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3849-301 MAY 2019
Mathematics
Pre-release material

Dates of examination
Paper 1 – Mathematical Modelling
Wednesday 15 May 2019 (09:30 – 11:00)
Paper 2 – Mathematical Comprehension and Communicating with Mathematics
Wednesday 22 May 2019 (09:30 – 11:30)

This booklet will be issued to centres two months in advance of the date of examination.

Candidates will be issued with a clean copy of this booklet for each paper sitting. Copies will be issued at the start of the examination session and collected at the end of the session. Candidates must **not** take their own copies of this booklet into the examination.

Centres should ensure that candidates are familiarised with the contexts and information contained in this booklet in preparation for the examination.

All examination questions will be based on this material.

This booklet contains a set of four documents
1) The Law of Retail Gravitation
2) Electric cars
3) Population data
4) Payday loans.

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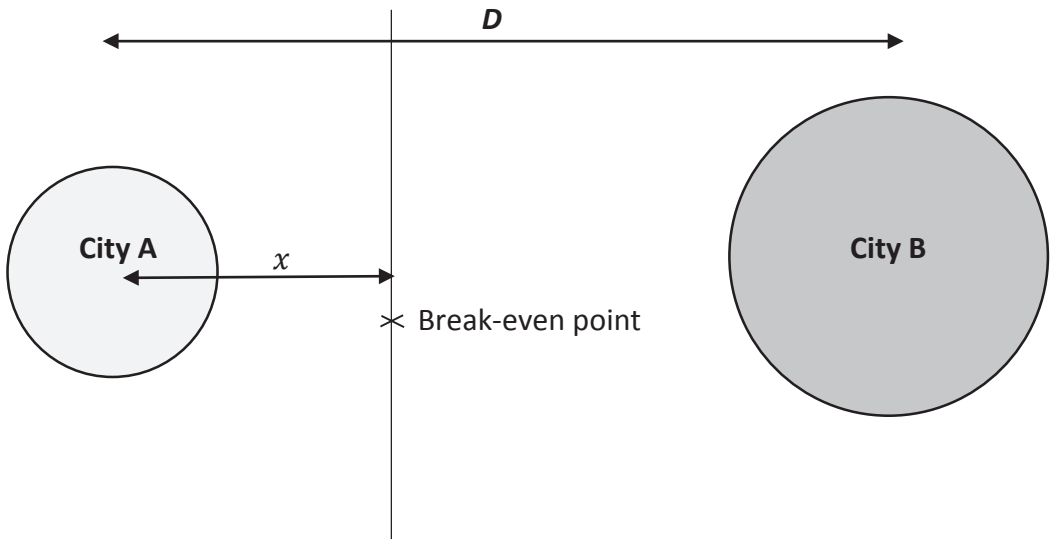
1) The Law of Retail Gravitation.

If you live somewhere between two cities, you have to decide which to visit for some activities, such as shopping for new clothes.

The Law of Retail Gravitation finds a point, the break-even point, which is a distance from each of the cities. This gives an estimate of the point where there is equal probability that you will visit either city. If you live on either side of this point you will go to the city on that side.

Figure 1 shows the law.



$$x = \frac{D}{1 + \sqrt{\frac{P_B}{P_A}}}$$

D is the distance between the two cities
x is the distance from City A to the break-even point
P_A is the population of City A
P_B is the population of City B

Figure 1. The Law of Retail Gravitation.

For example, the distance between Derby and Nottingham is 24 kilometres
The population of Derby is approximately 250,000.
The population of Nottingham is approximately 300,000

The break-even point is approximately 11.5 kilometres from Derby and therefore 12.5 kilometres from Nottingham.

2) Electric cars

There is a need to have cars that are more friendly towards the environment. In fact, it is the government's intentions that by 2040 all new cars and vans have zero emissions by being primarily electric vehicles. This should ensure significantly less air pollution and as we shift towards using more and more renewable power sources, such as wind and solar, to provide the vehicles with electricity. This will result in even more eco-friendly motoring.

