9210-215
Level 7 Post Graduate Diploma in Engineering
Modern control systems

Sample Paper

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, ruler

No additional data is attached

General instructions
- This paper consists of **eight** questions over four pages.
- Answer any **five** questions.
- This examination paper is of **three** hour duration.

1 a) A plant is described by $2\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = 4u(t)$.

Derive the open loop transfer function $G(s) = \frac{Y(s)}{U(s)}$ of the plant. (5 marks)

b) Determine the open loop poles of the plant and comment on the open loop plant stability. (5 marks)

c) Determine the step response of the open loop plant. (5 marks)

d) Determine the steady state response of the open loop plant. (5 marks)

2 a) An open loop plant is given by $G(s) = \frac{1}{s^2 + 5s + 4}$. Show that the closed loop transfer function through a feedback gain K is given by $G_c(s) = \frac{G_o(s)}{1 + KG_o(s)}$. (5 marks)

b) Determine the closed loop poles of the plant as a function of feedback gain K. (5 marks)

c) Determine the values for feedback gain K > 0 to for the following system responses

i) Critically damped.

ii) Oscillatory stable. (5 marks)

d) Calculate the condition for the feedback gain to make the closed loop plant unstable. (5 marks)

3 a) A second order control system is given by the transfer function $G(s) = \frac{24}{s^2 + 3s + 43}$.
Determine the following:

i) Natural undamped frequency ω_n. (2 marks)

ii) Damping ratio ζ. (2 marks)

iii) Nature of step response. (1 mark)

b) Describe how the response changes based on the value of damping ratio ζ. (5 marks)

c) Calculate the damped frequency, and determine the following for unit step input. (2 marks)

i) Peak time. (1 mark)

ii) 1% settling time. (1 mark)

iii) Peak overshoot. (1 mark)

d) Explain why a second order system is a popular choice in controller design. (5 marks)

4 a) A feedback control system through a single feedback gain is shown in Figure 4.

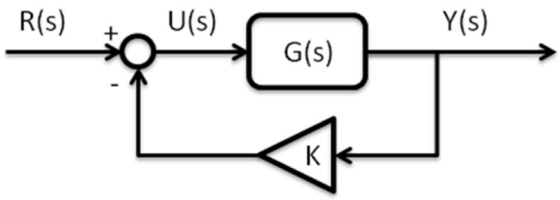

Figure 4

i) Derive the closed loop transfer function and show that closed loop poles can be positioned using feedback gain K. (2 marks)

ii) Derive the gain and phase equations for the closed loop poles. (4 marks)

b) For the open loop plant $G(s) = \dfrac{1}{(s+3)(s^2+6s+20)}$. Determine the following:

- i) Open loop poles.
- ii) Parts of the root locus on the real axis.
- iii) Asymptote angles.
- iv) Asymptote intersection point. (4 marks)

c) Sketch the root locus. (5 marks)

d) Determine the maximum stable feedback gain. (5 marks)

5 a) The Bode plots (Magnitude and frequency) of the plant $G(s) = \dfrac{32(s+2)}{s^4+2s^3+8s^2+12s+5}$ are shown in Figure 5.

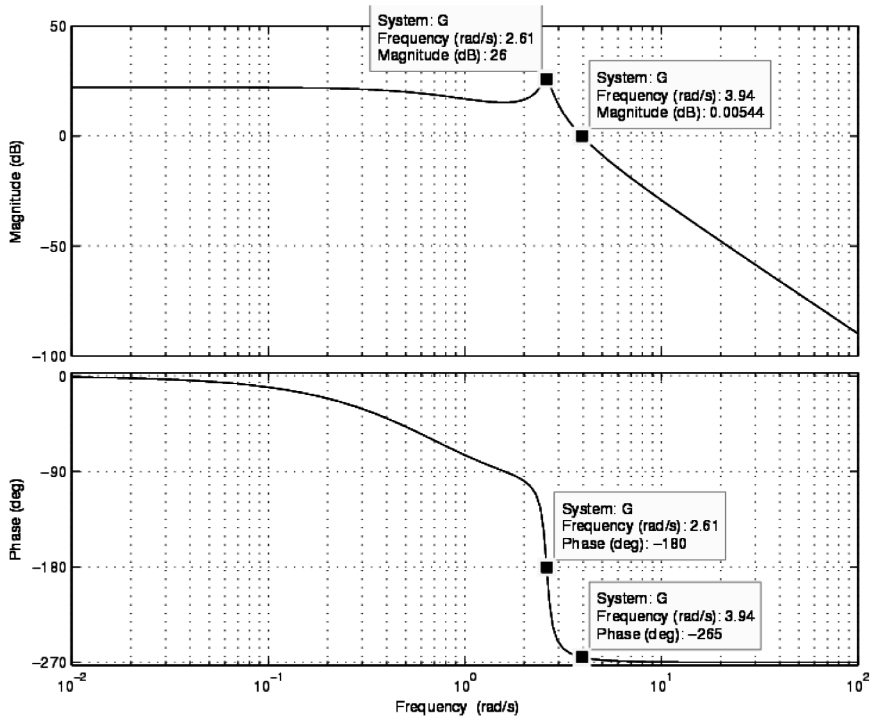

Figure 5

Determine from the plots the system bandwidth (2 marks)
And comment about the stability of the plant. (3 marks)

