9209-513 Resource materials

Candidates should familiarise themselves with this document throughout the course and will need to refer to a clean copy of this document in the exam. For the sample questions only the mathematical formulae needs to be referred to, not the Table of Laplace Transforms.

Short Table of Laplace Transforms

f(t)	$\boldsymbol{F(s) = \int_0^\infty f(t)e^{-st}dt}$
$af_1(t) + bf_2(t)$	$aF_1(s) + bF_2(s)$
$\frac{d}{dt}f(t)$	$sF(s) - f(0)$
$\frac{d^2}{dt^2}f(t)$	$s^2F(s) - sf(0) - \frac{df(t)}{dt}(0)$
Initial value: $f(t), t \to 0$	$sF(s), s \to \infty$
Final value: $f(t), t \to \infty$	$sF(s), s \to 0$
Unit step: *H*(*t*) $\frac{1}{s}$	
Constant: c	$\frac{c}{s}$
t	$\frac{1}{s^2}$
$\frac{1}{2}t^2$	$\frac{1}{s^3}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$te^{-\alpha t}$	$\frac{1}{(s+\alpha)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-\alpha t} \cos \omega t$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Mathematical Formulae Sheet

Taylor series expansion of $f(a + x)$:

$$f(a+x) = f(a) + \frac{x}{1!}f^{(1)}(a) + \frac{x^2}{2!}f^{(2)}(a) + \frac{x^3}{3!}f^{(2)}(a) + \cdots$$

Where x is the displacement measured from the fixed point a

where $f^{(n)}(a) =$ n'th derivative of $f(x)$ evaluated at $x = a$.

Maclaurin series expansion of $f(x)$:

This has the same expansion as for the Taylor series but with a = 0 thus,

$$f(x) = f(0) + \frac{x}{1!}f^{(1)}(0) + \frac{x^2}{2!}f^{(2)}(0) + \frac{x^3}{3!}f^{(2)}(0) + \cdots$$

Fourier series description of $f(x)$:

(a) for functions with period 2π

$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$, where

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \cos nx\, dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \sin nx\, dx$

(b) for functions $f(t)$ with period T in seconds

i.e. frequency in hertz $f_h = \frac{1}{T}$ or angular frequency $\omega = \frac{2\pi}{T}$

$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\ \cos n\omega t + b_n\ \sin n\omega t$, where

$a_0 = \frac{2}{T}\int_0^T f(t)\, dt$

$a_n = \frac{2}{T}\int_0^T f(t)\ \cos n\omega t\, dt$

$b_n = \frac{2}{T}\int_0^T f(t)\ \sin n\omega t\, dt$

Trapezoidal Rule using *n* subintervals of the interval [*a*,*b*] each of width *h*:

$\int_a^b f(x)\, dx\ \approx\ \frac{h}{2}[f(a) + f(b) + 2\ \sum_{k=1}^{n-1} f(a+kh)]$

Simpson's Rule with even number (n) of subintervals for [*a*,*b*], each of width *h*:

$$\int_a^b f(x)\,dx \;\approx \frac{h}{3}\left[f(a) + f(b) + 2\,\sum_{r=1}^{n-1} f(a+2rh) + 4\,\sum_{r=1}^{n} f(a+\{2r-1\}h)\right]$$

Euler numerical method for the solution of $\frac{dy}{dx} = f(x,y)$ using a step size *h*:

$$y_{n+1} = y_n + h\,f(x_n, y_n)$$

Improved Euler numerical method:

$y_{n+1}^0 = y_n + h\,f(x_n, y_n)$ then

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f^0(x_{n+1}, y_{n+1}^0)\right]$$