9210-200
Post Graduate Diploma in Engineering
Engineering analysis

Sample Paper

You should have the following for this examination
- one answer book
- pens, pencils, ruler

A Reference Booklet is attached

General instructions
- This examination paper is of **three hours** duration.
- This paper contains **nine (09)** questions.
- Answer **any five (05)** questions.
- The marks allocated to each question or parts of the question are shown in the brackets in the right hand margin. They are given for guidelines only.

1 a) i) Show that the half range Fourier sine series expansion of
$f(x) = \cos x, \quad 0 \le x \le \pi.$
Is given by
$\cos x = \sum_{n=1}^{\infty} b_n \sin nx$
where $b_n = \begin{cases} 0 & n \text{ odd} \\ \dfrac{4n}{\{n^2 - 1\}\pi} & n \text{ even} \end{cases}$ (4 marks)

b) A square plate of length π is electrically charged and potential $u = u(x, y)$, at a point (x, y), with reference to a co-ordinate system satisfies

$$c^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \,,\ 0 \le x, y \le \pi$$

where c is a constant.

i) Given that $u(0, y) = 0$ for $0 \le y \le \pi$, . $u(x, 0) = 0$ for $0 \le x \le \pi$,
Taking $c = 1$, and by using the variables separable method show that u satisfies $u(x, t) = B \sin \omega x \,.\sinh \omega y$,
where B is an arbitrary constant and ω a variable. (4 marks)

ii) If further, $u(\pi, y) = 0$, for $0 \le y \le \pi$, show that u can be written in the form
$u(x, y) = \sum_{n=0}^{\infty} B_n \sinh ny \sin nx$ (4 marks)

iii) Determine the first two terms of the complete solution for $u(x, y)$ given that $u(x, \pi) = f(x)$ with $f(x)$ as given in a). (4 marks)

c) Express the complete solution found in b iii) for the case $c = 2$. (4 marks)

2 a) i) The current *i* in the LRC circuit shown in Figure 2.1 satisfies differential equation
$L \dfrac{di}{dt} + Ri + \dfrac{1}{c} \int i(t)dt = E(t)$

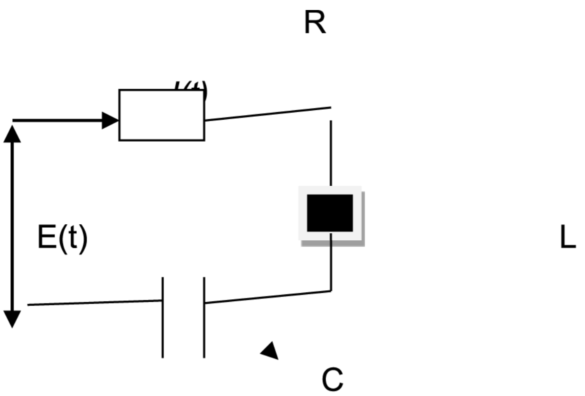


Figure 2.1

Show that *i* satisfies the above differential equation (2 marks)
Hence determine $i(t)$ for the following cases

ii) $L = 1$ H, $R = 2\,\Omega$, $C = 1/3$ F , and $E = e^{-t}$ V (6 marks)
with $i(0) = i'(0) = 0$

iii) $L = 1$ H, $R = 4\,\Omega$, $C = 1/4$ F , and $E = \sin t$ V (4 marks)

b) To solve the differential equation
$4x^2y'' + 4xy' - y = 0,$
with $y' = \dfrac{dy}{dx}$, $y'' = \dfrac{d^2y}{dx^2}$, it is assumed that $y = \sum_{i=0}^{\infty} a_n x^{c+n}$

i) Show that $c = \pm 1/2$. (4 marks)

ii) Hence determine the complete solution to the given differential equation. (4 marks)

3 The temperature u = u(x, t) of a metal rod is taken as follows: -

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{in } 0 < x < 1,\ t > 0$$

with initial and boundary conditions

$u(x,0) = 2x$ in $0 < x < 1/2$

$= 2(1 - x)$ in $1/2 < x < 1$

$u(0,t) = u(1,t) = 0$ $t > 0$

i) Derive a finite difference scheme for the numerical solution of the above problem and state the condition under which the solution will be stable. (2 marks)

ii) Determine u for $x = x_i = i \times \left(\frac{1}{4}\right)$, $i = 0, 1, 2, 3, 4,$

