You should have the following for this examination
• one answer book
• scientific calculator

No additional data is attached

General instructions
• This examination paper is of three hours duration.
• This paper contains nine (09) questions.
• Answer five (05) questions.
Section A

1 Three bars 1, 2 and 3 are connected together, as shown in Figure Q1 and a force of 2.4 N is applied in the x direction at node 2. Consider each element to be 600 mm long, Modulus of Elasticity $E = 20 \text{ MPa}$ and cross sectional area $A = 3 \text{ cm}^2$ for each element 1 and 2, Modulus of Elasticity $E = 10 \text{ MPa}$ and $A = 6 \text{ cm}^2$ for element 3, and nodes 1 and 4 are fixed. Determine the following.

a) Global stiffness matrix. 

b) Displacements of nodes 2 and 3.

c) Reactions at nodes 1 and 4.

![Figure Q1](image)

2 a) Considering a basic element in the x – y plane (as shown in Figure Q2a) with positive nodal forces, derive an equation to determine the axial stress in a bar if for a bar, the relationship between the local forces and the local displacements in the respective directions is given by

\[
\begin{bmatrix}
  f_{1x} \\
  f_{2x}
\end{bmatrix} = \frac{AE}{L} \begin{bmatrix}
  1 & -1 \\
  -1 & 1
\end{bmatrix} \begin{bmatrix}
  d_{1x}^* \\
  d_{2x}^*
\end{bmatrix}
\]

Where

\[
d_{1x}^* = d_{1x} \cos \theta + d_{1y} \sin \theta
\]

\[
d_{2x}^* = d_{2x} \cos \theta + d_{2y} \sin \theta
\]

![Figure Q2a](image)
b) Figure Q2b, shows a bar element for stress evaluation, which makes an angle of 60° degrees with the 'X' axis. If its cross sectional area $A = 4 \times 10^{-4}$ m$^2$, modulus of elasticity $E = 208$ GPa, and length $L = 2$ m, find the axial stress assuming the global displacements have been previously determined to be $d_{1x} = 0.20$ mm, $d_{1y} = 0.0$, $d_{2x} = 0.40$ mm and $d_{2y} = 0.60$ mm.  

![Figure Q2b](image)

3 a) Derive an expression for the stiffness matrix for a 2-D truss element.  
b) Derive the strain displacement matrix for a 1-D linear element and show that $\sigma = E[B][u]$.  

4 Consider the compound bar shown in Figure Q4, which is made of Aluminium (1-2) and Steel (2-3), with length 150 mm and 300 mm respectively and the following cross sectional areas and Moduli of elasticity.

<table>
<thead>
<tr>
<th>Material</th>
<th>Cross-sectional Area</th>
<th>Modulus of Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>$A_1 = 1000$ mm$^2$</td>
<td>$E_1 = 80 \times 10^9$ N/m$^2$</td>
</tr>
<tr>
<td>Steel</td>
<td>$A_2 = 200$ mm$^2$</td>
<td>$E_2 = 210 \times 10^9$ N/m$^2$</td>
</tr>
</tbody>
</table>

An axial load of $P = 150$ N is applied in the x direction. 
Applying Boundary conditions, determine 
a) nodal displacements  
b) stress in each material  
c) reaction forces.
The cantilever beam of length 'L' shown in Figure Q5 is subjected to a concentrated load 'P' at its free – end and an uniformly distributed load 'w' per unit length acting over the whole span of the beam. Determine,

a) the free – end displacements and

b) the nodal forces

![Figure Q5](image1)

The thin (steel) plate in Figure Q6 has uniform thickness, t = 1 cm. Its material has a Young's Modulus E = 200 GPa and weight density $\omega = 7850 \text{ kg/m}^3$. In addition to its self-weight, the plate is subjected to a point load $P = 100 \text{ N}$ at its midpoint. The area at the midpoint of the plate is 3 cm$^2$.

a) Model the plate with two finite elements.

b) Write down expressions for the element stiffness matrices and element body force vectors.

c) Assemble the structural stiffness matrix K and global load vector F.

d) Using the elimination approach, solve for the global displacement vector Q.

![Figure Q6](image2)
7 A 45° strain rosette is used to estimate stresses on a beam. The normal strains obtained are as given in Figure Q7. If the Modulus of elasticity and Poisson's ratio of the beam are 5 GPa and 0.25 respectively, determine the major and minor principal stresses. Assume plain strain condition. (20 marks)

8 Briefly explain the following:
   a) Constant strain triangle (5 marks)
   b) Serendipity family of element (5 marks)
   c) Isoparametric element (5 marks)
   d) The convergence criteria for non-linear solutions (5 marks)

9 a) State the purpose of the Gauss elimination method. (5 marks)
   b) Solve the simultaneous set of equations using the Gauss elimination method
      \[\begin{align*}
      2x_1 + 2x_2 + x_3 &= 9 \\
      2x_1 + x_2 &= 4 \\
      x_1 + x_2 + x_3 &= 6
      \end{align*}\] (15 marks)