3849-301 Level 3 Certificate in Using and Applying Mathematics

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Pre-release material

Sample assessment 3

This booklet will be issued to centres two months in advance of the date of examination.

Candidates will be issued with a clean copy of this booklet for each paper sitting. Copies will be issued at the start of the examination session and collected at the end of the session. Candidates must **not** take their own copies of this booklet into the examination.

Centres should ensure that candidates are familiarised with the contexts and information contained in this booklet in preparation for the examination.

All examination questions will be based on this material.

This booklet contains a set of four documents
1) Repaying loans
2) Supply and demand
3) Lottery numbers
4) Counting calories.

1) Repaying loans

If you take out a loan, for example from a bank or building society, you will be charged interest each month and the time it will take for you to pay back the loan will depend on the interest rate and how much you pay back each month. **Figure 1** shows typically how the loan amount outstanding (the amount you have left to repay) varies each month when you pay the same amount back each month.

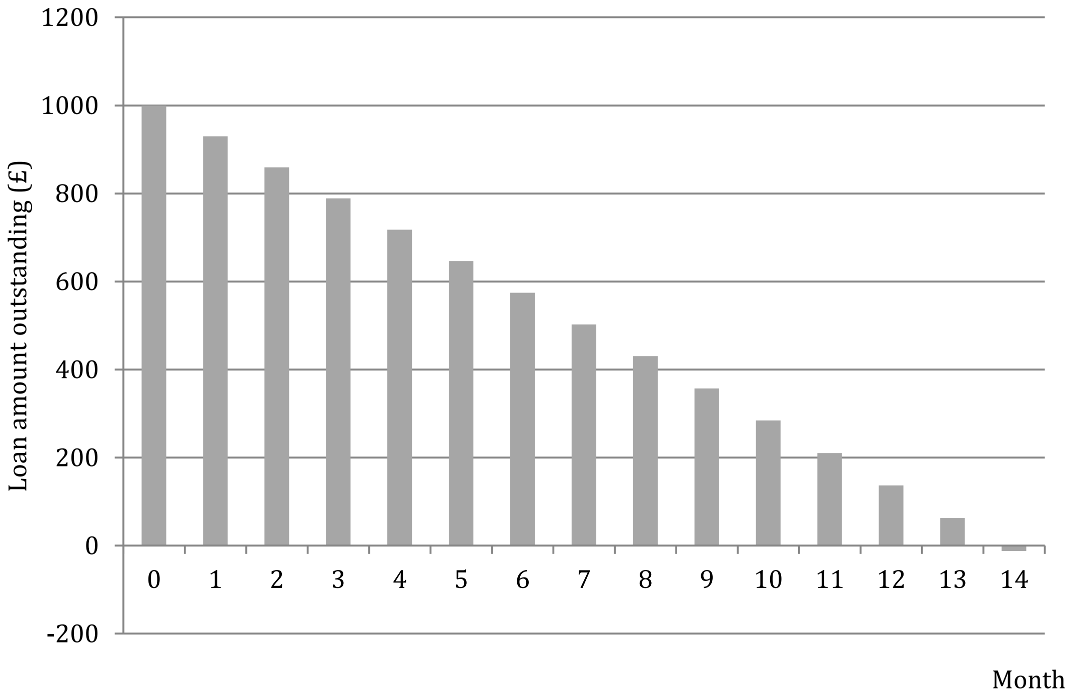

Figure 1. Graph showing how the loan amount outstanding varies each month when there is a fixed interest rate and a fixed amount is paid back each month.

The loan amount outstanding at the end of each month can be found using
$u_n = ku_{n-1} - c$

where

u_n represents the loan amount outstanding, in £, after n repayments.
$k = 1 + r$ and r is the monthly interest rate expressed as a decimal
c represents the fixed amount of the monthly repayment in pounds (£).

If the amount of the initial loan is £1000, the interest rate is fixed at 0.5% per month and the loan is repaid at a rate of £75 per month,
then $u_0 = 1000$, $k = 1.005$, $c = 75$ and
$u_n = 1.005 \times u_{n-1} - 75$
So $u_1 = 1.005 \times 1000 - 75 = 930$

The table in **Figure 2** shows how the loan amount outstanding varies with each passing month. These data have been used to plot the graph in **Figure 1**.