$t = t_j = j \times (1/64)$, $j = 0, 1, 2.$ (6 marks)

iii) Given that the finite difference scheme for the implicit method is given as

$- r\,u(x - h, t + k) + 2(1 + r)\,u(x, t + k) - r u(x + h, t + k)$

$= r\,u(x - h, t) + 2(1 - r)\,u(x, t) + r u(x + h, t)$

derive the following matrix for the above scheme, taking h = ¼, k = 1/16; and also by noting that u(x, t) is symmetric around x = ½, for all t.

$$\begin{bmatrix} 4 & -1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} u(1,k+1) \\ u(2,k+1) \end{bmatrix} = \begin{bmatrix} u(2,k) \\ 2u(1,k) \end{bmatrix}$$ (6 marks)

iv) Given that $\begin{bmatrix} 4 & -1 \\ -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 0.2857 & 0.0714 \\ 0.1429 & 0.2857 \end{bmatrix}$

obtain the numerical solution for

$x = x_i = i \times \left(\frac{1}{4}\right)$, $i = 0, 1, 2, 3, 4,$

$t = t_j = j \times (1/16)$, $j = 0, 1, 2.$ (6 marks)

4 a) Euler-Lagrange's equation is that given ø(x) satisfying necessary conditions an given F( ), I(ø) = Min $\int F(ø, Ø_x, x)dx$, satisfies (4 marks)

$$\frac{\partial F}{\partial ø} - \frac{d}{dx}\left[\frac{\partial F}{\partial ø_x}\right] = 0$$

Extend the above equation for a function F(ø, $Ø_x$ $Ø_y$, *x*, y)

b) A potential function V(*x*,*y*) satisfying Laplace's equation is defined on a triangular element with co-ordinates A, B and C as shown in Figure. Q 4.1.
Assuming V to take values V_1, V_2, and V_3 , at A, B, and C respectively, V satisfies

$$V(x,y) = \sum_{i=1}^{3} \alpha_i(x,y)V_i^e \, ; \, \alpha_i(xy) = \frac{1}{2A}[r_i + p_i x + q_i y]$$

and the shape matrix $C^{(e)}$ is given as $C^{(e)} = \{C_{ij}\}_{3x3}$ with $C_{ij}=(p_i p_j + q_i q_j)/(4A)$;
$r_i = x_2y_3 - x_3y_2$, $p_1 = y_2 - y_3$, $q_1 = x_3 - x_2$,
$r_2 = x_3y_1 - x_1y_3$, $p_2 = y_3 - y_1$, $q_2 = x_1 - x_3$.
$r_3 = x_1y_2 - x_2y_1$, $p_3 = y_1 - y_2$, $q_3 = x_2 - x_1$; A is the area of triangle.
It is given that V = [V_1, V_2, V_3] that minimizes W = (1/2) **V'CV** satisfies **CV'** = **0**.

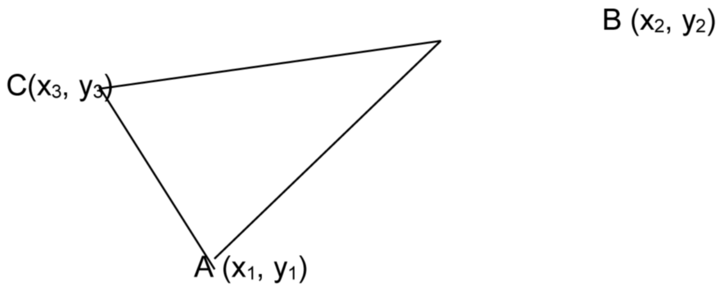


Figure Q 4.1

The potential V =V(x, y) satisfies the Laplace's equation in the region shown in Figure Q4.2, with dimensions.

