Level 3 Certificate in Using and Applying Mathematics

Pre-release material

This booklet will be issued to centres **two months** in advance of the date of examination.

Centres should issue a copy of this booklet to all candidates entered for the examination.

Centres should ensure that candidates are familiarised with the contexts and information contained in this booklet in preparation for the examination.

All examination questions will be based on this material.

Candidates may not take their own copies of this booklet into the examination. Copies will be issued at the start of each examination session.

Drug decay

Drugs can be considered to decay exponentially in the body, and therefore can be considered to have a half-life.

In such cases the amount of drug, A, in your body at time, t, can be modelled by the exponential function

$$A = A_0 e^{-\left(\frac{0.693}{T_{1/2}}\right)t}$$

where A_0 is the amount of drug in your body at time $t = 0$ and $T_{1/2}$ is the half-life of the drug.

When prescribed a drug by a doctor, or if taking a pain killer such as paracetamol, for example, it is usual to take some of the drug at regular intervals throughout the day.

For example:

- if you take a single 250 mg dose of a painkiller with a half-life of two hours there will be a level of 62.5 mg of the drug in your body after 4 hours
- if you take 250 mg of the painkiller every 4 hours, after 8 hours, having just taken the third dose of the painkiller, you will have a level of 328.125 mg of the drug in your body.

Speeding up

You will be aware that every so often world records in athletics are broken providing evidence that as the years pass it seems that humans can run faster, jump further and higher, and so on. Figure 1 demonstrates this for the 100 metres, as it shows graphs of the winning times for men and women in the Olympic Games for this event, since 1928. As you can see the time taken to win a gold medal is getting less over the years, although in every Olympic year men have outperformed women. Figure 1 shows an interesting trend: although both men and women are improving their performance, women are improving at a faster rate than men.

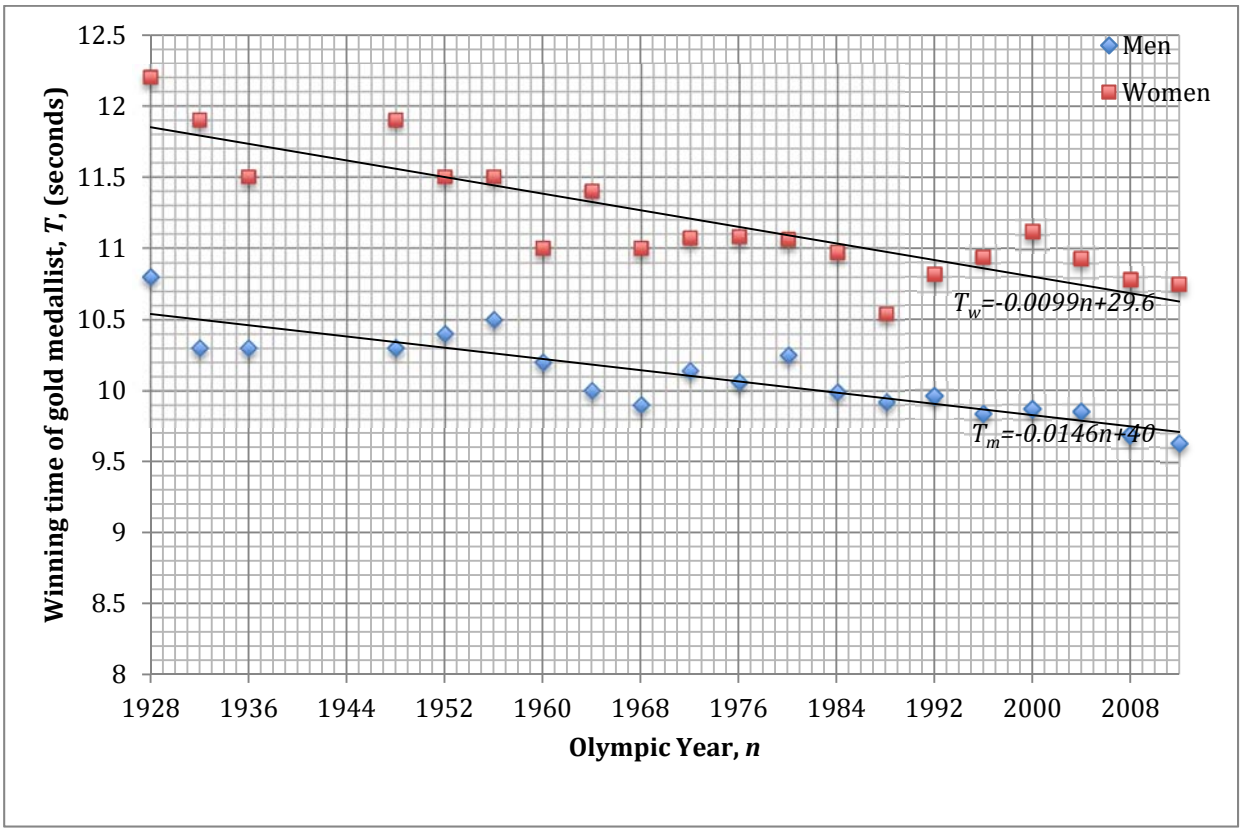

Figure 1. Graph showing gold medal winning times for 100 metre sprint at Olympic Games since 1928.