Month	Loan amount outstanding (£)
0	1000
1	930
2	859.65
3	788.95
4	717.89
5	646.48
6	574.71
7	502.59
. . .	. . .

Figure 2. Table showing how the loan amount outstanding of £1000 varies when the interest rate is fixed at 0.5% monthly and the amount paid back is fixed at £75 per month.

The graph in **Figure 3** shows how the loan amount outstanding varies each month for an initial loan of £500 with a fixed interest rate of 2% per month when paying back a fixed amount of £50 each month.

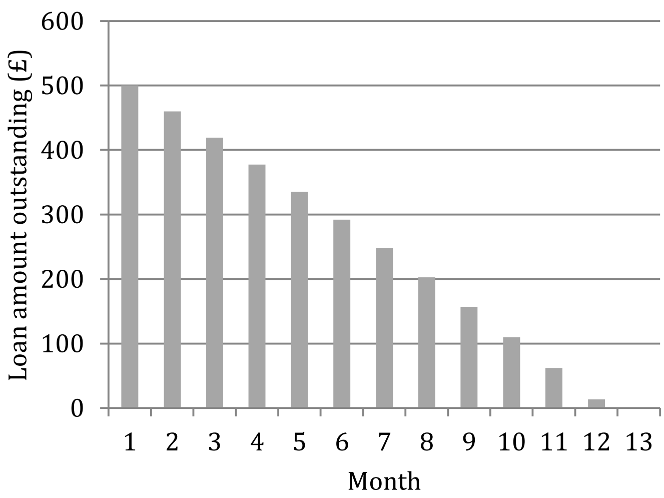

Figure 3. Graph showing how the amount outstanding on a loan of £500 varies each month with a fixed interest rate of 2% per month when paying back a fixed amount of £50 each month.

2) Supply and demand

Often as the price of something is increased people may be less likely to buy it and as its price falls people are more likely to buy it. Such behaviour can be represented by the graph in **Figure 4**. This is known as a demand curve.

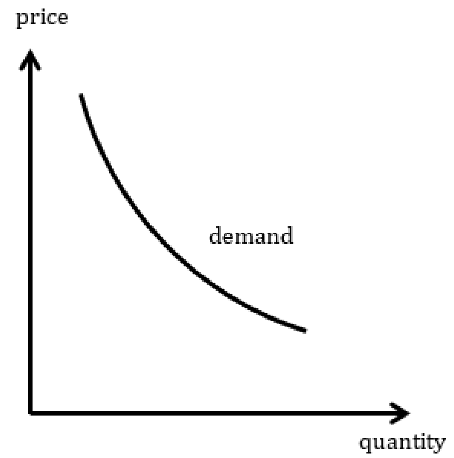

Figure 4. Demand curve.

Consider for example, fruit farmers who, when selling strawberries, want to maximise their profits. As the fruit farmers increase their prices demand falls. On the other hand as they decrease their prices demand rises.

If prices for strawberries are low, fruit farmers are most likely to supply fewer strawberries. Whereas, on the other hand, if prices for strawberries are high they will be willing to supply a lot more. Such behaviour can be represented by the graph in **Figure 5**. This is known as a supply curve.

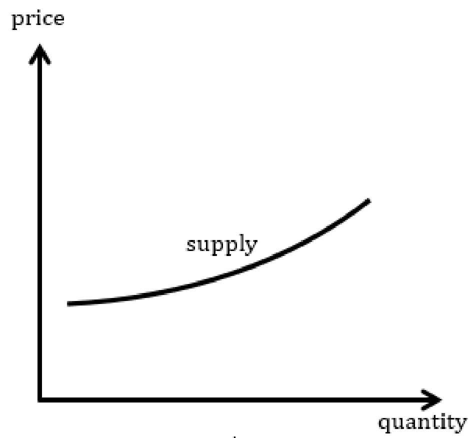

Figure 5. Supply curve.

Economists often bring the two graphs together in a single diagram showing both supply and demand curves (**Figure 6**). This shows that there is one point where the graphs of the supply and demand curves intersect. This point is known as the equilibrium point. At this point fruit farmers are supplying the exact quantity of strawberries that consumers want to buy at a certain price.

