City & Guilds

UNIT 518 THE DYNAMICS OF MACHINE SYSTEMS

Lesson 1: Kinematics of mechanisms		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**

Revision of basic concepts	Although there is a certain level of prerequisite knowledge required for entering this course the tutor will benefit from taking time at the beginning of the course to reinforce the basic equations.
	Whole-class teaching
	Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms:
	• Potential Energy (P.E.) and Kinetic Energy (K.E.),
	    ○ Potential Energy (P.E.) Work = mass x g x distance
	    ○ Kinetic Energy (K.E.) K.E. = $mv^2/2$
	• The difference between scalar and vector quantities
	    ○ Any physical quantity that requires a direction to be stated in order to define it completely is known as a vector quantity
	    ○ A scalar quantity, such as time, is adequately defined when the magnitude is given in the appropriate units
	• Force and motion
	    ○ Force, measured in newtons, is a vector quantity because its effect depends upon its magnitude and direction
	• How to determine the resultant of two coplanar vectors by using a vector triangle
	• How to calculate the resultant of two perpendicular vectors
	• How to resolve a vector into two perpendicular vector
	Show video.
	Small-group teaching
	Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements, tutor to circulate and correct as required.
		Books:
		Johnson, Keith, *New Physics for You*, Nelson Thorne 2011
		Zimba, Jason, *Force and Motion* Johns Hopkins University Press 2009
		Oxlade, Chris, *Forces and Motion*, Hodder Wayland 2008
		Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008
		Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013
		Website:
		www.metacafe.com/tags/Kinematics/page-3
		http://www.physicsclassroom.com/Shockwave-Physics-Studios
		www.revisionworld.co.uk?node/7814
		Practical equipment:
		Laboratory equipment for evaluating forces, velocity and acceleration

Lesson 2: Kinematics of mechanisms (continued)		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Revision of basic concepts (continued)	**Whole-class teaching** Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms: displacement, speed, velocity and acceleration, i.e.: • Displacement: the change of position of a body in a particular direction and is a vector quantity • Speed: ratio of distance to time taken by a moving body and is a scalar quantity • Velocity: the rate of motion in a given direction and is a vector quantity • Acceleration: the rate of change of velocity is a scalar quantity State and use the equations which represent uniformly accelerated motion in a straight line • v = u + at • s = 1/2(u+v)t • s = ut + ½at2 • v2 = u2 + 2as Where a is acceleration, s is distance, t is time, u is initial velocity and v is final velocity. **Small-group teaching** Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements, tutor to circulate and correct as required.	**Books:** Johnson, Keith, *New Physics for You*, Nelson Thorne 2011 Zimba, Jason, *Force and Motion* Johns Hopkins University Press 2009 Oxlade, Chris, *Forces and Motion*, Hodder Wayland 2008 Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Websites:** www.scienceaid.co.uk/physics/forces/motion.html http://www.physicsclassroom.com/Shockwave-Physics-Studios www.bbc.co.uk/learningzone/clips/contact Area http://www.YourOtherTeacher.com **Practical equipment:** Laboratory equipment for evaluating forces, displacement, velocity and acceleration

Lesson 3: Kinematics of mechanisms (continued)		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**

Revision of basic concepts (continued)	**Whole-class teaching**
	Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms:
	• State that mass is the property of a body which resists change in motion
	• State and apply the formula for density (D) of a material
	    ○ D = m/v, where D is density, m is mass and v is volume
	• State and apply the formula for force (F)
	    ○ F = ma, where a is acceleration, F is force and m is mass
	• Define the term newton
	    ○ Newton: the derived SI unit of force. The force required to give a mass of 1 kg an acceleration of 1 m/s2describe and apply the concept of weight as the effect of a gravitational field on mass
	• State and apply the formula for weight (W)
	    ○ W = mg, where W is weight, m is mass and g is acceleration due to gravity
	Small-group teaching
	Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements, tutor to circulate and correct as required.
		Books:
		Johnson, Keith, *New Physics for You*, Nelson Thorne 2011
		Zimba, Jason, *Force and Motion* Johns Hopkins University Press 2009
		Oxlade, Chris, *Forces and Motion*, Hodder Wayland 2008
		Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008
		Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013
		Websites:
		www.scienceaid.co.uk/physics/forces/motion.html
		http://www.physicsclassroom.com/Shockwave-Physics-Studios
		www.bbc.co.uk/learningzone/clips/contact Area
		http://www.YourOtherTeacher.com
		Practical equipment:
		Laboratory equipment for evaluating forces, density, weight, displacement, velocity and acceleration

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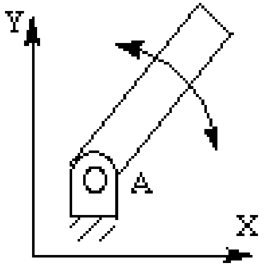
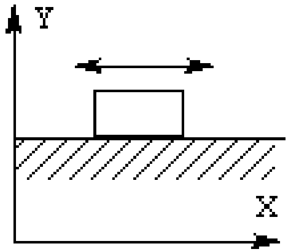
<table>
<tr><td colspan="2">Lesson 4: Kinematics of mechanisms (continued)</td><td>Suggested Teaching Time: 1 hour</td></tr>
<tr><td colspan="3">Learning Outcome 1: Understand the kinematics of mechanisms</td></tr>
<tr><th>Topic</th><th>Suggested Teaching</th><th>Suggested Resources</th></tr>
<tr><td>Revision of basic concepts (continued)</td><td>Whole-class teaching
Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms:
• State that the weight of a body may be considered as acting at a single point called the centre of gravity
• Explain that a couple as a pair of equal parallel forces tends to produce rotation only
• Define and use the moment of a force and the torque of a couple
  ○ Moment of a force: the tendency of a force to rotate a body
• State that for a system in equilibrium there is no resultant force and no resultant torque
• Define the term joule and apply the formula for work done (W)
• Explain the relationship between power (P), work done (W) and time (t)
  ○ W = Pt, where W is work done, P is power and t us time
Small-group teaching
Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements, tutor to circulate and correct as required</td><td>Books:
Johnson, Keith, New Physics for You, Nelson Thorne 2011
Zimba, Jason, Force and Motion Johns Hopkins University Press 2009
Oxlade, Chris, Forces and Motion, Hodder Wayland 2008
Doherty, J. J. J., An Elementary Text-book of Mechanics, (Kinematics and Dynamics), BiblioLife 2008
Wilson, Charles E. and Sadler, J. Peter, Kinematics and Dynamics of Machinery, Pearson 2013
Websites:
www.scienceaid.co.uk/physics/forces/motion.html
http://www.physicsclassroom.com/Shockwave-Physics-Studios
www.bbc.co.uk/learningzone/clips/contact Area
http://www.YourOtherTeacher.com
Practical equipment:
Laboratory equipment for evaluating forces, equilibrium and moments of force</td></tr>
</table>