This would suggest that, at some point in the future, women will run the 100 metres in the same time as men and eventually win their event in a faster time than men.

The year in which men and women will win the 100 metre Olympic finals in the same time can be predicted using the equations for the lines of best fit. These can be taken to be:
for men: $T_{\mathrm{m}} = -0.0146n + 40$
for women: $T_{\mathrm{w}} = -0.0099n + 29.6$

These equations provide two models that we can use to predict, for any year, *n*, the winning times for the Olympic 100 metres. Of course, in reality *n*, should be restricted to be an Olympic year, which means that when *n* is divided by four the result should be an integer value.

Solving these equations simultaneously suggests that in 2212 (n = 2212.766) men and women will run 100 metres in exactly the same time, 7.69 seconds. So in the Olympic Games of that year, or maybe in the next games, we might expect women to run faster than men.

The fact that women are improving at a faster rate than men is also demonstrated by the graph in Figure 2 which rather than showing the winning times shows the average speed of the gold medallists in each Olympic year. In this diagram the gradients of the lines of best fit for men and women, therefore, give a rate of change of speed (in metres per second per year). For men the value of this can be found to be 0.0096 and for women it can be found to be 0.0117.

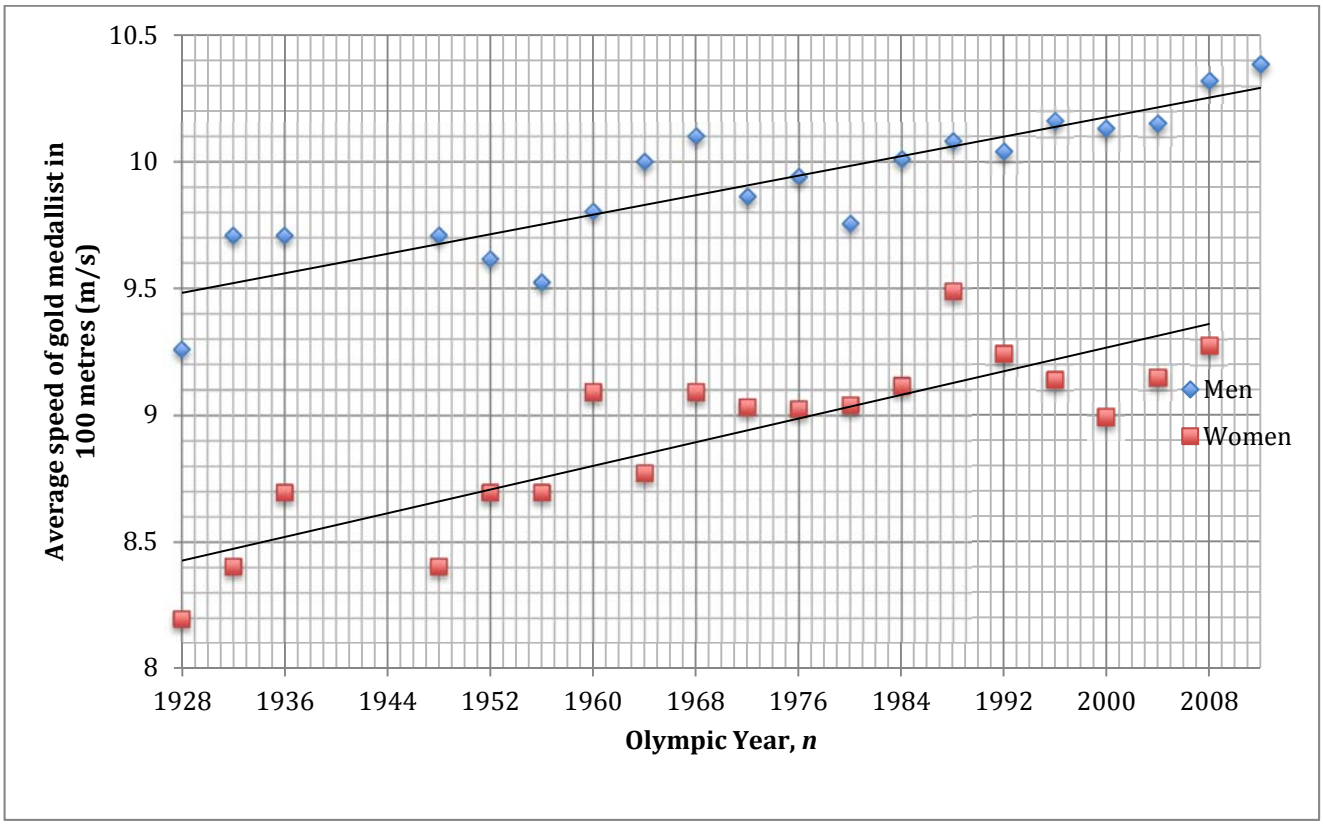

Figure 2. Graph showing gold medal winning average speeds for 100 metre sprint at Olympic Games since 1928.

A similar analysis can be carried out for other events. Figure 3, for example, is a graph showing gold medal winning times for men and women for the 10 000 metres at Olympic Games since 1988.

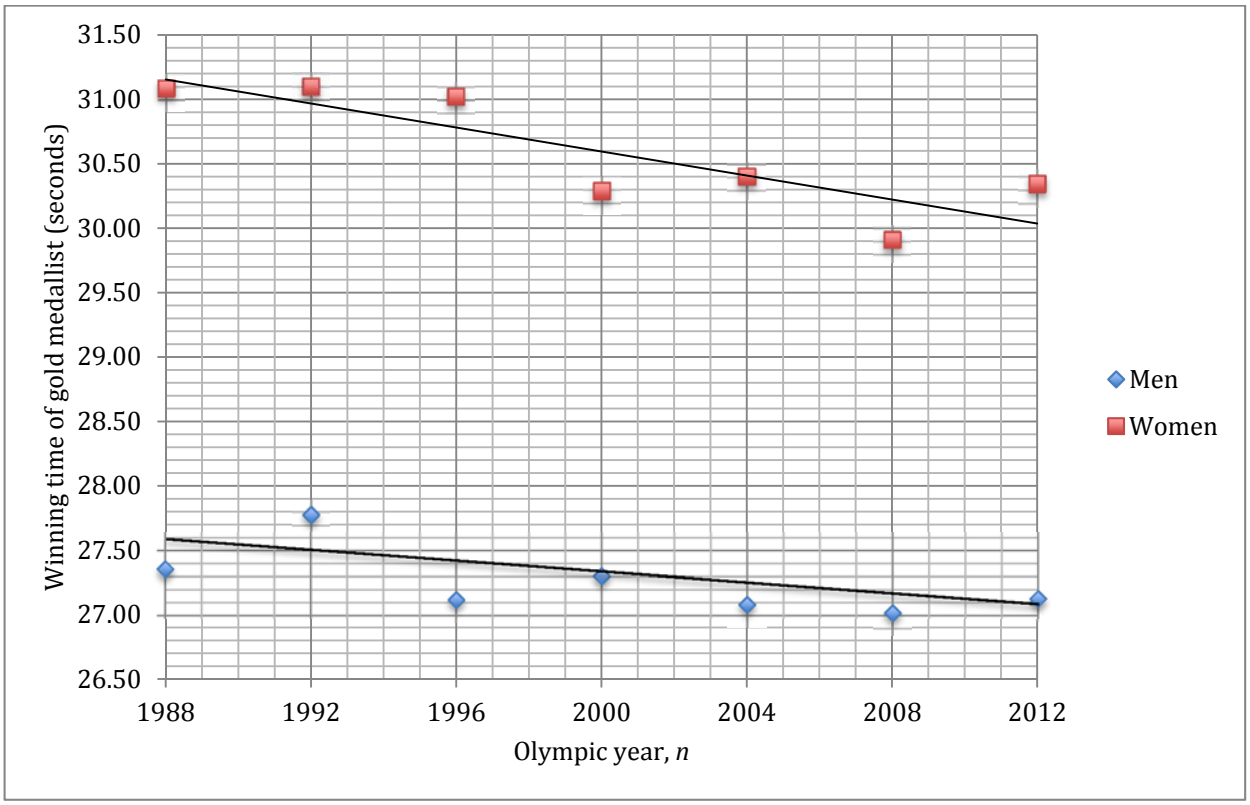

Figure 3. Graph showing gold medal winning times for the 10 000 metres events for men and women at the Olympic Games since 1988.