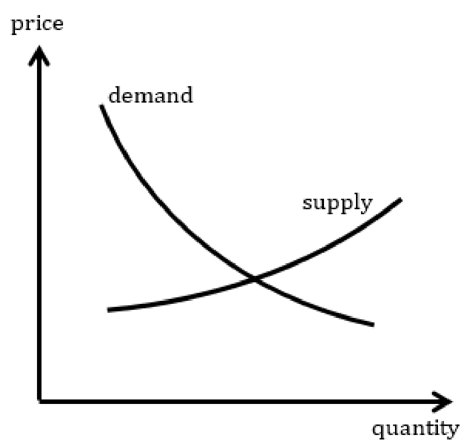

Figure 6. Supply and demand curves combined.

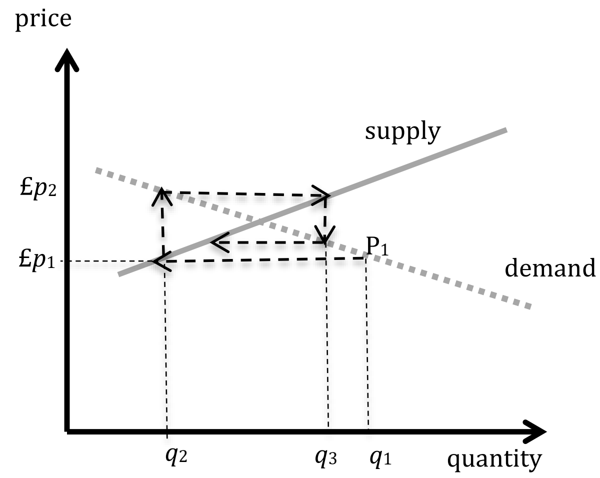

Figure 7. How supply and demand may affect prices.

It is demand that determines the price of goods. Consider for example, how a fruit farmer decides how to price his strawberries so that he can cover his costs and make a profit. If demand is low the farmer will need to increase prices, whereas if it is high he will have the opportunity to decrease them.

Suppose **Figure 7** shows a graph with both supply and demand curves for the sale of strawberries. (Note that here these have been modelled as straight lines).
In Month 1 demand was at point P_1 where a quantity q_1 were being sold at a price of £p_1. As a result in Month 2 farmers supplied a quantity q_2 at a price £p_1 and because of the demand for this quantity found that they were able to charge a price £p_2. In the following month farmers would therefore supply a quantity q_3 at a price £p_2 and because of the demand for this quantity would be able to charge an amount £p_3 and so on. Over a number of months the price would settle down at the point where supply and demand were in equilibrium at the intersection of the supply and demand curves.

Notice if you plot points (q_1, p_1), (q_2, p_1), (q_2, p_2), (q_3, p_2), (q_3, p_3), (q_4, p_3), (q_4, p_4) and so on you will get points that alternately lie on the demand and supply curves. This gives rise to the 'cobwebbing' shown in **Figure 7**. The term 'cobwebbing' is used because the diagram looks something like a spider's cobweb.