Lesson 5: Kinematics of mechanisms (continued)		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Revision of basic concepts (continued)	**Whole-class teaching** Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms: • Represent distance travelled, displacement, speed, velocity and acceleration using graphical methods • Determine the distance travelled by calculating the area under a speed-time graph • Determine velocity by using the gradient of a displacement: time graph • Determine speed by using the gradient of a displacement: time graph • Determine acceleration by using the gradient of a velocity: time graph **Small-group teaching** Split class into smaller groups and issue a series of questions covering the equations used so far. Where possible include practical elements, tutor to circulate and correct as required.	**Books:** Johnson, Keith, *New Physics for You*, Nelson Thorne 2011 Zimba, Jason, *Force and Motion* Johns Hopkins University Press 2009 Oxlade, Chris, *Forces and Motion*, Hodder Wayland 2008 Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Website:** www.scienceaid.co.uk/physics/forces/motion.html http://www.physicsclassroom.com/Shockwave-Physics-Studios www.bbc.co.uk/learningzone/clips/contact Area http://www.YourOtherTeacher.com **Practical equipment:** Laboratory equipment for evaluating displacement, velocity and acceleration

Lesson 6: Kinematic modelling		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Kinematic modelling of simple mechanisms (A.C. 1.1)	**Whole-class teaching** Explain kinematic modelling of simple mechanisms. Tutor to get the whole class involved in learner research and activity to cover the following principles and the meaning of the following terms: Reference frames • The movement of components of a mechanical system is analysed by attaching a reference frame to each part and determining how the reference frames move relative to each other. If the structural strength of the parts is sufficient then their deformation can be neglected and rigid transformations used to define this relative movement. Degrees of freedom • The degrees of freedom (DOF) of a rigid body is defined as the number of independent movements it has e.g. a rigid body on a plane has 3 DOF. The bar can be translated along the x axis, translated along the y axis, and rotated about its centroid. Rigid body links • Two or more rigid bodies in space are collectively called a rigid body system. We can hinder the motion of these independent rigid bodies with kinematic constraints. Kinematic constraints are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system	**Books:** Johnson, Keith, *New Physics for You*, Nelson Thorne 2011 Zimba, Jason, *Force and Motion* Johns Hopkins University Press 2009 Oxlade, Chris, *Forces and Motion*, Hodder Wayland 2008 Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Websites:** www.metacafe.com/tags/Kinematics/page-3 http://www.physicsclassroom.com/Shockwave-Physics-Studios

Topic	Suggested Teaching	Suggested Resources

Kinematic modelling of simple mechanisms (A.C. 1.1)	• Revolute and prismatic joints (known as lower pairs in planar mechanisms)
	    ○ A rigid body in a plane has three independent motions, two translational and one rotary; introducing either a revolute pair or a prismatic pair between two rigid bodies removes two degrees of freedom.
	
	
	
	
	• Discuss Gruebler's equation
	    ○ $F=3(n-1) - 2l - h$
	        ▪ Where
	        F = total degrees of freedom in the mechanism
	        n = number of links (including the frame)
	        l = number of lower pairs (one degree of freedom)
	        h = number of higher pairs (two degrees of freedom)
	• Kinematic chains
	    ○ A link is defined as a rigid body having two or more pairing elements which connect it to other bodies for the purpose of transmitting force or motion. In every machine, at least one link either occupies a fixed position relative to the earth or carries the machine as a whole along with it during motion. This link is the frame of the machine and is called the fixed link. The combination of links and pairs without a fixed link is not a mechanism but a kinematic chain.
		Books:
		Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008
		Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013
		Websites::
		http://www.cs.cmu.edu/~rapidproto/mechanisms/chpt4.html
		http://kmoddl.library.cornell.edu/model.php?m=reuleaux
		Practical equipment:
		Examples of:
		• Resolute and prismatic joins
		• Kinematic chains
		• Planar kinematic mechanisms
		• Spatial kinematic mechanisms
		Including working examples of:
		• Four-bar linkage
		• Crank and rocker
		• Drag link
		• Slider-crank
		• Scotch yoke
		• Quick-return

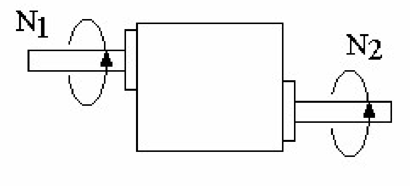
Topic	Suggested Teaching	Suggested Resources

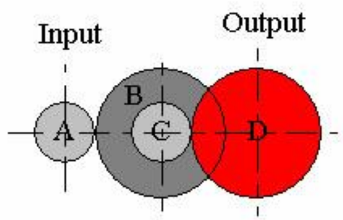
Kinematic modelling of simple mechanisms (A.C. 1.1)	○ Planar kinematic mechanisms
	  ▪ In a planar mechanisms, all of the relative motions of the rigid bodies are in one plane or in parallel planes
	○ Spatial kinematic mechanisms
	  ▪ If there is any relative motion that is not in the same plane or in parallel planes, the mechanism is called the spatial mechanism
	○ Discuss the terms general motion and relative motion
	• Tutor to get the whole class involved in learner research and activity to cover the modelling of the following systems:
	  ▪ Four-bar linkage
	  ▪ Crank and rocker
	  ▪ Drag link
	  ▪ Slider-crank
	  ▪ Scotch yoke
	  ▪ Quick-return
		Website:
		http://kmoddl.library.cornell.edu/model.php?m=reuleaux