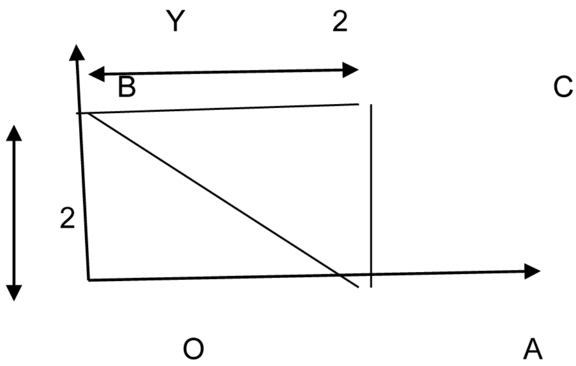


Figure Q 4

i) Determine the shape matrix for triangle OAB, with order of vertices as O ,A, B. (6 marks)

ii) Given the shape matrix for triangle ACB in that order as

$$\begin{bmatrix} 0.5 & -0.5 & 0 \\ -0.5 & 1.0 & -0.5 \\ 0 & -0.5 & 0.5 \end{bmatrix}$$

determine the assembled matrix of the system. (5 marks)

Iii) If V = 1, 2 at the vertices O and A, respectively, use the finite element method to find V at the vertices B and C. (5 marks)

5 a) A factory manufactures items of products A, and B, by utilizing machinery and labour. Profit made for a single item together with machine hours, and numbers of labour units required for an item of the two products are as shown in Table Q5.1. It is required to determine how many items of each product are to be manufactured in order to maximize the total profit per day.

	A	B
Profit per item (monetary units)	8	7
Machine hours needed per item	2	3
Labor units needed per item	4	3

Table Q5.1

Further, the total machine time available for one day is 80 hours and the total units of labour available for a day is 100. Further the management had decided to produce not more than 20 items of product A for a day.

i) Write down the linear program model for the above problem. (4 marks)

ii) Obtain the graphical solution to the above problem. (4 marks)

	A	B	C
Profit per item (monetary units)	8	7	6
Machine hours needed per item	2	3	2
Labor units needed per item	4	3	1

Table Q5.2

b) The management had decided to introduce another product C and amended the profit , machine and labour hours as shown in Table Q5.2. Further, the total machine time available for one day is amended as 100 hours and the total units of labour available for a day as 120. Further, the management had decided to produce not more than 20 items of product A per day

i) Write down the linear program model for the above problem. (4 marks)

ii) By performing one iteration show how to solve the problem in i). (8 marks)

6 a) Show by performing two iterations each of the following two methods to determine the value of x at which the function
$x^2 - 2\cos x - 2x$
Is a minimum at $0 < x < 1$.

i) Binary search method. (5 marks)

ii) Fibonacci method using $F_n = 13$, $n = 6$, given the following Fibonacci values. (5 marks)

Value of n	**Fibonacci Number F_n**
4	5
5	8
6	13

b) The transient currents (i_j, $j = 1, 2, 3$) in the LC network shown in Figure Q6 satisfy differential equations

$$Li_1'' + \frac{1}{C}(i_1 - i_2) = 0$$

$$Li_2'' + \frac{1}{C}(i_2 - i_3) - \frac{1}{C}(i_1 - i_2) = 0$$

$$Li_3'' + \frac{1}{C}i_3 - \frac{1}{C}(i_2 - i_3) = 0 ,$$

$$\text{with} \quad \frac{d^2 i_j}{dt^2} = i_j''$$

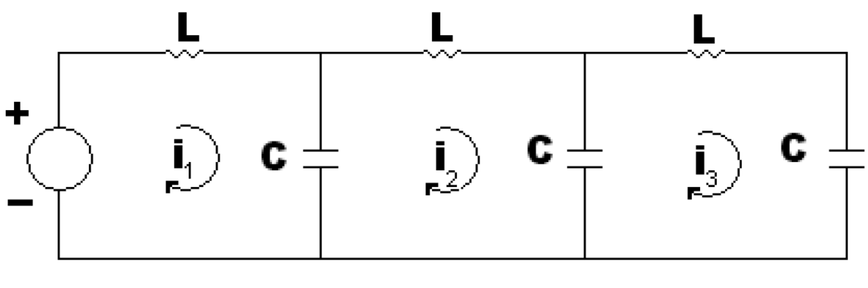


Figure Q6

i) By taking $C = L = 1$ and $i_j'' = -\omega^2 i_j$, $j = 1, 2, 3$ show that above system of equations can be written as
$\mathbf{Bi} = \lambda\,\mathbf{i}$
where $\mathbf{B}$ is a symmetric matrix and determine $\mathbf{B}$. (5 marks)

ii) By taking the initial vector (-1 1 -1) and by doing at least two iterations show how to determine the maximum frequency ω. (5 marks)

7 a) i) Obtain the Fourier Transform F{f(x)} of the unit step function.