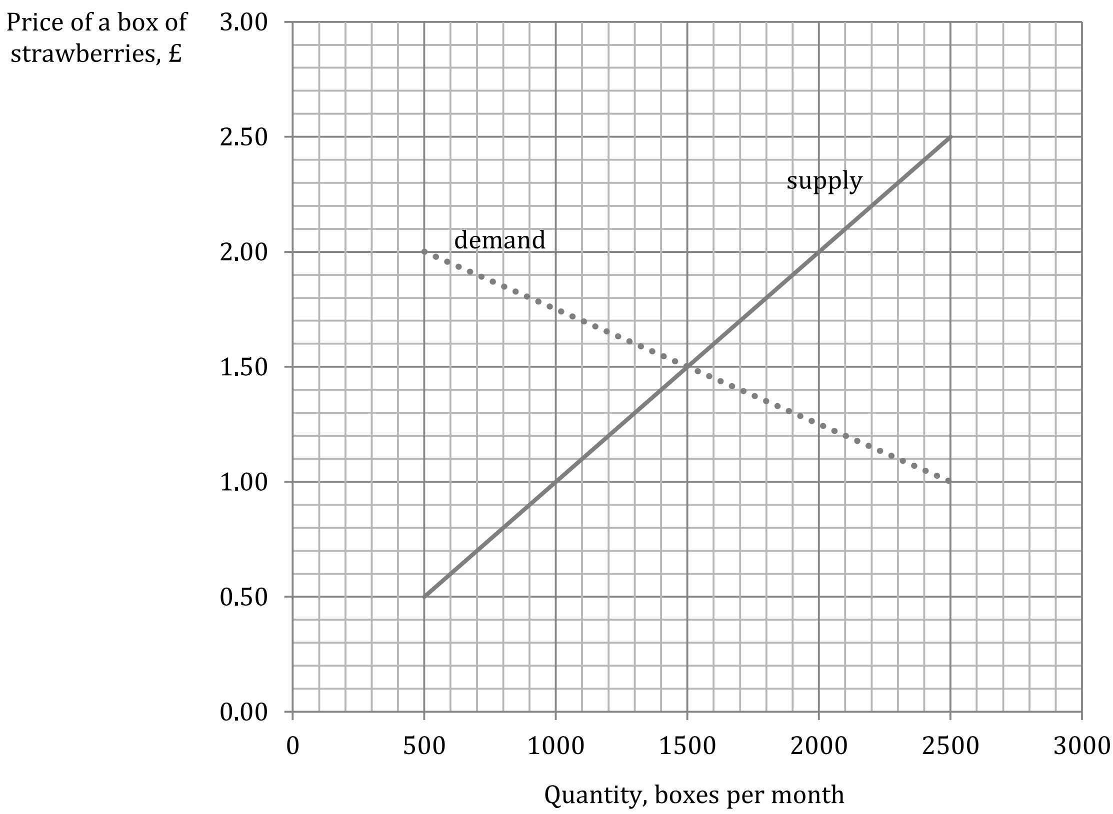

Figure 8. Supply and demand curves for strawberries.

Figure 8 shows both supply and demand curves (modelled as straight lines in this case) for the sale of boxes of strawberries. From this we can see that if a box of strawberries is priced at £1 the farmer is willing to supply 1000 boxes per month but customers are willing to buy (demand) more than this, 2500 per month.

3) Lottery numbers

Mathematical ideas can be used in many ways to make sense of the national lottery.

A relatively simple first question to ask is, "What are the chances of winning?"

The lottery draw involves randomly selecting six balls from 59 that are numbered from 1 to 59.

If these six numbers have been selected by a player (in any order) on their lottery ticket then they win the jackpot. To work out the probability of winning the jackpot we need to know how many different ways six balls can be selected from the 59 different balls. This is found by calculating

$$\frac{59x58x57x56x55x54}{6x5x4x3x2x1} = 45{,}057{,}474$$

meaning that there is a one in approximately 45 million chance of winning the jackpot.

To understand why you need to carry out this particular calculation think about the situation of selecting just two balls from three balls that are numbered 1, 2 and 3. There are three different balls that could be the first selected and just two remaining - either of which could be the second ball selected. This is shown in the table in **Figure 9**.

1st ball	2nd ball
3	2
	1
2	3
	1
1	3
	2

Figure 9. Table showing different ways of selecting two balls out of three.

This means that there are 6 different ways of selecting the first and second balls. If, however, we are only interested in what numbers are on the two balls it doesn't matter which order the balls are selected. Hence there are $\frac{3\times 2}{2\times 1} = 3$ ways of selecting the two balls to get different combinations of numbers. In this calculation 3x2 gives the total number of different ways of selecting the first and then the second ball. This is divided by 2x1 because for each pair of balls there are two different ways of selecting the first of the two numbers and then only one way of selecting the second number remains.

Some people think that some of the balls occur in the draw more frequently than others and base their choices on seeking numbers that are more likely to be drawn. It is indeed the case that some numbers have been drawn more frequently than others but this doesn't mean that the draw is fixed. This is exactly what we would expect. Like many things that occur randomly in nature we find that the distribution of how frequently the different balls occur might be modelled by a normal distribution.