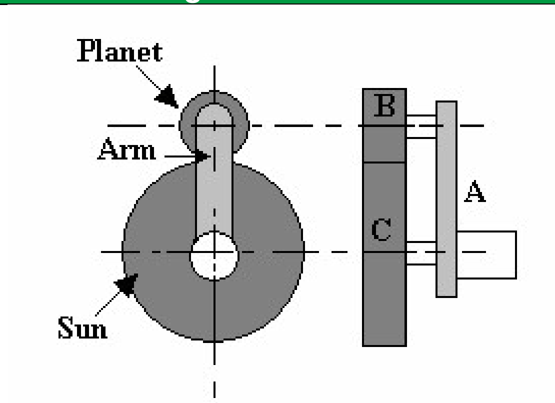
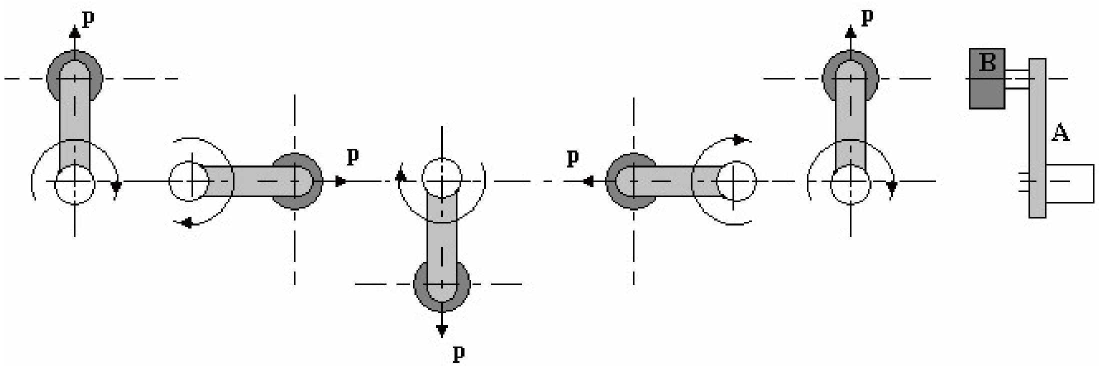
Lesson 7: Evaluation of velocities in kinematic mechanisms by graphical analysis		**Suggested Teaching Time:** 1 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Evaluate velocities in kinematic mechanisms by graphical analysis (A.C. 1.2)	**Velocity diagrams** This involves the construction of diagrams which need to be done accurately and to a suitable scale. Students should use: a Drawing board, ruler, compass, protractor, and triangles or a suitable CAD Package which the students are familiar with. **Tutor-led learning** Learner research and activity on the concepts of: absolute and relative velocity, tangential velocity, radial velocity and the motion of a crank, con-rod and piston. Tutor should demonstrate the drawing of the different types of diagrams and then get the students to solve example questions using the graphical method, tutor to assist individual students, and correcting errors as required. Diagram types to include: instantaneous centres; relative velocities; velocity and acceleration diagrams for the following types of mechanisms: • Four-bar linkage • Crank and rocker • Drag link • Slider-crank • Scotch yoke • Quick-return	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Software:** Basic CAD programme **Practical equipment:** Drawing board, ruler, compass, protractor and triangles. Examples of resolute and prismatic joins, kinematic chains, planar kinematic mechanisms, and spatial kinematic mechanisms, including working examples of mechanisms listed in text. **Websites:** http://www.freestudy.co.uk/dynamics/velaccdiag.pdf https://www.youtube.com/watch?v=IzaZ38Rn9Tk

Lesson 8: Evaluation the accelerations in kinematic mechanisms by graphical analysis		**Suggested Teaching Time:** 8 hour
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Evaluate **the accelerations** in kinematic mechanisms by graphical analysis (A.C. 1.3)	**Acceleration diagrams** This involves the construction of diagrams which need to be done accurately and to a suitable scale. Students should use: a Drawing board, ruler, compass, protractor, and triangles or a suitable CAD Package with which the students are familiar. Tutor-led learning. learner research and activity on the concepts of: Centripetal (radial) Acceleration, Tangential acceleration and Coriolis Acceleration. Tutor should demonstrate the drawing of the different types of drawings and then get the students to solve example questions using the graphical method, tutor to assist individual students, and correcting errors as required. Diagram types to include Instantaneous centres; relative velocities; velocity and acceleration diagrams for the following types of mechanisms: • Four-bar linkage • Crank and rocker • Drag link • Slider-crank • Scotch yoke • Quick-return	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Software:** Basic CAD programme **Practical equipment:** Drawing board, ruler, compass, protractor, and triangles Examples of Resolute and Prismatic joins, kinematic chains, planar kinematic mechanisms, and spatial kinematic mechanisms. These are to include working examples of the mechanism listed in the text. **Website:** http://www.freestudy.co.uk/dynamics/velaccdiag.pdf

Lesson 9: Evaluation of the motions in kinematic mechanisms by mathematical analysis		**Suggested Teaching Time:** 8 hours
Learning Outcome 1: Understand the kinematics of mechanisms		
Topic	**Suggested Teaching**	**Suggested Resources**
Evaluate the motions in kinematic mechanisms by mathematical analysis (A.C. 1.4)	**Whole-class teaching** To cover the relationship between displacement, velocity and acceleration: • Displacement (x) = R(sin ωt) • Velocity (v) = dx/dt = ωR cos(ωt) • Acceleration (a) = dv/dt = $-\omega^2$R sin(ωt) Tutor-led discussion on how we can use mathematics to solve the problems given in previous lesson rather than using diagrams The tutor should work through typical examples of calculations covering the different equations and the learners should then work through other examples of such calculations. These examples should cover the following types of for the following types of mechanisms: • Four-bar linkage • Crank and rocker • Drag link • Slider-crank • Scotch yoke • Quick-return The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained.	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Practical equipment:** Examples of: • Resolute and Prismatic joins • Kinematic chains • Planar kinematic mechanisms • Spatial kinematic mechanisms These are to include working examples of the mechanism listed in the text. **Website:** http://www.freestudy.co.uk/dynamics/velaccdiag.pdf

Lesson 10: Gear trains		**Suggested Teaching Time:** 1 hour
Learning Outcome 2: Understand the dynamics of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
Analyse the operation of a gear train in a machine (A.C. 2.1)	Tutor-led group teaching on the concept of gears, how and why they are used progressing from simple gear trains to compound gear trains and introducing the three types of epicyclic gearboxes. Tutor to cover the topics in bold using material below headings. Basic gear box theory:  Consider a simple schematic of a gear box with an input and output shaft. **Gear box ratio** The ratio of the gear box is defined as $G.R. = \frac{\text{INPUT } SPEED}{\text{OUTPUT SPEED}} = \frac{N_1}{N_2}$ N is usually in rev/min but the ratio is the same whatever units of speed are used. If angular velocity is used then $G.R. = \frac{INPUTSPEED}{OUTPUT\ SPEED} = \frac{\omega_1}{\omega_2}$ **Torque and efficiency** The power transmitted by a torque $T\ Nm$ applied to a shaft rotating at $N\ rev/min$ is given by $\text{S.P.} = \frac{2\pi NT}{60}$ In an ideal gear box, the input and output powers are the Same so $\frac{2\pi N_1 T_1}{60} = \frac{2\pi N_2 T_2}{60}$ $N_1 T_1 = N_2 T_2$ $\frac{T_2}{T_1} = \frac{N_1}{N_2} = G.R.$ It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as: $\eta = \frac{power\ Out}{Power\ in} = \frac{2\pi N_2 T_2 \times 60}{2\pi N_1 T_1 \times 60} = \frac{N_2 T_2}{N_1 T_1} =$	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Practical equipment:** Practical boards and equipment demonstrating different layouts of gears with the option of changing gear ratios **Website:** http://www.freestudy.co.uk/dynamics/gears.pdf http://ocw.uc3m.es/ingenieria-mecanica/machine-theory/lab-reports/analisis-of-gear-trains/at_download/file www.asee.org/public/conferences/1/papers/838/download

