City&Guilds

9210-220
Post Graduate Diploma in Mechanical Engineering
Computational mechanics using finite element method

Sample Paper

You should have the following for this examination
- one answer book
- scientific calculator

No additional data is attached

General instructions
- This examination paper is of **three hours** duration.
- This paper contains **nine (09)** questions.
- Answer **five (05)** questions.

Section A

1 Three bars 1, 2 and 3 are connected together, as shown in Figure Q1 and a force of 2.4 N is applied in the x direction at node 2. Consider each element to be 600 mm long, Modulus of Elasticity E = 20 MPa and cross sectional area A = 3 cm^2 for each element 1 and 2, Modulus of Elasticity E = 10MPa and A = 6 cm^2 for element 3, and nodes 1 and 4 are fixed. Determine the following.

a) Global stiffness matrix. (7 marks)

b) Displacements of nodes 2 and 3. (6 marks)

c) Reactions at nodes 1 and 4. (7 marks)

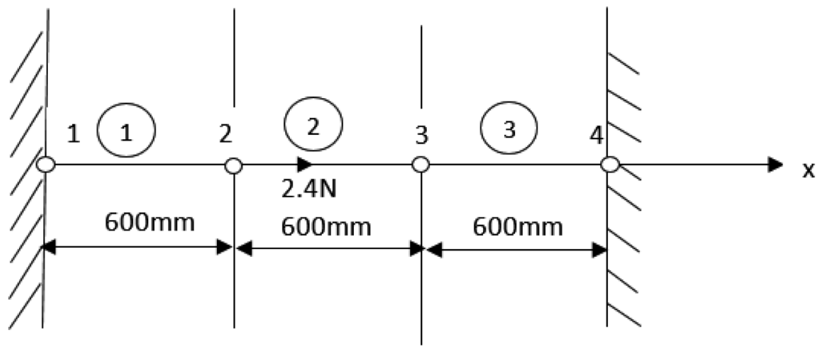

Figure Q1

2 a) Considering a basic element in the x – y plane (as shown in Figure Q2a) with positive nodal forces, derive an equation to determine the axial stress in a bar if for a bar, the relationship between the local forces and the local displacements in the respective directions is given by
Where

$$\begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} d^{\bullet}{}_{1x} \\ d^{\bullet}{}_{2}X \end{Bmatrix}$$

Where

$d^{\bullet}{}_{1x} = d_{1x} \, Cos\theta + d_{1y} Sin\theta$

$d^{\bullet}{}_{2x} = d_{2x} \, Cos\theta + d_{2y} Sin\theta$ (10 marks)

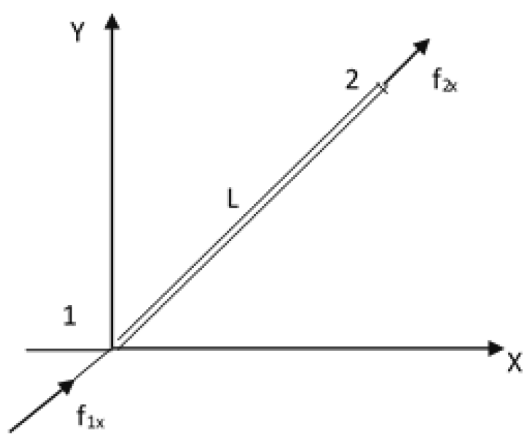

Figure Q2a

b) Figure Q2b, shows a bar element for stress evaluation, which makes an angle of 60° degrees with the 'X' axis. If its cross sectional area A = 4 x 10^{-4} m^2, modulus of elasticity E = 208 GPa, and length L = 2 m, find the axial stress assuming the global displacements have been previously determined to be d_{1x} = 0.20 mm, d_{1y} = 0.0, d_{2x} = 0.40 mm and d_{2y} = 0.60 mm.` (10 marks)

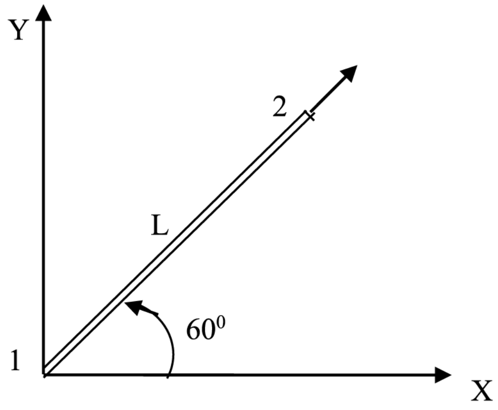

Figure Q2b

3 a) Derive an expression for the stiffness matrix for a 2-D truss element. (10 marks)

b) Derive the strain displacement matrix for a 1-D linear element and show that $\sigma = E[B]\{u\}$. (10 marks)

4 Consider the compound bar shown in Figure Q4, which is made of Aluminium (1-2) and Steel (2-3), with length 150 mm and 300 mm respectively and the following cross sectional areas and Moduli of elasticity.

