

**9210-21%**

## **Level 7 Post Graduate Diploma in Engineering**

Fields and network theory

GLA d'Y DUdYf

**You should have the following for this examination**

- one answer book
- non-programmable calculator
- pen, pencil, ruler

**No additional data is attached**

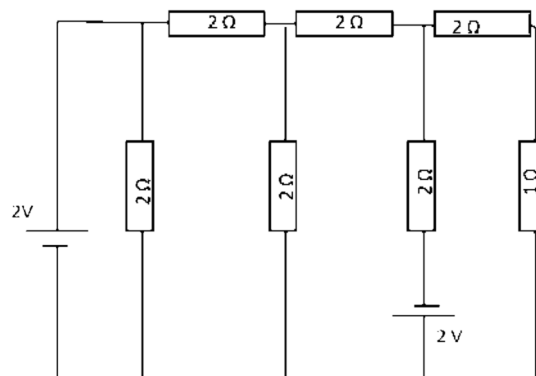
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### **General instructions**

- This paper consists of **eight** questions.
- Answer **five** questions only, selecting at least **two** from each section.

## Section A

- 1 a) i) Identify a set of independent loops and mark corresponding loop currents on a diagram for the network shown in Figure Q1 without making modifications to the network. (3 marks)
- ii) Write a sufficient number of mesh equations to obtain all branch currents. Solution of the equations is not required. (4 marks)
- b) Use nodal analysis to solve the network in Figure Q1 to obtain node voltages. (7 marks)
- c) Explain the following statements.
- i) In certain situations it is easier to solve a network using mesh analysis rather than using nodal analysis. (3 marks)
- ii) Sparsity can be effectively used to reduce computer time in solving large networks. (3 marks)



**Figure Q1**

- 2 a) Using a suitable notation, derive an expression for the r.m.s. value of a periodic, non-sinusoidal voltage consisting of the fundamental, the 3rd harmonic and the 5th harmonic. (4 marks)
- b) A 50 Hz ac voltage waveform spoiled with 3rd and 5th harmonics is given by  $v(t) = 100 \sin \omega t + 20 \sin 3\omega t + 10 \sin 5\omega t$ . When this voltage is applied across a pure capacitance of  $C$ , an r.m.s. current of 0.85 A is measured.
- i) Calculate the r.m.s. voltage across the capacitor. (2 marks)
- ii) What is the apparent value of the capacitor based on r.m.s values of voltage and current? (4 marks)
- iii) Calculate the true capacitance  $C$ . (4 marks)
- iv) The same voltage applied across a non-linear resistance produces a current  $i(t) = .40 \sin \omega t + 5 \sin 3\omega t + 2 \sin 5\omega t$ . Calculate the power consumed by the resistor. (4 marks)
- v) What is the effective resistance of the resistor in iv) above? (2 marks)

- 3 a) Calculate the Thevenin's equivalent of the network shown in Figure Q3 as seen from the terminals a and b. (10 marks)

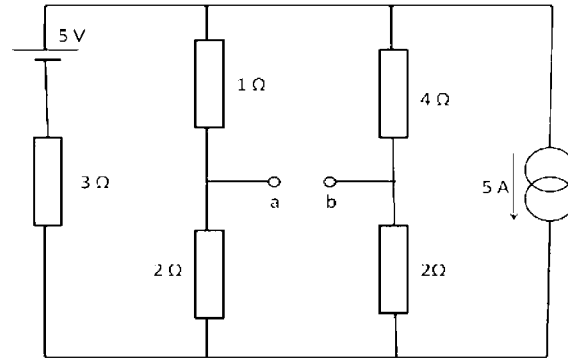


Figure Q3

- b) A 50-turn coil of axial length 8 mm is wound around the centre of a long ferrite rod having a diameter of 10 mm. Ferrite material has a relative permeability of 12. The coil is placed in a magnetic field with magnetic flux density of  $10^{-10}$  T (r.m.s. value) and a frequency of 200 kHz in such a way that the induced voltage has its maximum value. Determine the Norton equivalent circuit of the coil placed in the magnetic field in this manner. (Hint: Ignore the resistance of the coil and calculate the inductance of the coil either using a standard equation or from first principles.) (10 marks)