Topic	Suggested Teaching	Suggested Resources
Analyse the operation of a gear train in a machine (A.C. 2.1)	**Compound gears** Compound gears are simply a chain of simple gear trains with the input of the second being the output of the first. A chain of two pairs is shown below. Gear B is the output of the first pair and gear C is the input of the second pair. Gears B and C are locked to the same shaft and revolve at the same speed.  The velocity of each tooth on A and B are the same so $\omega_A t_A = \omega_B t_B$ as they are simple gears. Likewise for C and D, $\omega_C t_C = \omega_D t_D$. $\frac{\omega_A}{t_A} = \frac{\omega_B}{t_B}$ *and* $\frac{\omega_C}{t_C} = \frac{\omega_D}{t_D}$ $\omega_A = \frac{t_B \omega_B}{t_A}$ *and* $\omega_C = \frac{t_D \omega_D}{t_C}$ $\omega_A \omega_C = \frac{t_B \omega_B}{t_A} \times \frac{t_D \omega_D}{t_C} = \frac{t_B t_D}{t_A t_C} \times \omega_B \omega_D$ $\frac{\omega_A \omega_C}{\omega_B \omega_D} = \frac{t_B t_D}{t_A t_C}$ Since Gears B and C are on the same shaft $\omega_B = \omega_C$ $\frac{\omega_A}{\omega_D} = \frac{t_B t_D}{t_A t_C} = G.R$ Gears B and D are the driven gears. Gears A and C are the driver gears. It follows that: • Gear ratio = product of driven teeth/product of driving teeth This rule applies regardless of how many pairs of gears there are.	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Practical equipment:** Practical boards and equipment demonstrating different layouts of gears with the option of changing gear ratios **Website:** http://www.freestudy.co.uk/dynamics/gears.pdf http://ocw.uc3m.es/ingenieria-mecanica/machine-theory/lab-reports/analisis-of-gear-trains/at_download/file www.asee.org/public/conferences/1/papers/838/download

Topic	Suggested Teaching	Suggested Resources
Analyse the operation of a gear train in a machine (A.C. 2.1)	Epicyclic means one gear revolving upon and around another. The design involves planet and sun gears as one orbits the other like a planet around the sun  **Basic theory** The diagram shows a gear B on the end of an arm A. Gear B meshes with gear C and revolves around it when the arm is rotated. B is called the planet gear and C the sun First consider what happens when the planet gear orbits the sun gear; observe point p and you will see that gear B also revolves once on its own axis. Any object orbiting around a centre must rotate once. Now consider that B is free to rotate on its shaft and meshes with C. Suppose the arm is held stationary and gear C is  	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Practical equipment:** Practical boards and equipment demonstrating different layouts of gears with the option of changing gear ratios **Websites:** http://www.freestudy.co.uk/dynamics/gears.pdf http://ocw.uc3m.es/ingenieria-mecanica/machine-theory/lab-reports/analisis-of-gear-trains/at_download/file www.asee.org/public/conferences/1/papers/838/download

rotated once. B spins about its own centre and the number of revolutions it makes is the ratio t_C/t_B. B will rotate by this number for every complete revolution of C Now consider that C is unable to rotate and the arm A is revolved once. Gear B will revolve $t_C/t_B + 1$ because of the orbit. It is this extra rotation that causes confusion

One way to get round this is to imagine that the whole system is revolved once. Then identify the gear that is fixed and revolve it back one revolution. Work out the revolutions of the other gears and add them up. The following tabular method makes it easy. Suppose gear C is fixed and the arm A makes one revolution. Determine how many revolutions the planet gear B makes.

Step 1 is to revolve everything once about the centre.

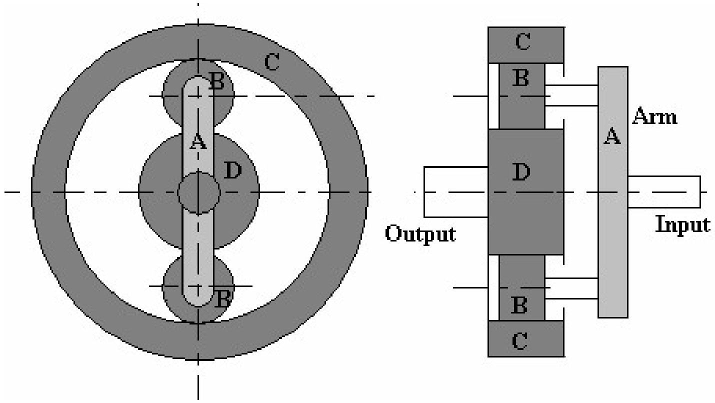
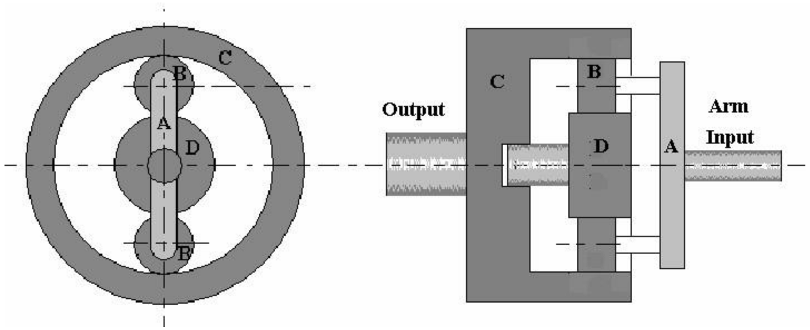
Step 2 identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of B.

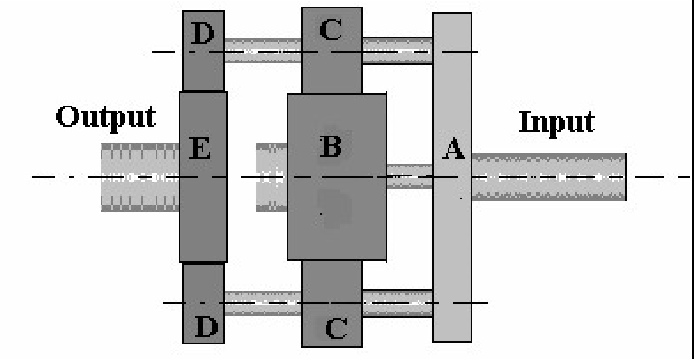
Step 3 is simply add them up and we find the total revs of C is zero and for the arm is 1.