Figure 2. Electric car charging

At the moment electric car batteries are relatively expensive and inefficient. However, battery technology is being developed and improved constantly so that in a few years it is expected that electric cars will be able to run for longer and travel further (and at a lower cost) on a single charge.

It is perhaps not well known that electric cars, in general, have much better acceleration than conventional petrol or diesel powered cars. That is, the rate at which their speed increases is quicker. The table in **Figure 3** gives results of measuring the distance, in metres, travelled by an electric car moving in a straight line, every 5 seconds with the car starting from rest.

Time (seconds)	Distance (metres)
0	0
5	30
10	100
15	200
20	320
25	460
30	590
35	730

Figure 3. Distance-time data for an electric car starting at rest and travelling in a straight line for its first 35 seconds of travel.

This data is plotted as a graph in **Figure 4**.

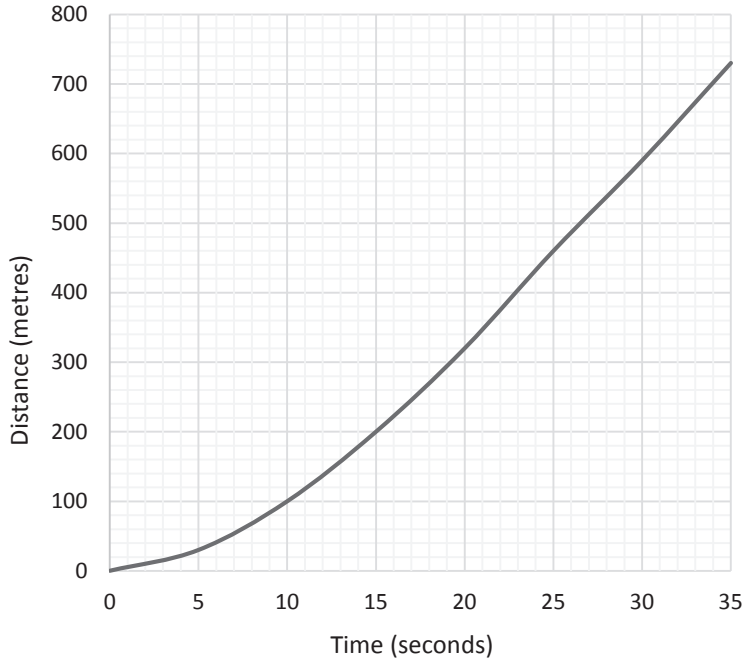


Figure 4. Graph showing the distance travelled by an electric car in a straight line, starting from rest.

The **average speed,** v ms^{-1}, over a certain time interval of something moving in this way, is found by dividing the distance, *d* metres, travelled in an interval of time, *t* seconds. So, for example, in the case of the first 35 seconds of travel of the car here its average speed v ms^{-1} is given by $v = \frac{730}{35} = 20.9$ ms^{-1}.

Of course, this is an average speed over a relatively long time period. If we want to use the data here to know the speed of the car at any particular instant we need to use a method that finds the speed by finding the average speed over a very brief time period. For example, if we want to know the speed of the car after it has been travelling for 5 seconds we could find the speed by finding how far it has travelled from when the time was 5 seconds to 5.5 seconds after the start of the motion. However, we do not have the distances recorded for these times.

One way to proceed is to find a function *d*(*t*) for distance, *d* metres, in terms of time, *t* seconds, after the car starts to move. This provides a model of the situation and allows us to find an approximate distance travelled at any time. The graph in **Figure 5** shows a function plotted to model the situation for the first 10 seconds of travel of the car. You can see that this is a relatively good fit to the data for $0 \leq t \leq 10$.

