

## **9210-220 Post Graduate Diploma in Mechanical Engineering** Computational mechanics using finite element method

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## You should have the following for this examination

No additional data is attached

- one answer book
- scientific calculator

## **General instructions**

- This examination paper is of **three hours** duration.
- This paper contains nine (09) questions.
- Answer **five (05)** questions.

## Section A

- 1 Three bars 1, 2 and 3 are connected together, as shown in Figure Q1 and a force of 2.4 N is applied in the x direction at node 2. Consider each element to be 600 mm long, Modulus of Elasticity E = 20 MPa and cross sectional area A = 3 cm<sup>2</sup> for each element 1 and 2, Modulus of Elasticity E = 10MPa and A = 6 cm<sup>2</sup> for element 3, and nodes 1 and 4 are fixed. Determine the following.
  - a) Global stiffness matrix.
  - b) Displacements of nodes 2 and 3.
  - c) Reactions at nodes 1 and 4.

(7 marks) (6 marks) (7 marks)



Figure Q1

a) Considering a basic element in the x – y plane (as shown in Figure Q2a) with positive nodal forces, derive an equation to determine the axial stress in a bar if for a bar, the relationship between the local forces and the local displacements in the respective directions is given by Where

$$\begin{cases} f_{1x} \\ f_{2x} \end{cases} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{cases} d^{\bullet}_{1x} \\ d^{\bullet}_{2}X \end{cases}$$

Where

 $d^{\bullet}_{1x} = d_{1x} \cos\theta + d_{1y} \sin\theta$  $d^{\bullet}_{2x} = d_{2x} \cos\theta + d_{2y} \sin\theta$ 

(10 marks)



Figure Q2a

b) Figure Q2b, shows a bar element for stress evaluation, which makes an angle of 60° degrees with the 'X' axis. If its cross sectional area  $A = 4 \times 10^{-4} \text{ m}^2$ , modulus of elasticity E = 208 GPa, and length L = 2 m, find the axial stress assuming the global displacements have been previously determined to be  $d_{1x} = 0.20 \text{ mm}$ ,  $d_{1y} = 0.0$ ,  $d_{2x} = 0.40 \text{ mm}$  and  $d_{2y} = 0.60 \text{ mm}$ .

(10 marks)



- 3a)Derive an expression for the stiffness matrix for a 2-D truss element.(10 marks)b)Derive the strain displacement matrix for a 1-D linear element and show<br/>that  $\sigma = E[B]\{u\}$ .(10 marks)
- 4 Consider the compound bar shown in Figure Q4, which is made of Aluminium (1-2) and Steel (2-3), with length 150 mm and 300 mm respectively and the following cross sectional areas and Moduli of elasticity.

Aluminium	Steel
$A_1 = 1000 \text{ mm}^2$	$A^2 = 200 \text{ mm}^2$
E <sub>1</sub> = 80 x 109 N/m <sup>2</sup>	$E_1 = 210 \text{ x} 109 \text{ N/m}^2$

An axial load of P = 150 N is applied in the x direction. Applying Boundary conditions, determine

- a) nodal displacements
- b) stress in each material
- c) reaction forces.



(12 marks) (4 marks) (4 marks)

- 5 The cantilever beam of length 'L' shown in Figure Q5 is subjected to a concentrated load 'P' at its free end and an uniformly distributed load 'w' per unit length acting over the whole span of the beam. Determine,
  - a) the free end displacements and
  - b) the nodal forces

(10 marks) (10 marks)

(5 marks)

(4 marks)

(4 marks)

(7 marks)



**Figure Q5** 

- 6 The thin (steel) plate in Figure Q6 has uniform thickness, t = 1 cm. Its material has a Young's Modulus E = 200 GPa and weight density  $\omega$  = 7850 kg/m<sup>3</sup>. In addition to its self-weight, the plate is subjected to a point load P = 100 N at its midpoint. The area at the midpoint of the plate is 3 cm<sup>2</sup>.
  - a) Model the plate with two finite elements.
  - b) Write down expressions for the element stiffness matrices and element body | force vectors.
  - c) Assemble the structural stiffness matrix K and global load vector F.
  - d) Using the elimination approach, solve for the global displacement vector Q.



**Figure Q6** 

A 45° strain rosette is used to estimate stresses on a beam. The normal strains obtained are as given in Figure Q7. If the Modulus of elasticity and Poisson's ratio of the beam are 5 GPa and 0.25 respectively, determine the major and minor principal stresses. Assume plain strain condition.

(20 marks)





Briefly explain the following: 8 a) Constant strain triangle (5 marks) Serendipity family of element (5 marks) b) C) Isoparametric element (5 marks) The convergence criteria for non-linear solutions d) (5 marks) 9 a) State the purpose of the Gauss elimination method. (5 marks) Solve the simultaneous set of equations using the Gauss elimination method b)  $2x_1 + 2x_2 + 1x_3 = 9$  $2x_1 + 1x_2 = 4$  $1x_1 + 1x_2 + 1x_3 = 6$ (15 marks)