9210-224
Level 7 Post Graduate Diploma in Engineering
Dynamics of mechanical systems

Sample Paper

You should have the following for this examination
- one answer book
- non-programmable calculator
- pen, pencil, drawing instruments
- ordinary graph sheets

No additional data is attached

General instructions
- This paper consists of **eight** questions.
- Answer any **five** questions.
- All questions carry equal marks.

1 Figure Q1 shows two uniform thin rigid rods OA and AB freely hinged at A. The end O is freely hinged to a fixed point as shown. The rods are of equal length and the mass of rod AB is twice that of rod OA. When the rods are at rest in the vertical position, an impulse of magnitude *P* is applied at the mid-point of rod AB and perpendicular to AB.

a) Show that the magnitudes of the initial angular velocities of the rods OA and AB are equal. (12 marks)

b) Determine the ratio of initial kinetic energies of the rods. (8 marks)

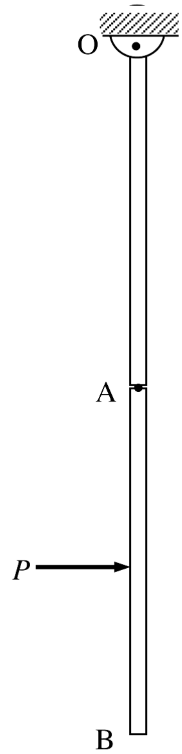

Figure Q1

2 A machine of mass *m* is mounted on a rigid foundation through an isolator. The isolator may be assumed to be consisting of a parallel combination of a spring of stiffness *k* and a dashpot having a damping ratio β. The machine produces an excitation force $P \sin \omega t$, where *P* is the amplitude and ω is the frequency.

The instantaneous displacement (x) at steady state of the machine in vibratory motion is given by

$$x = \frac{P}{k}\frac{1}{\sqrt{(1-r^2)^2 + 4\beta^2 r^2}} \sin(\omega t - \phi)$$

where $r = \frac{\omega}{\omega_n}$, $\omega_n = \sqrt{\frac{k}{m}}$, and $\phi = \tan^{-1}\left(\frac{2\beta r}{1-r^2}\right)$

a) Show that the transmissibility ratio (T) of the force transmitted to the foundation at steady state is given by

$$T = \left[\frac{1 + 4\beta^2 r^2}{(1-r^2)^2 + 4\beta^2 r^2}\right]^{1/2}$$ (8 marks)

b) A machine of mass 1000 kg operates at a speed of 1500 rev/min and produces a repeating force of amplitude of 20 kN. The machine is isolated from the rigid foundation by means of an undamped isolator. Determine the maximum stiffness of the isolator if the force transmitted to the foundation is limited to 2500 N. (6 marks)

c) If an isolator with damping ratio of 0.3 is used in place of the undamped isolator in Q2 (b), what would be the maximum stiffness of the isolator? (6 marks)

3 A light elastic uniform shaft of torsional stiffness K carries a rotor of moment of inertia I, at each end. It is found that this two rotor torsional system has a natural frequency of torsional vibration of 20 rad/s.
A third rotor of moment of inertia I is mounted on the shaft at a distance equal to one third of the length of the shaft from the left end.

a) Determine natural frequencies of torsional vibrations of the three rotor system. (15 marks)

b) Determine the mode shape corresponding to the highest frequency. (5 marks)

4 Figure Q4 shows a three degree of freedom vibratory system consisting of two uniform rigid rods each of length L and mass 3 m, two springs each of stiffness k and a mass 2 m.

a) Determine the equations of motion for small displacements of the vibratory system using Newton's second law. Verify the results obtained by using Lagrange's equation. (12 marks)

b) Obtain the frequency equation and determine the natural frequencies in terms of m, k, L and g. (8 marks)