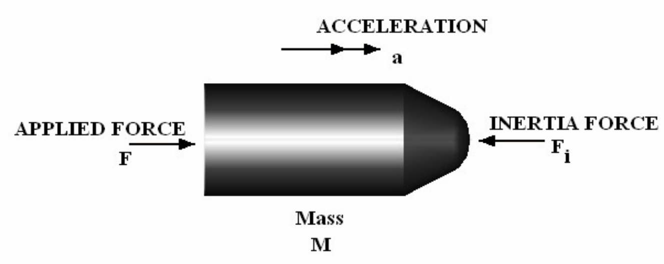
Step	Action	A	B	C	
1	Revolve all once	1	1	1	
2	Revolve C by -1 rev	0	$+{}^{t_C}/_{t_B}$	-1	
3	Add	1	$1+{}^{t_C}/_{t_B}$	0	

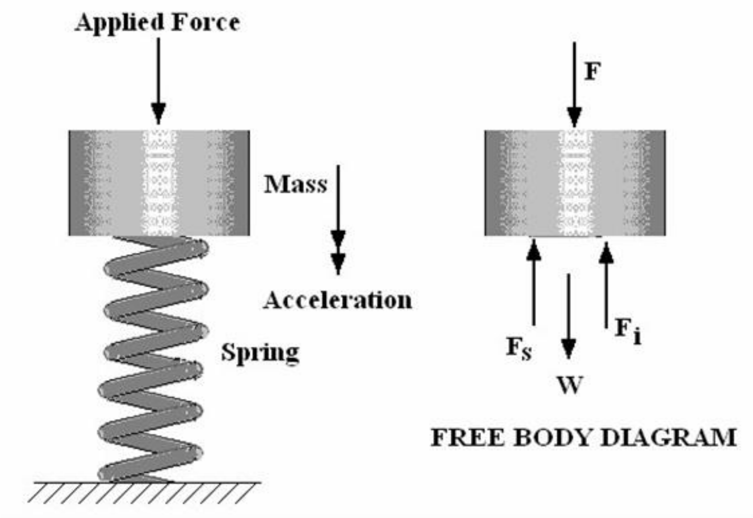
See http://www.freestudy.co.uk/dynamics/gears.pdf for worked examples

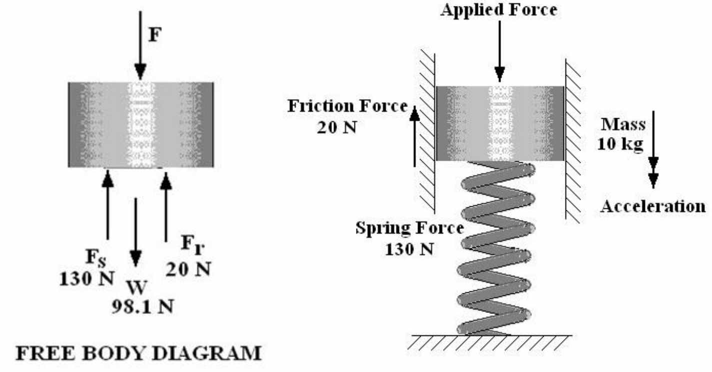
UNIT 518 THE DYNAMICS OF MACHINE SYSTEMS

Topic	Suggested Teaching	Suggested Resources
Analyse the operation of a gear train in a machine (A.C. 2.1)	The design so far considered has no identifiable input and output. We need a design that puts an input and output shaft on the same axis. This can be done several ways  **Method 1** The arm is the input and gear D is the output. Gear C is a fixed internal gear and is normally part of the outer casing of the gear box. There are normally four planet gears and the arm takes the form of a cage carrying the shafts of the planet gears. Note that the planet gear and internal gear both rotate in the same direction. See http://www.freestudy.co.uk/dynamics/gears.pdf for worked examples	**Books:** Doherty, J. J. J., *An Elementary Text-book of Mechanics, (Kinematics and Dynamics)*, BiblioLife 2008 Wilson, Charles E. and Sadler, J. Peter, *Kinematics and Dynamics of Machinery*, Pearson 2013 **Practical equipment:** Practical boards and equipment demonstrating different layouts of gears with the option of changing gear ratios **Websites:** http://www.freestudy.co.uk/dynamics/gears.pdf http://ocw.uc3m.es/ingenieria-mecanica/machine-theory/lab-reports/analisis-of-gear-trains/at_download/file www.asee.org/public/conferences/1/papers/838/download
	Method 2 In this case the sun gear D is fixed and the internal gear C is made into the output See http://www.freestudy.co.uk/dynamics/gears.pdf for worked examples 	

	Method 3 In this design a compound gear C and D is introduced. Gear B is fixed and gears C rotate upon it and around it. Gears C are rigidly attached to gears D and they all rotate at the same speed. Gears D mesh with the output gear E  See http://www.freestudy.co.uk/dynamics/gears.pdf for worked examples	
	The tutor should work through typical examples of calculations covering the different equations and the learners should then work through other examples of such calculations. These examples should cover the following types of gearing: simple gear trains, compound gear trains and the three types of epicyclic gearboxes The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained	

Lesson 11: The forces in a machine		**Suggested Teaching Time:** 3 hours
Learning Outcome 2: Understand the dynamics of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
Analyse the forces in machines (A.C. 2.2)	**Tutor-led discussion** How other forces affect the machines that we have been looking at so far, discussion to include: **D'Alembert's principle in dynamics** When a body accelerates, the applied force has to overcome the inertia. The inertia force resisting acceleration (or deceleration) is equal and opposite of the applied force. This means that total force acting on the body is zero. This is similar to the ideas of conservation of energy and momentum. Consider the free body diagram of an accelerating body. The total force acting on the body is in equilibrium even though it is accelerating so we have: $F + F_I = 0$  We know that F_I will be negative in value (to the left) but when using symbols we always put plus as in this case. Evaluation of the numbers will yield the negative figure. Since no other forces are involved, F = Ma = - F i so F i = - Ma The inertia force always opposes acceleration so it is always negative when evaluated.	**Books:** Ambekar, Ashok G., *Mechanism and Machine Theory*, Prentice Hall of India Pvt. Ltd, ISBN-10: 8120331346, ISBN-13: 978-8120331341, 2011 Bapaiah, G., *Machine Dynamics (DOM)*, Vol 2, Mechanical Engineering Monograph Series, Indian Institute of Technology Madras Lal, Jagdish, *Theory of Machines and Mechanisms*, Metropolitan Book Co. Pvt. Ltd., ISBN: 97881200002722 Mabie, Hamilton H., and Ocvirk, Fred W., *Mechanisms and Dynamics of Machinery*, John Wiley and Sons, 1957 Rattan, S. S., *Theory of Machines*, Tata McGraw-Hill 2009, ISBN-10: 007014477X, ISBN-13: 978-0070144774 **Websites:** http://ocw.metu.edu.tr/pluginfile.php/6467/mod_resource/content/6/ch7/index.htm http://elearning.vtu.ac.in/17/e-Notes/10ME54/Unit1-SRJ.pdf