b) i) Calculate the required absolute value of gain K, so that the plant KG(s) has 10dB gain margin. (2 marks)

ii) Describe how the system response changes as a result of this modification. (3 marks)

c Determine the DC gain of the modified plant, and the phase at 2 rad/s. (5 marks)

d Referring to the Bode plots in Figure 5, calculate the phase requirement of a suitable compensator to have a 45° phase margin while keeping the original bandwidth unchanged. (5 marks)

6 a) Briefly explain the characteristics of proportional, derivative, and integral control actions of popular PID controller. (5 marks)

b) The Bode plots of the plant $G(s) = \dfrac{(s+5)}{s^4 + 16s^3 + 41s^2 + 28s + 4}$ are shown in Figure 6,

in which a phase lag of –180° is observed at 1.81 rad/s.

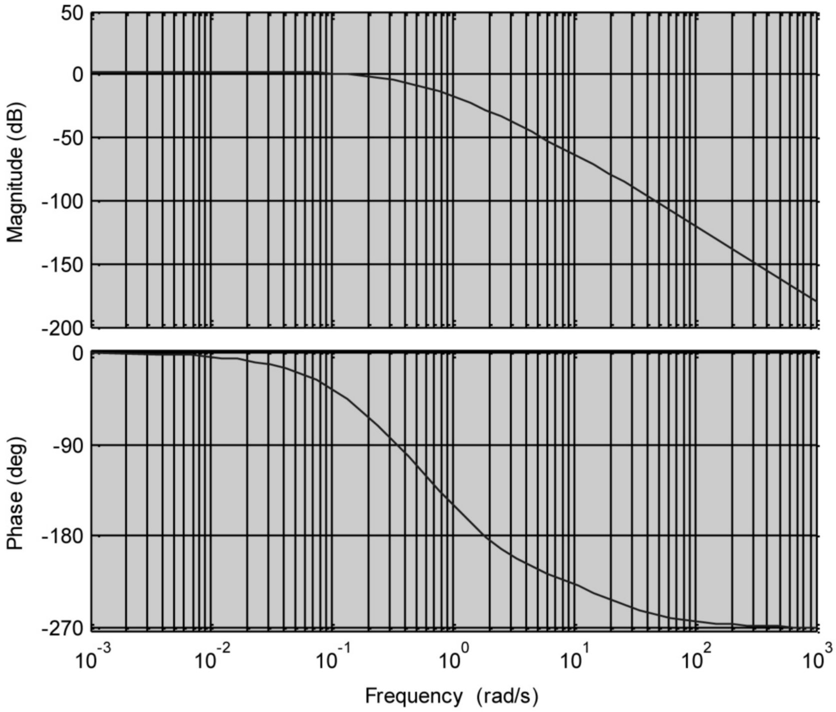

Figure 6

controller	K_P	K_I	K_D
P	$0.5GM_{co}$	-	-
PI	$0.45GM_{co}$	$1.2\frac{K_P}{T_{co}}$	-
PID	$0.6GM_{co}$	$2\frac{K_P}{T_{co}}$	$0.125\ K_P T_{co}$

Table 6: Zeigler-Nichols PID tuning chart

Determine the followings:

i) Gain at crossover frequency GM_{co}. (2 marks)

ii) Period of crossover frequency T_{co}. (3 marks)

c) Calculate PID gains using Zeigler-Nichols method for the above plant. (5 marks)

d) Describe why the PID controller is the most popular in industrial process control. (5 marks)

7 a) Describe the following concepts in modern control systems.

i) Observability. (2 marks)

ii) Controllability. (3 marks)

b) Develop the state space model for the system described by the differential equation $\ddot{y}(t) + 5\dot{y}(t) + 3y(t) = 2u(t)$ and identify the following: (2 marks)

i) System matrix ***A(t)***.

ii) Drive matrix ***B(t)***.

iii) Output matrix ***C(t)***. (3 marks)

c) Analyze the observability of the system. (5 marks)

d) Analyze the controllability of the system. (5 marks)

8 a) A plant is modelled by the following state space model.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = [1 \;\; 0]$$

Determine the system transfer function. (5 marks)

b) Derive the state space model with the state feedback control $u(t) = -K\mathbf{x(t)}$. (5 marks)

c) Determine the characteristic equation of the system with the state feedback control. (5 marks)

d) Calculate the state feedback gains for the control system to have poles at -5 and -3. (5 marks)