- 4 a) State Tellegen's theorem for a n-branch network. (4 marks)  
 b) Show that the 3-branch network shown in Figure Q4a satisfies Tellegen's theorem. (4 marks)

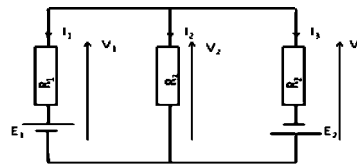


Figure Q4a

- c) Compare the networks in Figures Q4b and Q4c with the network in Figure Q4a and verify that all three are adjoint networks of each other. Apply Tellegen's theorem for currents in network Q4b with voltages in network Q4a and repeat this with currents in Network Q4c and voltages in network Q4a. (6 marks)

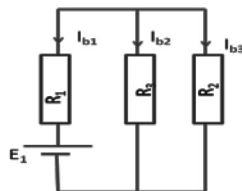


Figure Q4b

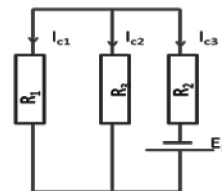


Figure Q4c

- d) Analyze the equations in b) and c) above and evaluate the consistency of Tellegen's theorem with the superposition theorem. (6 marks)

## Section B

- 5 a) Two infinite thin conducting plates A and B are placed horizontally parallel to each other separated by a distance  $2d$ . The upper half of the space between the plates is filled with a charge free dielectric material of relative permittivity  $\epsilon_r$ . The lower space is filled with another insulating material having a relative permittivity of 1 and uniform charge density of  $\rho$ . The bottom plate B is earthed and is at zero potential.
- Give a sketch of this arrangement. (4 marks)
  - Taking plate B as the x-axis, find the y-component of the electric flux density  $D(x,y)$  as a function of  $y$  for the space between the plates. (4 marks)
  - Also give the Electric field  $E(x,y)$  and the potential  $V(x,y)$  as a function of  $y$ . (4 marks)
  - Calculate the surface charge density on the plate B. (4 marks)
- b) A point charge  $q$  is placed along the x-axis at the point  $(10,0)$  in front of a conducting sphere having a radius of 5. The centre of the sphere is the origin. Find the position and size of the image charge that could be used to obtain the electric field of this arrangement. (4 marks)
- 6 a) Explain how the spin movement of electrons in an atom contributes to the magnetic dipole moments of the atom. (4 marks)
- b) i) Prove that the M-H relationship for paramagnetic materials is given through  $M = N\beta \tanh\left(\frac{\mu_0\beta H}{kT}\right)$ , where  $N$  is the number of spins per unit volume,  $\beta$  the magnetic dipole moment associated with the spin and  $T$  the absolute temperature (5 marks)
- ii) Extend the above relationship in for ferromagnetic materials and obtain the Curie law  $\chi_m = \frac{C}{T-\theta}$  for ferromagnetic materials. (5 marks)
- c) The magnetic circuit shown in Figure Q6 is formed from a thick plate. All the distances shown are in mm. The coils shown in the diagram are connected in series. The current directions are as shown in the diagram and the current is such that each coil produces 2000 A-turns. Calculate the magnetic flux density in the central limb if the plate material has a relative permeability of 1000. (6 marks)