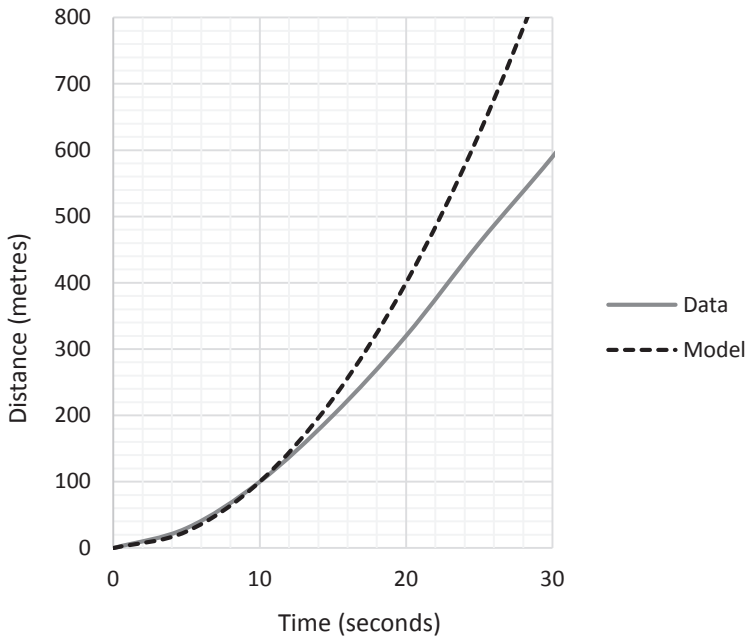


Figure 5. Graph showing how the distance travelled by an electric car in a straight line, may be modelled by a function for $0 \leq t \leq 10$, where t is the time in seconds after the car starts from rest.

The function that has been used is simply $d = t^2$. Using the method suggested, the speed of the car after 5 seconds, that is using values of d for t = 5.5 and t = 5, is approximately 10.5 ms^{-1}.

Of course, now that we have a function that we can use to model how the distance of the car varies with time we can use any time interval we wish to find an average speed. For example, we could find a better approximation for the speed of the car at t = 5 using a time interval of say 0.2 seconds. This gives the speed to be 10.2 ms^{-1}.

More generally, use a time interval $5.0 \leq t \leq 5 + \delta$ where δ is a small value. For example, we could use $\delta = 0.01,\ \delta = 0.001.\ \delta = 0.0001$ and so on.
If we do this we can see that the value for the speed of the car approaches 10 ms^{-1}.

Once we have found a function to model the situation, as we have done in this case, at least for the first ten seconds or so, we can then find values for the speed at a particular time using techniques such as this. Using more advanced mathematics than we will use here, it can be shown that the speed, v ms^{-1}, can be taken to be $v = 2t$, where t is the time in seconds after the car starts from rest.

As we can see the function we have been using, $d = t^2$, is only useful for the early stages of the car's motion. **Figure 6** suggests that the later part of the motion might be better modelled using a linear function.