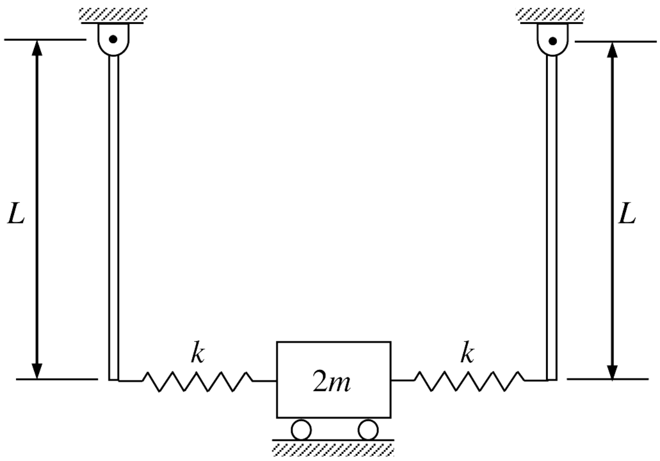

Figure Q4

5 a) Show that the whirling speed (ω_c) of a light horizontal uniform shaft carrying a disc at its midpoint and rotating at a speed ω is given by

$$\omega_c^2 = \omega^2 \left(1 + \frac{e}{y}\right)$$

where e is the eccentricity of the mass centre of the disc from the shaft axis and y is the deflection at the centre of the shaft. (6 marks)

b) A uniform horizontal shaft of diameter 20 mm rotates in long fixed bearings 1.0 m apart and a disc of mass 22 kg is fixed at mid span. The mass centre of the disc is 0.3 mm from the shaft axis. Determine:

i) critical speed of the shaft (6 marks)

ii) range of speeds within which the shaft should not be allowed to run in order to keep the maximum bending stress of the shaft below 120 N/m^2. (8 marks)

Modulus of elasticity of the material of the beam – 200 GN/m^2
You may assume that for a uniform shaft of length L with fixed ends and subjected to a point load P at its centre,

i) the deflection at the centre = $\frac{PL^3}{192EI}$, and

ii) the maximum bending moment = $\frac{PL}{8}$

See next page

6 A uniform beam of mass m, flexural rigidity EI and length L is simply supported at each end. The dynamic deflection curve of the beam when it is undergoing transverse vibration is of the form, $y = a \sin\left(\frac{\pi x}{L}\right)$, where y is the deflection of the beam at a distance from one end and is a constant.

a) Using Rayleigh's method, determine the natural frequency of transverse vibration of the beam. (14 marks)

b) If a mass 2 m is now placed at the centre of the beam, what would be the new frequency of transverse vibration of the beam? Assume that the dynamic deflection curve remains unchanged. (6 marks)

7 The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s+4)(s+6)}$$

where K is the gain of the controller.

a) Draw the root locus as a function of K and determine the range of K for which the system is stable. (12 marks)

b) Using the root locus you have drawn, show that $s = -8$ is a closed loop pole of the system for $K = 64$. Find the other closed poles for $K = 64$. (2 marks)

c) Determine the damping ratio of the closed loop system when $K = 64$. (2 marks)

d) Determine the value of K for critical damping of the closed loop system. (2 marks)

e) Determine the value of K corresponding to 2% settling time of 4 s. (2 marks)

8 a) A simple pendulum of length L and mass m_2 is hinged at the centre of mass m_1 as shown in Figure Q8. The mass m_1 slides on a frictionless horizontal surface under the action of a force u.

i) Derive the non-linear equations of motion of the masses m_1 and m_2. (4 marks)

ii) Linearise the equations of motion assuming θ to be a small angle. Represent the system in state space, considering both θ and x as outputs of the system. (8 marks)

b) The state space representation of a type one servo system is given below.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

It is required to have closed loop poles of this system at desired locations by using the state feedback control $u = -Kx$ where K is the state feedback gain matrix. Find the elements of the matrix K, if the closed loop poles are located at $s = -2 \pm 2j$ and $s = -5$. (8 marks)

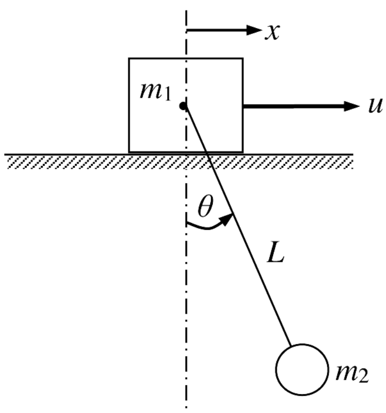

Figure Q8