For this event the lines of best fit have the following equations:
for men: $T_m = -0.0466n + 123.8$
for women: $T_w = -0.0199n + 67.16$

Again solving these simultaneously suggests that men and women will run the 10000 metres in the same time during 2121.

This provides some evidence to support the interesting phenomenon that women appear to be improving more quickly than men in longer distance events than they are in sprinting.

Making a quick buck

The Boston Post EXTRA

DECLARES PONZI IS NOW HOPELESSLY INSOLVENT

Publicity Expert Employed by "Wizard" Says He Has Not Sufficient Funds to Meet His Notes—States He Has Sent No Money to Europe Nor Received Money From Europe Recently

NEW MOVE TO FOIL MANNIX

POISON AS ANTIDOTE FOR LOVE

HOLD SOLDIER FOR MURDER

Money in Banks Taken From Investors

Tells of Visits to Officials Last Monday

TREMONT DAY

Charles Ponzi was an Italian who emigrated to the United States where he put his mind and talents to making a fortune. His money making scheme is now world famous as, although it made him very rich in the short run, it led to his downfall and prison. His 'Ponzi scheme' was fraudulent as it promised investors unbelievable returns for their money that was in fact paid for by unsuspecting later investors.

Ponzi promised 50% interest after 45 days or 100% interest after 90 days. He was in effect offering to double an investment after just 90 days!

In the early days after starting up the investment scheme Ponzi was able to pay out such levels of interest as his 'get rich quick' scheme attracted many investors keen to double their money in such a short space of time. Initially he had set up the scheme to raise money to fund another scheme which would make money by buying and selling at advantageous exchange rates between Italy and the United States. This soon became problematic but he persisted with his investment scheme and found that in the first few months he attracted so many investors so quickly that he was able to pay the interest promised to those who wished to withdraw their money from the scheme.

To see how it works a simple model is helpful.
Suppose each investor invests $100 and cashes in their investment after 90 days. That is they take their original investment plus 100% interest out of the scheme. Assume that at the start of the scheme Ponzi manages to attract 100 investors so he has to be able to have available the original $10,000 at the end of 90 days and an additional $10,000. If during this period 200 new investors can be attracted the original 100 investors can be paid what is owed.

The spreadsheet table in Figure 4 shows the required number of investors, investment amounts and amount paid out every 90 days for the case of this simplified model.

	A	B	C	D	E	F	G
1	n	day	new investors	cumulative no. investors	total invested (this period)	Interest payable	Total paid out
2	0	0	100	100	$10,000		
3	1	90	200	300	$20,000	$10,000	$20,000
4	2	180	400	700	$40,000	$20,000	$40,000
5	3	270	800	1500	$80,000	$40,000	$80,000
6	4	360	1600	3100	$160,000	$80,000	$160,000
7	5	450	3200	6300	$320,000	$160,000	$320,000
8	6	540	6400	12700	$640,000	$320,000	$640,000
9	7	630	12800	25500	$1,280,000	$640,000	$1,280,000
10	8	720	25600	51100	$2,560,000	$1,280,000	$2,560,000
11	9	810	51200	102300	$5,120,000	$2,560,000	$5,120,000
12	10	900	102400	204700	$10,240,000	$5,120,000	$10,240,000
13	11	990	204800	409500	$20,480,000	$10,240,000	$20,480,000
14	12	1080	409600	819100	$40,960,000	$20,480,000	$40,960,000
15	13	1170	819200	1638300	$81,920,000	$40,960,000	$81,920,000
16	14	1260	1638400	3276700	$163,840,000	$81,920,000	$163,840,000

Figure 4. A simple model of Ponzi's investment scheme.

Figure 5 shows the formula in each cell of the spreadsheet:

	A	B	C	D	E	F	G
1	n	day	new investors	cumulative no. investors	total invested (this period)	Interest payable	Total paid out
2	0	0	100	100	=C2*100		
3	1	90	=G3/100	=D2+C3	=C3*100	=E2	=E2+F3
4	2	=B3+90	=G4/100	=D3+C4	=C4*100	=E3	=E3+F4
5	3	=B4+90	=G5/100	=D4+C5	=C5*100	=E4	=E4+F5
6	4	=B5+90	=G6/100	=D5+C6	=C6*100	=E5	=E5+F6
7	5	=B6+90	=G7/100	=D6+C7	=C7*100	=E6	=E6+F7
8	6	=B7+90	=G8/100	=D7+C8	=C8*100	=E7	=E7+F8
9	7	=B8+90	=G9/100	=D8+C9	=C9*100	=E8	=E8+F9
10	8	=B9+90	=G10/100	=D9+C10	=C10*100	=E9	=E9+F10
11	9	=B10+90	=G11/100	=D10+C11	=C11*100	=E10	=E10+F11
12	10	=B11+90	=G12/100	=D11+C12	=C12*100	=E11	=E11+F12
13	11	=B12+90	=G13/100	=D12+C13	=C13*100	=E12	=E12+F13
14	12	=B13+90	=G14/100	=D13+C14	=C14*100	=E13	=E13+F14
15	13	=B14+90	=G15/100	=D14+C15	=C15*100	=E14	=E14+F15
16	14	=B15+90	=G16/100	=D15+C16	=C16*100	=E15	=E15+F16