	The principle can be extended to many other areas of work. Consider a mass supported by a spring. If a force F is applied downwards as shown, it will accelerate the mass downwards but it will also have to overcome the force exerted by stretching the spring. The mass also has a weigh W = Mg that acts downwards. Applying D'Alembert's principle, F + Fs + Fi + W = 0  We know the spring force is up and the inertia force is up and will be opposite in sign to F and W when evaluated but when setting up the equation everything is assumed positive and we put in negative values when they the actual figures are known. Another force that might be considered is friction and this always opposes movement. The next example explains this.	

Lesson 11: The forces in a machine		**Suggested Teaching Time:** 3 hours
Learning Outcome 2: Understand the dynamics of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
	Worked example A spring is fixed at the left end and has a sliding mass of 10 kg at the other as shown. A force of 100 N is applied to move the mass. At the given instant shown, the spring force is 130 N. A friction force of 20 N also exists. Draw the free body diagram and calculate the acceleration of the mass at that instant. **Solution** The spring force acts to the left. The friction force and the inertia force oppose movement and both act to the left. It follows that: $F + F_i + F_s + {}_{Fr} + W = 0$ F = -100 N (Down) W = -Mg = -10 x 9.81 = -98.1 N (Down) F_s = 130 N (Up) F_r = 20 N (Up) -100 + Fi + 130 + 20 - 98.1 = 0 Fi = 48.1 N (Up)  Fi = -Ma 48.1 = -10 a ⇒a = (48.1)/(-10) = -4.81m/s2 negative so down as expected. There is often confusion about the signs to use. If the value is unknown and represented by a symbol, then make it plus (even if sure the result is going to be minus) and it will come out minus to confirm your assumption as in this case.	

Lesson 12: Analysis of torque in machines		**Suggested Teaching Time:** 2 hours
Learning Outcome 2: Understand the dynamics of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
Analyse the torque in machines (A.C. 2.3)	Discuss the relationship between torque power and energy: $W = \int_{\theta_1}^{\theta_2} T\, d\theta$ • Where W is work, Tis torque, and θ_1 and θ_2 represent (respectively) the initial and final angular positions of the body $P = T\omega$ • Where P is power, T is torque, ω is the angular velocity $Power = \frac{Force \times linear\ Distance}{time} = \frac{\left(\frac{Torque}{r}\right) \times (r \times angular\ speed \times t)}{t}$ $= torque \times angular\ speed$ The tutor should work through typical examples of calculations covering the different equations, including both input and output torque, and the learners should then work through other examples of such calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained.	**Books:** Ambekar, Ashok G., *Mechanism and Machine Theory*, Prentice Hall of India Pvt. Ltd, ISBN-10: 8120331346, ISBN-13: 978-8120331341, 2011 Bapaiah, G., *Machine Dynamics (DOM)*, Vol 2, Mechanical Engineering Monograph Series, Indian Institute of Technology Madras Lal, Jagdish, *Theory of Machines and Mechanisms*, Metropolitan Book Co. Pvt. Ltd., ISBN: 97881200002722 Mabie, Hamilton H., and Ocvirk, Fred W., *Mechanisms and Dynamics of Machinery*, John Wiley and Sons, 1957 Rattan, S. S., *Theory of Machines*, Tata McGraw-Hill 2009, ISBN-10: 007014477X, ISBN-13: 978-0070144774

UNIT 518 THE DYNAMICS OF MACHINE SYSTEMS

Lesson 13: Flywheels		**Suggested Teaching Time:** 2 hours
Learning Outcome 2: Understand the dynamics of machines		
Topic	**Suggested Teaching**	**Suggested Resources**

Analyse the operation of a flywheel in a machine
(A.C. 2.4)
	Discuss the concept of a reciprocating machine and how it exerts erratic torque on to a shaft.
	
	Bring in concept of Kinetic energy and how it could be stored in a flywheel:
	• K.E. = I $\omega^2/2$
	  ◦ I is moment of inertia given by formula I = Mk^2
	  ◦ ω is the angular velocity in rad/s
	  ◦ k is the radius of gyration in meters
	  ◦ M is the mass of the wheel
	• For a plain disk I = $MR^2/2$ where R is the outer radius
	• When a rotating body changes speed , the angular acceleration is related to the moment of inertia and the applied torque by the formula T=Iα. Where α is the angular acceleration in Rad/s^2
	
	Develop these concepts into the cyclic torque diagram for a machine and how we can then use these to carry out an energy analysis of a flywheel.
	
	The tutor should work through typical examples of calculations covering the different equations, and the learners should then work through other examples of such calculations.
	
	The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained.
		Books:
		Ambekar, Ashok G., *Mechanism and Machine Theory*, Prentice Hall of India Pvt. Ltd, ISBN-10: 8120331346, ISBN-13: 978-8120331341, 2011
		Bapaiah, G., *Machine Dynamics (DOM)*, Vol 2, Mechanical Engineering Monograph Series, Indian Institute of Technology Madras
		Lal, Jagdish, *Theory of Machines and Mechanisms*, Metropolitan Book Co. Pvt. Ltd., ISBN: 97881200002722
		Mabie, Hamilton H., and Ocvirk, Fred W., *Mechanisms and Dynamics of Machinery*, John Wiley and Sons, 1957
		Rattan, S. S., *Theory of Machines*, Tata McGraw-Hill 2009, ISBN-10: 007014477X, ISBN-13: 978-0070144774

UNIT 518 THE DYNAMICS OF MACHINE SYSTEMS

<table>
<tr><td colspan="2">Lesson 14: Balancing rotating masses</td><td>Suggested Teaching Time: 3 hours</td></tr>
<tr><td colspan="3">Learning Outcome 3: Understand the need for machine balancing</td></tr>
<tr><th>Topic</th><th>Suggested Teaching</th><th>Suggested Resources</th></tr>
<tr><td>Analyse balancing of rotating masses in a machine
(A.C. 3.1)</td><td>Tutor-led discussion on the concepts of balancing and why we need it. Explain the difference between static balancing and dynamic balancing:
Static balance: occurs when there is no resultant centrifugal force and the centre of gravity is on the axis of rotation
Dynamic balance: occurs when there is no resulting turning moment along the axis
Tutor-led instruction to cover a simple rotating disk e.g. in machine where the centre of gravity is not the same as the centre of rotation there will be a single out of balance force and the force applied can be calculated using earlier calculations C.F.= $M \omega^2 r$ in order to balance this an equal and opposite force is needed such that $M_1 \omega^2 r_1 = M_2 \omega^2 r_2$

Develop this into a machine with several masses in one transverse plane, drawing Mr diagrams and solving the missing vector.