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

(3 marks)

ii) Hence determine the Fourier Transform of the unit impulse function $\gamma(0)$. (2 marks)

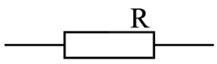
iii) Given that $F\{e^{-|x|}\} = \dfrac{2}{\omega^2 + 1}$,find the inverse Fourier Transform of

$$F(\omega) = \frac{12 + \omega^2}{4 + \omega^2}$$

(5 marks)

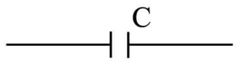
b) Properties and Fourier Transforms of electrical elements are given as follows

Resister



$$v(t) = Ri(t)$$
$$v(w) = RI(w)$$

Capacitor



$$i(t) = C\frac{dv}{dt}$$
$$F(w) = jwCV(w)$$

For the circuit shown in Figure Q7

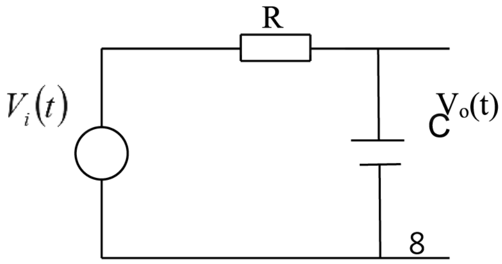


Figure Q7

i) Show that the transfer function = $\dfrac{V_0(w)}{V_i(w)} = \dfrac{1/j\omega c}{R + 1/j\omega c}$ (5 marks)

ii) Hence determine the output when the input is a unit impulse. (5 marks)

8 a) i) Bottles made by a company are expected to have diameters which are normally distributed with mean 10 cm and standard deviation 0.2 cm. Show that the probability a bottle taken at random from the produce of the company will have diameter between 9.8 and 10.2 cm is approximately 0.7. (4 marks)

ii) Taking that a bottle with diameter falling outside the range 9.8 to 10.2 cm to be defective determine the probability that out of 5 bottles picked at random from the produce of this company there will be, (i) no bottles, (ii) 1 bottle, (iii) more than 1 bottle, that are defective. (6 marks)

iii) If for each 5 bottles produced, the company makes a loss of $. 100, and 200 respectively, for 1 defective bottle, and more than 1 defective bottles, determine the average extra loss made for each set of 5 bottles. (4 marks)

b) The quality of bottles of the same type manufactured by two companies A and B were assessed as follows: (6 marks)

	Company A	**Company B**
Good	30	20
Bad	20	10

Examine at 95 % confidence limit whether the products of the two companies have the same quality standards.

[A contingency table is given as T = $\{a_{ij}\}_{nr X nc}$, with $fr_i = \sum_{j=1}^{nc} a_{ij}$

$fc_j = \sum_{i=1}^{nr} a_{ij}$ and f = sum of all a_{ij} then probability matrix P = $\{ p_{ij}\}_{nr X nc}$

with $p_{ij} = fr_i \times fc_j/f$ and $\chi^2 = \Sigma\Sigma\dfrac{(a_{ij} - p_{ij})^2}{p_{ij}}$ and

degree of freedom = (nr-1) × (nc-1)].

9 a) i) Write down without deriving normal equations to fit a straight line to pairs of values (x_i , y_i) , i = 1, 2, …, n. (2 marks)

ii) If n is an odd integer and $x_{i+1} - x_i = h$, a constant for all i = 1, 2, …, n − 1 show how normal equations can be modify to make calculations easy. (3 marks)

b) Figures of electrical power production from renewable sources for years 2000 to 2010, in USA is given in Table Q 9 as total energy resulting from uses of hydro, geo- thermal, bio-mass, solar and wind, sources.
Followed by results of ANOVA analysis of the values. It is required
To fit a realistic linear model to forecast the total production.

i) From Tables of Part A of the ANOVA analysis explain briefly why some power sources can be dropped from the linear model. (3 marks)

ii) Using results of Part B of the analysis write down a linear equation to model total production as a function of energy production from hydro and bio-mass sources. (3 marks)

iii) Using results of Part C of the analysis write down a linear equation to model hydro power production as a function of year number. (3 marks)

iv) Using results of Part D of the analysis write down a linear equation to model bio-mass power production as a function of year number. (3 marks)

v) Using results of iii) and iv) above estimate possible total power production for 2019. (3 marks)