Aluminium	Steel
A_1 = 1000 mm^2	A^2 = 200 mm^2
E_1 = 80 x 109 N/m^2	E_1 = 210 x 109 N/m^2

An axial load of P = 150 N is applied in the x direction.
Applying Boundary conditions, determine

a) nodal displacements (12 marks)
b) stress in each material (4 marks)
c) reaction forces. (4 marks)

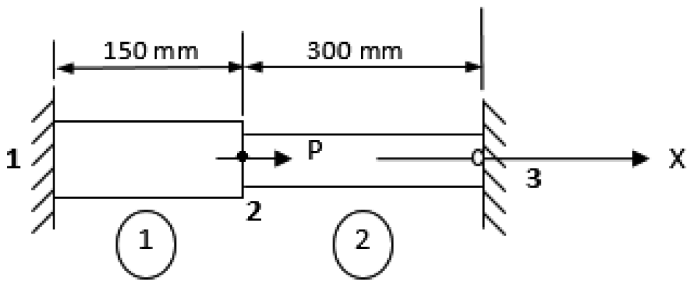

Figure Q4

5 The cantilever beam of length 'L' shown in Figure Q5 is subjected to a concentrated load 'P' at its free – end and an uniformly distributed load 'w' per unit length acting over the whole span of the beam. Determine,

a) the free – end displacements and (10 marks)

b) the nodal forces (10 marks)

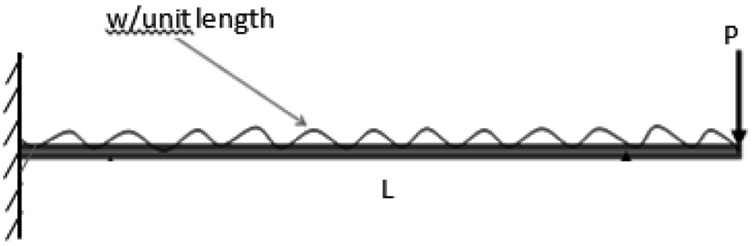

Figure Q5

6 The thin (steel) plate in Figure Q6 has uniform thickness, t = 1 cm. Its material has a Young's Modulus E = 200 GPa and weight density ω = 7850 kg/m^3. In addition to its self-weight, the plate is subjected to a point load P = 100 N at its midpoint. The area at the midpoint of the plate is 3 cm^2.

a) Model the plate with two finite elements. (5 marks)

b) Write down expressions for the element stiffness matrices and element body | force vectors. (4 marks)

c) Assemble the structural stiffness matrix K and global load vector F. (4 marks)

d) Using the elimination approach, solve for the global displacement vector Q. (7 marks)

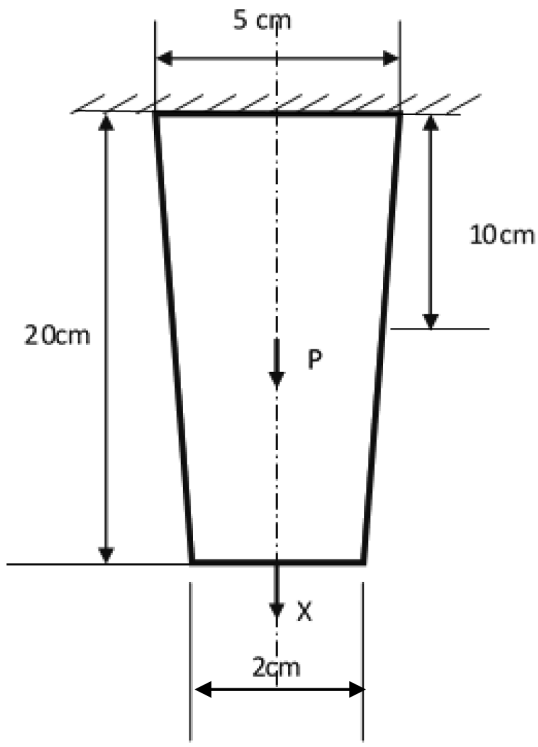

Figure Q6

7 A 45° strain rosette is used to estimate stresses on a beam. The normal strains obtained are as given in Figure Q7. If the Modulus of elasticity and Poisson's ratio of the beam are 5 GPa and 0.25 respectively, determine the major and minor principal stresses. Assume plain strain condition. (20 marks)

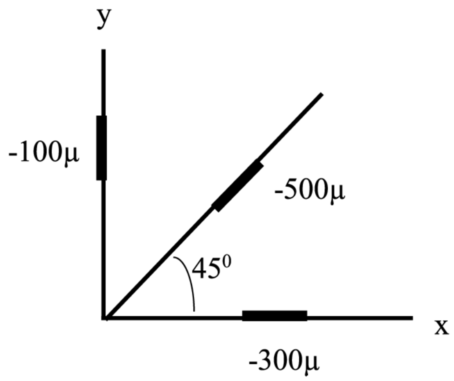

Figure Q7

8 Briefly explain the following:

a) Constant strain triangle (5 marks)

b) Serendipity family of element (5 marks)

c) Isoparametric element (5 marks)

d) The convergence criteria for non-linear solutions (5 marks)

9 a) State the purpose of the Gauss elimination method. (5 marks)

b) Solve the simultaneous set of equations using the Gauss elimination method

$2x_1 + 2x_2 + 1x_3 = 9$

$2x_1 + 1x_2 = 4$

$1x_1 + 1x_2 + 1x_3 = 6$ (15 marks)