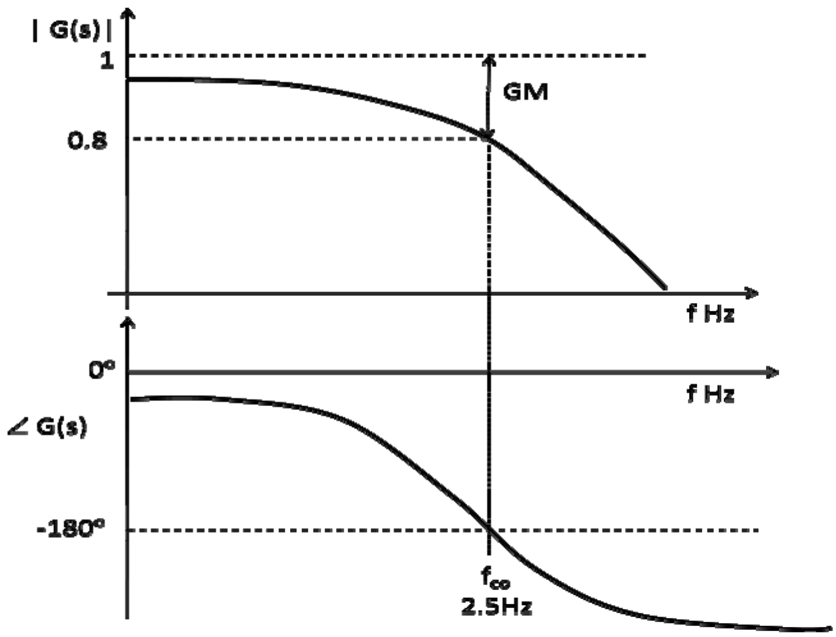

Figure 6a

A PID controller is to be designed for the plant.

controller	K_P	K_I	K_D	T_I	T_D
P	$0.5GM$	-	-	-	-
PI	$0.45GM$	$1.2\frac{K_P}{T_{co}}$	-	$0.8T_{co}$	-
PID	$0.6GM$	$2\frac{K_P}{T_{co}}$	$0.125\ K_P T_{co}$	$0.5\ T_{co}$	$0.125\ T_{co}$

Figure 6b

Use the Zeigler-Nichols PID tuning table given in Figure 6b and determine the following values.

i) Proportional gain.
ii) Derivative gain.
iii) Integral gain. (9 marks)

c) Write the PID controller transfer function with the gains calculated in 6b above. (3 marks)

d) Draw the feedback control system with the PID controller. (2 marks)

7 a) Define the observability and controllability of systems. (4 marks)

b) A plant is modelled by the following differential equation.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = u(t)$$

Derive the state space model of the plant. (8 marks)

c) Check whether the plant is observable. (4 marks)

d) Check whether the plant is controllable. (4 marks)

8 a) Describe briefly the following terms in relation to sensors and transducers.

i) Resolution. (2 marks)

ii) Dynamic range. (2 marks)

iii) Response. (2 marks)

iv) Calibration. (2 marks)

b) Describe the features of an OPAmp that qualify it as a key element in analogue controller design. (3 marks)

c) The following OpAmp circuit is used to measure the weight of three storage containers.

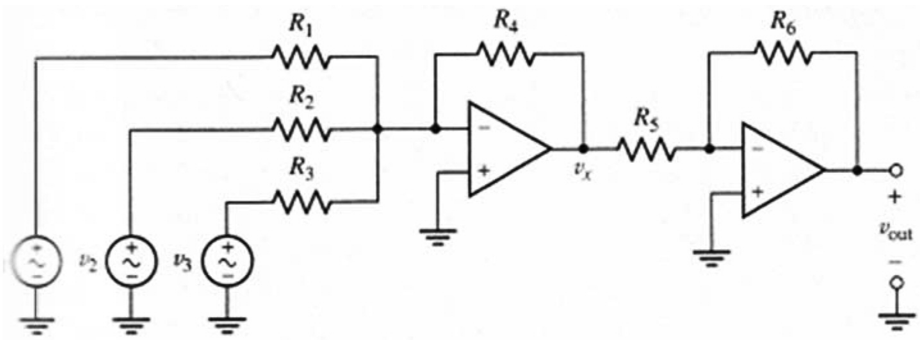

Figure Q8

Describe the functionality of the circuit. (3 marks)

d) Sensitivity of the sensors is 0.5 mV/kg. At a given moment, the three containers contain 75 kg, 90 kg, and 65 kg, respectively. Calculate the following assuming $R_1 = R_2 = R_3 = R_4 = 2.5\ k\Omega$, $R_5 = 25\ \Omega$, and $R_6 = 3\ \Omega$.

i) Output voltages of three sensors.

ii) Gain of two OpAmps.

iii) Voltage output of the circuit. (6 marks)

9 a) Describe the favourable features of analogue controllers compared with digital controllers. (4 marks)

b) Describe how analogue to digital converter (ADC) and digital to analogue converter (DAC) can be used to control an analogue plant using a digital controller. (4 marks)

c) Describe how sampling and zero-order hold device in digital control systems reduce relative stability of a feedback control system. (4 marks)

d) Explain how digitization can reduce accuracy of a feedback control system. (4 marks)

e) Digitize the analogue controller $u(t) = 3\dot{e}(t) + 1.5e(t)$ using a sampler with sampling interval 0.5 s. (4 marks)