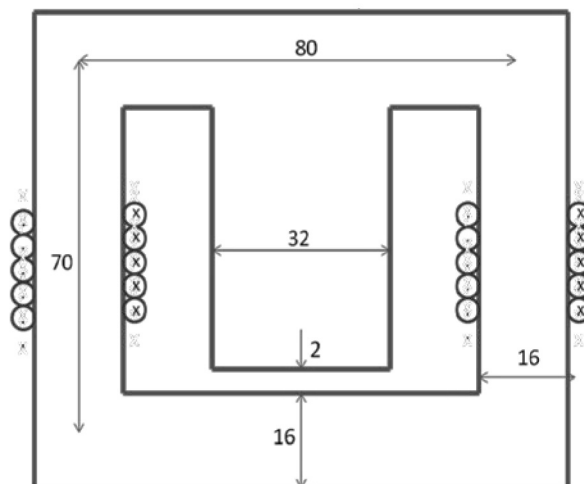


Figure Q6

- 7 The electrons emitted by an electron gun are accelerated in the horizontal direction for them to achieve a horizontal velocity of  $v$ . The beam then enters the space between two parallel plates 10 cm apart at a point just above the bottom plate. The plates are positioned in such a way that by applying a variable potential difference  $V$  across the plates, an electric field in the vertical direction can be created. A fixed uniform magnetic field also exists between the plates and is directed perpendicular to the electron beam and the electric field produced by the parallel plates. The electrons are un-deflected when the potential difference between plates  $V$  is equal to 10 kV. It is also observed that for  $V = 0$ , the beam takes a semi-circular path between the plates just avoiding both the plates.

$$\begin{aligned} \text{electron charge} &= 1.6 \times 10^{-19} \text{ C,} \\ \text{electron rest mass} &= 9.1 \times 10^{-31} \text{ kg} \end{aligned}$$

- a) Draw two diagrams showing the path of the electron beam between the plates for the two different values of  $V$ . In your diagrams, you must indicate the directions of the magnetic and electric fields between the plates. (4 marks)
- b) Write equations considering the deflective forces acting upon the beam for  $V = 0$  and for  $V = 10 \text{ kV}$ . (6 marks)
- c) Calculate the value of the magnetic flux density  $B$  and the velocity of the electrons entering the magnetic field area. (6 marks)
- d) Calculate the electric potential accelerating the electrons. (4 marks)
- 8 a) Grid points in a two dimensional rectangular grid spanning a charge free dielectric medium are considered. Consecutive grid points in  $x$ -direction as well as in  $y$ -direction are 'h' apart from each other. If the voltages at the points  $(0,h)$ ,  $(h,2h)$ ,  $(2h,h)$  and  $(h,0)$  are denoted by  $V_{01}$ ,  $V_{12}$ ,  $V_{21}$  and  $V_{10}$  respectively, obtain an approximation for the voltage  $V_{11}$  at the point  $(h,h)$  using Laplace's equation. (5 marks)
- b) Cross section at the centre of a capacitor on  $x$ - $y$  plane is shown in Figure Q8. The length of the capacitor in the  $z$ -direction is large compared to its  $x$ - and  $y$ -dimensions. The top and bottom electrodes are at voltages 10 V and 0 V respectively. It is required to find the voltages at points 1 to 13 marked on Figure Q8 using Laplace's equation in its numerical form. The voltage at the point  $k$  is denoted by  $V_k$ .
- i) Considering the symmetry of the arrangement predict  $V_4$ ,  $V_5$ ,  $V_6$ ,  $V_7$ ,  $V_8$ ,  $V_9$  and  $V_{10}$ . (2 marks)
- ii) What can you say about how  $V_1$  and  $V_3$  are related? Establish a similar relationship between  $V_{11}$  and  $V_{13}$ . (4 marks)
- iii) Using the equation derived in a or otherwise, show that  $V_2 + V_{12} = 10$ . and also that  $V_1 + V_{11} = 10$ . (4 marks)
- iv) Calculate  $V_{12}$  by selecting 4 points around point 12 where the potentials are known, Find all the other unknown potentials. (5 marks)

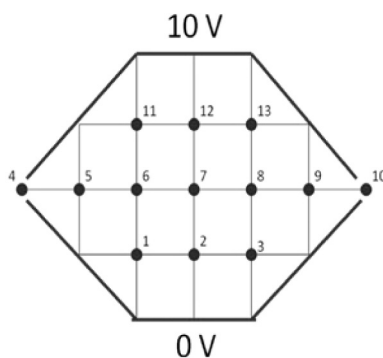


Figure Q8