To investigate this we need to look at what happens over a large number of draws of the lottery balls. The number of balls has only relatively recently been increased to 59 from the 49 which were used from November 1994 until October 2015. The table in **Figure 10** shows the number of times each of the balls 1 – 49 had appeared in the first 2065 draws. These data are grouped in **Figure 11** and a histogram drawn of the grouped data in **Figure 12**.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
234	241	249	248	246	256	250	239	267	258	269	252	215	245	238	223	257

18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
258	251	215	228	252	282	260	269	242	266	254	250	274	249	249	277	249

35	36	37	38	39	40	41	42	43	44	45	46	47	48	49		
267	239	230	275	268	279	232	252	267	274	247	240	263	261	264		

Figure 10. Table showing the number of times each lottery ball was drawn as a winning ball in the first 2065 draws when 49 balls were used.

Range of no. draws	215-224	225-234	235-244	245-254	255-264	265-274	275-284
No. of balls	3	4	6	14	8	10	4

Figure 11. Table showing grouped frequency data for the number of times each of the 49 lottery balls was drawn in the National Lottery in the first 2065 draws using 49 balls.

It turns out that this can be modelled by a normal distribution using the mean and standard deviation of the data. These values can be found using either the actual data or the grouped data. Using the actual data we can calculate that (mean = 252.8, standard deviation = 16.2); using the grouped data we can calculate mean = 253 and standard deviation = 16.11. Of course these values are very close to each other but not quite the same.

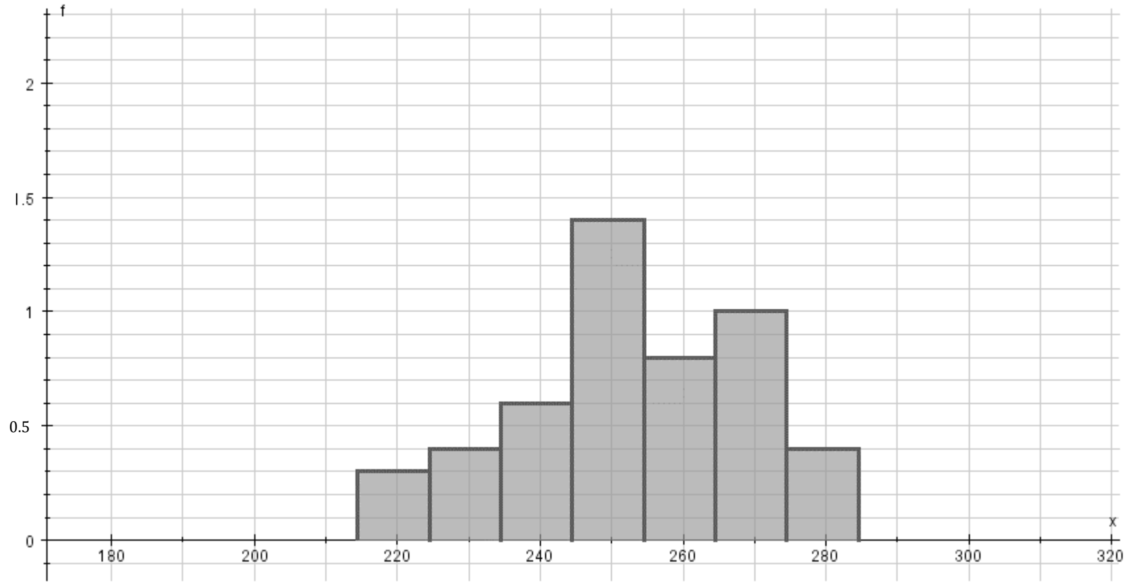

Figure 12. Histogram of the grouped frequency data for the number of times each of the 49 lottery balls has been drawn in the National Lottery in the first 2065 draws.