Figure 5. Formulae behind the simple model of Ponzi's investment scheme.

The problem with the scheme is evident if you look at the cumulative number of investors required after say 12 periods, that is 819,100. After only two more periods this has risen to over 3 million. It seems clear that the scheme is going to

run into trouble, since even in a large country like the United States it will eventually be difficult to attract enough investors.

However, because the scheme is so attractive, Ponzi attracted many investors in the early stages: more than the number needed to pay those who wanted to withdraw their investment. Some investors also left their money in the scheme for longer than 90 days. This meant that Ponzi was able to divert some of their money to fund his increasingly lavish lifestyle. However, as soon as it became difficult to attract enough investors to pay back those who had already invested the scheme collapsed.

It would be possible to run a similar scheme without offering such a high interest rate. For example Figure 6 shows the situation for an interest rate of 50%. This reduces the number of investors needed at the start of any new 90-day period but will still eventually run into trouble for the same reasons as when the interest rate was 100%.

	A	B	C	D	E	F	G
1	n	day	new investors	cumulative no. investors	Total invested (this period)	Interest payable	Total paid out
2	0	0	100	100	$100,000		
3	1	90	150	250	$150,000	$50,000	$150,000
4	2	180	300	550	$300,000	$75,000	$30,000
5	3	270	600	1150	$600,000	$150,000	$60,000
6	4	360	1200	2350	$1,200,000	$300,000	$120,000
7	5	450	2400	4750	$2,400,000	$600,000	$240,000
8	6	540	4800	9550	$4,800,000	$1,200,000	$480,000
9	7	630	9600	19150	$9,600,000	$2,400,000	$960,000
10	8	720	19200	38350	$19,200,000	$4,800,000	$1,920,000
11	9	810	38400	76750	$38,400,000	$9,600,000	$3,840,000
12	10	900	76800	153550	$76,800,000	$19,200,000	$7,680,000
13	11	990	153600	307150	$153,600,000	$38,400,000	$15,360,000
14	12	1080	307200	614350	$307,200,000	$76,800,000	$30,720,000
15	13	1170	614400	1228750	$614,400,000	$153,600,000	$61,440,000
16	14	1260	1228800	2457550	$1,228,800,000	$307,200,000	$122,880,000

Figure 6. A simplified model of a modified version of Ponzi's investment scheme (paying 50% interest).

Crime data

The data below is extracted from the *Crime Survey for England and Wales, Office for National Statistics*.

England and Wales

Offence group	**Jan-12 to Dec-12**	**Jan-13 to Dec-13**
	Number of incidents (000s):	
Vandalism	**1,811**	**1,540**
Vehicle vandalism	1,243	1,060
Other vandalism	568	480
Burglary	**633**	**610**
With entry	374	361
Attempts	259	249
With loss	261	235
No loss (including attempts)	372	374
Vehicle-related theft	**1,073**	**964**
Theft from vehicles	792	744
Theft of vehicles	92	65
Attempts of and from	188	155
Bicycle theft	**482**	**377**
Other household theft	**1,355**	**1,017**
Unweighted base – household crime	*36,595*	*36,315*
Theft from the person	**575**	**565**
Snatch theft from person	58	66
Stealth theft from person	517	500
Other theft of personal property	**1,023**	**939**
All violence	**1,981**	**1,537**
Wounding	482	344
Assault with minor injury	504	405
Assault without injury	748	632
Robbery	247	156
Violence with injury	1,074	808
Violence without injury	907	728
Domestic violence	387	315
Acquaintance	705	493
Stranger	631	568
Mugging (robbery + snatch theft)	305	222
Unweighted base – personal crime	*36,625*	*36,354*
All acquisitive crime	**5,388**	**4,629**
Household acquisitive crime	3,543	2,968
Personal acquisitive crime	1,845	1,660
ALL HOUSEHOLD CRIME	**5,354**	**4,508**
ALL PERSONAL CRIME	**3,580**	**3,041**
Unweighted base	*36,625*	*36,354*