The tutor should work through typical examples of calculations covering the different equations, and the learners should then work through other examples of such calculations.

The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained.

Students to be given the opportunity to carry out practical activities involving the balancing of a simple shaft and flywheel assembly.</td><td>Practical equipment:
Laboratory equipment to evaluate the effects of out-of-balance rotating masses in a simple shaft and flywheel assembly

Laboratory equipment to evaluate the effects of out-of-balance rotating masses in a simple machine having masses in different transverse planes

Website:
https://www.youtube.com/watch?v=I3jE-PXV-68</td></tr>
</table>

Analyse balancing of rotating masses in a machine (A.C. 3.1)	Tutor-led discussion to consider machines that are not quite so simple and therefore have masses in different transverse planes. The centrifugal force produced is $F=Mr\omega 2$ The turning moment about the reference plane = T.M. = Fx = $Mr\omega 2x$ For dynamic and static balance we must work out the resultant turning moment and add masses at appropriate points to cancel it out. The appropriate points will be on two planes not coplanar with any of the original masses. This involves drawing two vector diagrams and since ω is common to all vectors we can again take $\omega=1$ and draw vectors representing Mr and Mrx The tutor should work through typical examples of calculations covering the different equations, and the learners should then work through other examples of such calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained. Students to be given the opportunity to carry out practical activities involving the balancing of a simple shaft and flywheel assembly.	

Lesson 15: Balancing reciprocating masses	**Suggested Teaching Time:** 3 hours
Learning Outcome 3: Understand the need for machine balancing	

Topic	Suggested Teaching	Suggested Resources
Analyse balancing of reciprocating masses in a machine (A.C. 3.2)	**Group discussion** Recap lessons learned about slider-crank mechanisms discussing the general layout of piston con rod and crankshaft machine within the machine. Revisit the acceleration equation and work through a couple of examples as revision. **Group teaching** Split the class into several groups and present them with the data for several reciprocating engines and get the groups to produce graphs showing displacement, velocity and acceleration against angle. Discuss results of graphs and how as n gets larger the nearer the results get to being harmonic. Explain using the close approximation for acceleration how the inertia force required to accelerate can be given by $F = M\omega r^2 R\left[\cos(\theta) + \frac{\cos(2\theta)}{n}\right]$ and how this may be thought of as two separate forces Primary forces $F_P = M\omega^2 R\cos(\theta)$ Secondary forces $F_S = M\omega r^2 R\left[\frac{\cos(2\theta)}{n}\right]$	**Practical equipment:** Reciprocating machine with laboratory equipment for measuring vibration

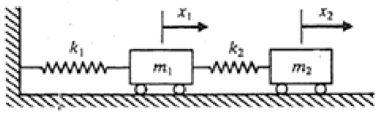
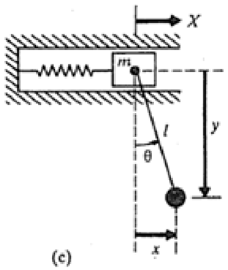
	The tutor should work through typical examples of calculations covering a single reciprocating mass, and the learners should then work through other examples of such calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained **Group discussion** Develop this into a machine with several reciprocating masses in one transverse plane, and discussing complexity of arrangements. The tutor should work through typical examples of primary force calculations for multiple cylinder machines. They should then discuss the complexity of calculating the secondary forces in multiple piston machines and show the development of the formula for calculating the rotating masses. $M\omega 2(R/n)\cos 2\theta = M_S(2\omega)^2 R_S \cos 2\theta = 4 M_S \omega^2 R \cos 2\theta$ The tutor should work through typical examples of secondary force calculations for multiple cylinder machines. The learners should then work through examples of both primary and secondary force calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained. Tutor-led group learning to introduce different methods of balancing including the Lanchester balancer, reciprocating balance and contra rotating masses	

UNIT 518 THE DYNAMICS OF MACHINE SYSTEMS

Lesson 16: Vibration		**Suggested Teaching Time:** 2 hours
Learning Outcome 4: Understand the vibration of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
Explain the causes of vibration in a simple machine system (A.C. 4.1)	**Tutor-led discussion** What is meant by vibration; its causes. Vibration can result from a number of conditions, acting alone or in combination. Problems may be caused by auxiliary equipment, not just the primary equipment. **Imbalance** could be caused by manufacturing defects (machining errors, casting flaws) or maintenance issues (deformed or dirty fan blades, missing balance weights) The effect that machine speed has on the vibration and what effects vibration may have on the machine. **Misalignment**/shaft runout. Vibration can result when machine shafts are out of line. Angular misalignment occurs when the axes of (for example) a motor and pump are not parallel. When the axes are parallel but not exactly aligned, the condition is known as parallel misalignment. Misalignment may be caused during assembly or develop over time, due to thermal expansion, components shifting or improper reassembly after maintenance. The resulting vibration may be radial or axial (in line with the axis of the machine) or both. **Wear**. As components such as ball or roller bearings, drive belts or gears become worn, they may cause vibration. When a roller bearing race becomes pitted, for instance, the bearing rollers will cause a vibration each time they travel over the damaged area. A gear tooth that is heavily chipped or worn, or a drive belt that is breaking down, can also produce vibration. **Looseness**: Vibration that might otherwise go unnoticed may become obvious and destructive if the component that is vibrating has loose bearings or is loosely attached to its mounts. This may or may not be caused by the underlying vibration. Looseness can allow any vibration present to cause damage, such as further bearing wear, wear and fatigue in equipment mounts and other components.	**Websites:** http://www.reliableplant.com/Read/24117/introduction-machinery-vibration http://www.proviso-systems.co.uk/images/stories/pdf/beginners_guide.pdf

