## 9210-220 <br> Post Graduate Diploma in Mechanical Engineering Computational mechanics using finite element method

## 6DP SOBDSHU

You should have the No additional data is attached following for this examination<br>- one answer book<br>- scientific calculator

## General instructions

- This examination paper is of three hours duration.
- This paper contains nine (09) questions.
- Answer five (05) questions.


## Section A

1 Three bars 1, 2 and 3 are connected together, as shown in Figure Q1 and a force of 2.4 N is applied in the $x$ direction at node 2 . Consider each element to be 600 mm long, Modulus of Elasticity $E=20 \mathrm{MPa}$ and cross sectional area $A=3 \mathrm{~cm}^{2}$ for each element 1 and 2 , Modulus of Elasticity $E=10 \mathrm{MPa}$ and $A=6 \mathrm{~cm}^{2}$ for element 3 , and nodes 1 and 4 are fixed. Determine the following.
a) Global stiffness matrix.
b) Displacements of nodes 2 and 3 .
c) Reactions at nodes 1 and 4.


Figure Q1
2 a) Considering a basic element in the $x$ - y plane (as shown in Figure Q2a) with positive nodal forces, derive an equation to determine the axial stress in a bar if for a bar, the relationship between the local forces and the local displacements in the respective directions is given by
Where
$\left\{\begin{array}{l}f_{1 x} \\ f_{2 x}\end{array}\right\}=\frac{A E}{L}\left[\begin{array}{rr}1 & -1 \\ -1 & 1\end{array}\right]\left\{\begin{array}{l}d^{\bullet}{ }_{1 x} \\ d^{\bullet}{ }_{2} x\end{array}\right\}$
Where
$d^{\bullet} 1 x=d_{1 x} \cos \theta+d_{1 y} \sin \theta$
$d^{\bullet} 2 x=d_{2 x} \cos \theta+d_{2 y} \sin \theta$


Figure Q2a
b) Figure Q2b, shows a bar element for stress evaluation, which makes an angle of $60^{\circ}$ degrees with the ' $X$ ' axis. If its cross sectional area $A=4 \times 10^{-4} \mathrm{~m}^{2}$, modulus of elasticity $E=208 \mathrm{GPa}$, and length $L=2 \mathrm{~m}$, find the axial stress assuming the global displacements have been previously determined to be
$d_{1 x}=0.20 \mathrm{~mm}, d_{1 y}=0.0, d_{2 x}=0.40 \mathrm{~mm}$ and $d_{2 y}=0.60 \mathrm{~mm}$.


Figure Q2b
3 a) Derive an expression for the stiffness matrix for a 2-D truss element.
b) Derive the strain displacement matrix for a 1-D linear element and show that $\sigma=E[B]\{u\}$.

4 Consider the compound bar shown in Figure Q4, which is made of Aluminium (1-2) and Steel (2-3), with length 150 mm and 300 mm respectively and the following cross sectional areas and Moduli of elasticity.

Aluminium
$\mathrm{A}_{1}=1000 \mathrm{~mm}^{2}$

> Steel
$\mathrm{E}_{1}=80 \times 109 \mathrm{~N} / \mathrm{m}^{2}$

$$
\mathrm{A}^{2}=200 \mathrm{~mm}^{2}
$$

$$
\begin{aligned}
& \mathrm{E}_{1}=210 \times 109 \mathrm{~N} / \mathrm{m}^{2}
\end{aligned}
$$

An axial load of $P=150 \mathrm{~N}$ is applied in the $x$ direction.
Applying Boundary conditions, determine
a) nodal displacements
(12 marks)
b) stress in each material
c) reaction forces.


Figure Q4

5 The cantilever beam of length ' L ' shown in Figure Q5 is subjected to a concentrated load ' $P$ ' at its free - end and an uniformly distributed load ' $w$ ' per unit length acting over the whole span of the beam. Determine,
a) the free - end displacements and
b) the nodal forces


Figure Q5
6 The thin (steel) plate in Figure Q6 has uniform thickness, $\mathrm{t}=1 \mathrm{~cm}$. Its material has a Young's Modulus $\mathrm{E}=200 \mathrm{GPa}$ and weight density $\omega=7850 \mathrm{~kg} / \mathrm{m}^{3}$. In addition to its self-weight, the plate is subjected to a point load $\mathrm{P}=100 \mathrm{~N}$ at its midpoint. The area at the midpoint of the plate is $3 \mathrm{~cm}^{2}$.
a) Model the plate with two finite elements.
b) Write down expressions for the element stiffness matrices and element body force vectors.
c) Assemble the structural stiffness matrix K and global load vector $F$.
d) Using the elimination approach, solve for the global displacement vector Q .


Figure Q6

7 A $45^{\circ}$ strain rosette is used to estimate stresses on a beam. The normal strains obtained are as given in Figure Q7. If the Modulus of elasticity and Poisson's ratio of the beam are 5 GPa and 0.25 respectively, determine the major and minor principal stresses.
Assume plain strain condition.


Figure Q7
8 Briefly explain the following:
a) Constant strain triangle
b) Serendipity family of element
c) Isoparametric element
d) The convergence criteria for non-linear solutions

9 a) State the purpose of the Gauss elimination method.
b) Solve the simultaneous set of equations using the Gauss elimination method

$$
\begin{aligned}
& 2 x_{1}+2 x_{2}+1 x_{3}=9 \\
& 1 x_{1}+1 x_{2}+1 x_{3}=6
\end{aligned} \quad 2 x_{1}+1 x_{2}=4
$$