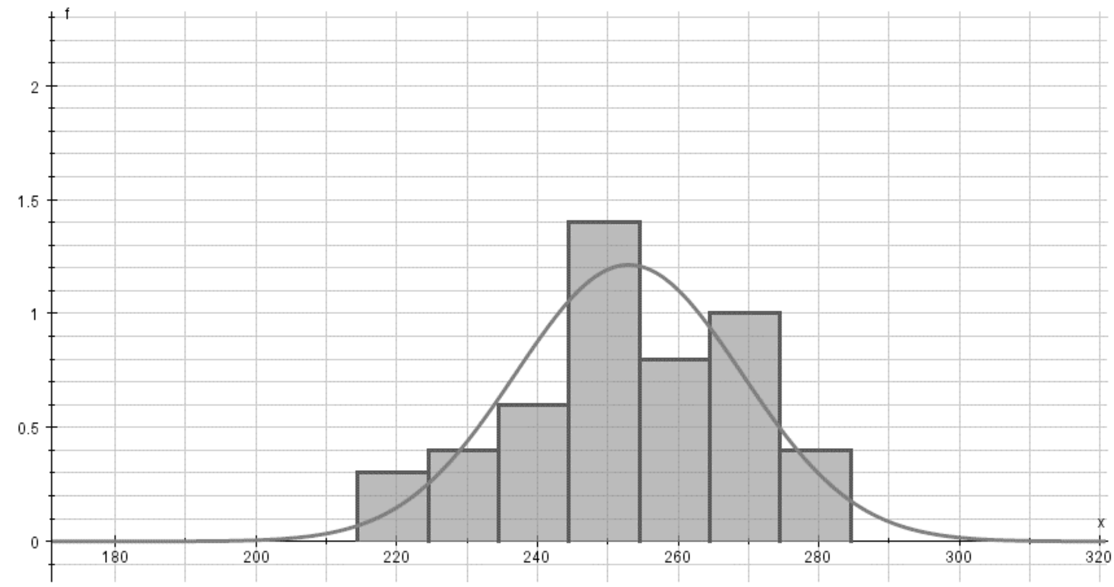

Figure 13. Normal distribution superimposed on the histogram of grouped frequency data for the number of times each of the 49 lottery balls was drawn in the first 2065 draws of the National Lottery.

Figure 13 shows the histogram of the grouped frequency data redrawn with a normal distribution superimposed taking the mean to be 253 and the standard deviation to be 16.11. This illustrates that the theoretical model approximately fits the grouped frequency data although perhaps after many more lottery draws the fit would be closer.

For the case where a lottery is held with 49 balls such a normal distribution as a model will allow us to answer (only approximately) questions such as “what is the probability that a ball is drawn more than 270 times in 2065 draws of the National lottery?” It also allows us to make statements such as “we can expect about two-thirds of the balls to be drawn between 237 and 269 times in 2065 draws of the National lottery”.

4) Counting calories

Your basal metabolic rate, **BMR**, is a measure of how many calories[1] you burn just being alive and awake. It depends on your gender, age, weight and body height.

The formulae below in **Figure 14** tell us how to calculate the basal metabolic rate for men and women.

	BMR	**Average weight**	**Average height**
Women	BMR = 655 + (4.35 x weight in pounds) + (4.7 x height in inches) – (4.7 x age in years)	143.5 pounds	64.5 inches
Men	BMR = 66 + (6.23 x weight in pounds) + (12.7 x height in inches) – (6.8 x age in years)	174 pounds	69.5 inches

Figure 14. Table showing how to calculate the basal metabolic rate for men and women.

Your active metabolic rate, AMR, depends on how much exercise you typically do in a day. Your AMR is simply found by multiplying your BMR by an activity factor. Typical activity levels are given in the table below (**Figure 15**).

Activity	**Activity factors**
Sedentary (little or no exercise)	1.2
Lightly active (light exercise/work 1-3 days per week)	1.375
Moderately active (moderate exercise/work 3-5 days per week)	1.55
Very active (hard exercise/work 6-7 days a week)	1.725
Extra active (very hard exercise/work 6-7 days a week)	1.9

Figure 15. Table giving activity factors for typical activity levels.

So for example if your BMR is 1,500 and you are lightly active then your AMR is 1,500 x 1.375 = 2,062.5. That means you need 2,062.5 calories per day to maintain your weight.

[1] The word calorie is almost always used instead of the more precise scientific term kilocalorie.

It is often suggested that you need to take in 500 fewer calories per day if you want to lose 1 pound a week. Conversely if your food intake is 500 calories per day over your AMR then you will put on weight at a rate of 1 pound per week.

The data in **Figure 16** below shows typically how many calories you consume when eating some fast foods sold by a fast food retailer.

Fast food	**Calories**
Quarter pounder with cheese	516
Italian sandwich	446
Bacon Double Cheeseburger	479
Chicken fillet meal	761
Classic club salad	390

Figure 16. Table showing typically how many calories you consume when eating some fast foods sold by a fast food retailer.