Topic	Suggested Teaching	Suggested Resources
Explain the causes of vibration in a simple machine system (continued) (A.C. 4.1)	**Tutor-led discussion** Degrees of freedom recapping info learned in kinematic modelling lesson. Tutor to introduce the concept of free vibration and forced vibration. Under the topic of free vibration the tutor should discuss a pendulum, an object bobbing in the water and a weight on a spring, simple harmonic motion (SHM), emphasising that it occurs naturally without energy being added to the vibrating system and dies away with time as the energy is dissipated. While discussing these bring in natural frequency and the idea of dampening to reduce vibration. Tutor-led group learning, tutor to introduce the concept of forced vibration, develop discussion to cover the concepts of phase and resonance; where the phase relationship between the driving oscillation and the oscillation of the object being driven is different at different frequencies. Below resonance they are in phase with each other. At resonance the phase relationship is 90º or π/2 rad. Above resonance the phase relationship is 180o or π rad.	**Books:** Ramamurti, Viswanatha, *Mechanical Vibration Practice with Basic Theory*, Alpha Science International, 2000 ISBN 0849309751, 9780849309755 Krylov, Victor V., *Noise and Vibration from High-speed Trains*, Thomas Telford, 2001, ISBN 0727729632, 9780727729637 Norton, Michael Peter, Karczub, D. G., *Fundamentals of Noise and Vibration Analysis for Engineers*, Cambridge University Press, 2003, ISBN 0521499135, 9780521499132 **Website:** http://physicsnet.co.uk/a-level-physics-as-a2/further-mechanics/forced-vibrations-resonance/

Lesson 17: Vibration analysis		**Suggested Teaching Time:** 2 hours
Learning Outcome 4: Understand the vibration of machines		
Topic	**Suggested Teaching**	**Suggested Resources**
Analyse a system with one degree of freedom (A.C. 4.2)	Recap the following calculations: • Displacement (x) = R(sin ωt) • Velocity (v) = dx/dt = ωR cos(ωt) • Acceleration (a) = dv/dt = $-\omega^2R\sin(\omega t)$ Set up a pendulum and get students to discuss what will happen when you move the pendulum by applying a force and then remove the force. Show calculations for restoring torque and inertia torque and balance of moments Work through typical examples of pendulum type calculations, the learners should then work through further examples of these calculations. Provide feedback on the answers obtained and repeat the process until consistent answers are obtained. Set up a spring/mass system and get students to discuss what will happen when you raise and release the weight. Work through typical examples of spring mass system type calculations, the learners should then work through further examples of these calculations. Provide feedback on the answers obtained and repeat the process until consistent answers are obtained Set up a shaft/flywheel system and get students to discuss what will happen when you apply a force to the flywheel and release it Work through typical examples of shaft/flywheel system type calculations, the learners should then work through further examples of these calculations. Provide feedback on the answers obtained and repeat the process until consistent answers are obtained.	**Books:** Ramamurti, Viswanatha, *Mechanical Vibration Practice with Basic Theory*, Alpha Science International, 2000 ISBN 0849309751, 9780849309755 **Practical equipment:** Pendulum Stopwatch Spring/Mass system Shaft/flywheel system **Websites:** https://www.youtube.com/watch?v=dRkJuVh9hF0 https://www.youtube.com/watch?v=YbFgNsM6r44

Topic	Suggested Teaching	Suggested Resources
Analyse a system with one degree of freedom (A.C. 4.2)	**Tutor-led discussion** Recap the contents of the lesson and the concept of free vibration and apply to a car's suspension and what it would be like to wait for the vibration to die down naturally. Introduce concept of vibration dampening and cover the term critical damping The tutor should work through typical examples of dampened system type calculations, the learners should then work through further examples of these calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained	**Books:** Ramamurti, Viswanatha, *Mechanical Vibration Practice with Basic Theory*, Alpha Science International, 2000 ISBN 0849309751, 9780849309755 **Practical equipment:** Stopwatch Spring/Mass system shaft/flywheel system Different methods of dampening vibration Laboratory equipment to illustrate systems with two degrees of freedom Simple machine with turbine and compressor on a single shaft to demonstrate theory **Website:** http://www.brown.edu/Departments/Engineering/Courses/En4/Notes/vibrations_mdof/vibrations_mdof.htm http://www.freestudy.co.uk/dynamics/holzer.pdf http://www.freestudy.co.uk/dynamics/holzer.pdf.
Analyse the normal modes of vibration in a system with two degrees of freedom (A.C. 4.3)	 (a)  (c) $m_1 \frac{d^2x_1}{dt^2} + (k_1 + k_2)x_1 - k_2x_2 = 0$ $m_2 \frac{d^2x_2}{dt^2} - k_2x_1 + (k_2 + k_3)x_2 = 0$ Recap calculations covered in previous lessons and adapt to systems with two degrees of freedom as illustrated. The tutor should work through typical examples of calculations for systems with two degrees of freedom, both dampened and un dampened, the learners should then	

	work through further examples of these calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained	
Analyse torsional vibration of a multi-mass system using Holzer's method (A.C. 4.4)	**Tutor-led group discussion** Expand principles learned about a simple flywheel and shaft arrangement to a shaft with multiple disks, introducing concepts of: • Multiple inertia • Torsion in shafts • Simple harmonics The tutor should work through typical examples of calculations for systems with multiple masses using the Holzer method, the learners should then work through further examples of these calculations. The tutor should provide feedback on the answers obtained and repeat the process until consistent answers are obtained.	

Lesson 18: Vibration reduction		**Suggested Teaching Time:** 2 hours
Learning Outcome 4: Understand the vibration of machines		
Topic	**Suggested Teaching**	**Suggested Resources**

Evaluate **methods for reducing vibration** in a machine
(A.C. 4.4)
	Tutor-led group discussion
	What methods could be used to reduce vibration? Build on material covered to date. Topics to cover include:
	• Reducing harmonic forces
	  ○ When a shaft rotates it may go into transverse oscillations, if the shaft is out of balance the resulting centrifugal forces will cause the shaft to vibrate. When the shaft rotates at a speed equal to the natural frequency of transverse oscillations, this vibration becomes very large. It also occurs at multiples of the resonant speed. This can be very damaging to heavy rotary machines such as turbine generator sets and the system must be carefully balanced to reduce the effect and designed to have a natural frequency different to the speed of rotation.
	• Vibration isolation
	  ○ Vibration will transfer from a machine to surrounding structures if it is bolted directly to the structure, this is why car engines are mounted on rubber blocks to reduce the transmission of vibration to the car.
		Books:
		Norton, Michael Peter, Karczub, D. G., *Fundamentals of Noise and Vibration Analysis for Engineers*, Cambridge University Press, 2003, ISBN 0521499135, 9780521499132
		
		Practical equipment:
		Practical machinery that can be adapted to reduce vibration
		
		Website:
		https://www.youtube.com/watch?v=xktZSII_bfY
		http://www.freestudy.co.uk/dynamics/damped%20vibrations.pdf
		http://www.freestudy.co.uk/dynamics/damped%20vibrations.pdf
		http://www.freestudy.co.uk/d225/t14.pdf