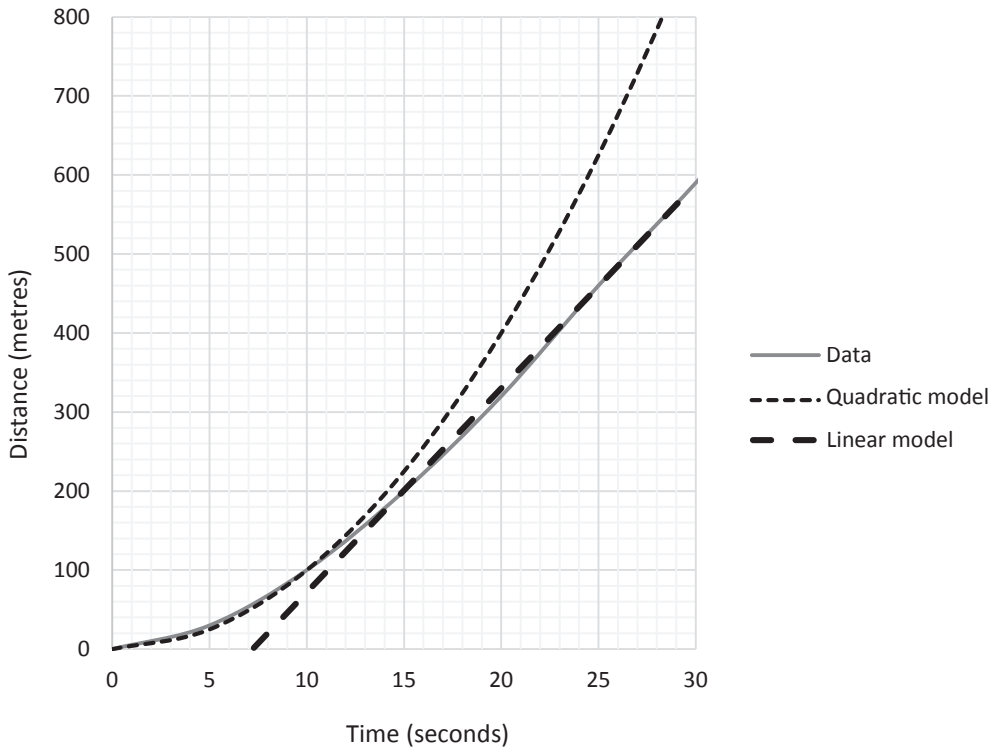


Figure 6. Graph showing how the distance travelled by an electric car in a straight line, may be modelled by a quadratic function for $0 \leq t \leq 10$, and a linear function for $15 \leq t \leq 30$, where t is the time in seconds after the car starts from rest.

The equation of the linear function shown in **Figure 6** is $d = 26t - 190$. This appears to be a useful fit to the data for $15 \leq t \leq 30$. In this case the speed of the car is simply given by the gradient of the straight line. That is, the speed is 26 ms^{-1}.

In the UK speeds are usually given in miles per hour rather than metres per second.
1 mile per hour = 0.447 metres per second.
So the speed of the car in the interval $15 \leq t \leq 30$ is approximately 58 miles per hour.

3) Population data

Every year the Office for National Statistics (ONS) provides many sets of data that help inform the direction of policy at every level, by the national government through to cities and towns. For example, every year, the government provides sets of data that inform how different authorities plan to provide a wide range of services such as schools.

The population changes depend on the number of births and deaths in the population (natural change) and also depends on the number of people immigrating into the country and the number of people emigrating from the country.

Below is just a very small amount of the data and how it was reported by the ONS in mid-2017. Much of this is based on estimates made by the team of statisticians at the ONS.

- The population of the UK at 30 June 2017 exceeded 66 million people (66,040,200 with a confidence interval of +/- 0.2%), an increase of 392,000 people since mid-2016.
- This growth rate (0.6%) is the lowest since mid-2004.
- 41% of the population growth occurred from natural change (births minus deaths), 59% through net international migration and an increase of 2,700 people in the armed forces population based in the UK.
- Nearly 12 million UK residents, or 18.2% of the population were aged 65 years and over in mid-2017.

Further details are given in the report about how the population is changing across different areas of the country, including in the different countries that make up the UK as a whole. For example, the Table in **Figure 7** shows how the population has changed between 2016 and 2017.

	Population 2017	Share of UK population	Increase on 2016	Percentage change since 2016	Percentage change since 2007
England	55,619,400	84.2%	351,400	0.64%	8.2%
Wales	3,125,200	4.7%	12,100	0.39%	4.0%
Scotland	5,424,800	8.2%	20,100	0.37%	4.9%
Northern Ireland	1,870,800	2.8%	8,700	0.47%	6.2%
UK	66,040,200	100.0%	392,200	0.60%	7.7%

Figure 7. Population growth for UK countries, mid-2017

In the year to mid-2017 international migration, natural change and other changes have all decreased at the same time. Breaking this down further shows that in the year to mid-2017 there were:

- 572,000 international immigrants (a 12% decrease on the previous year)
- 342,000 international emigrants (a 9% increase on the previous year)
- 762,000 births, 19,000 fewer than the previous year (a 2% decrease)
- 602,000 deaths, 14,000 more than the previous year (a 2% increase)

The graph in **Figure 8** shows how the mid-year estimated population has changed over a longer timespan (1971 -2017)

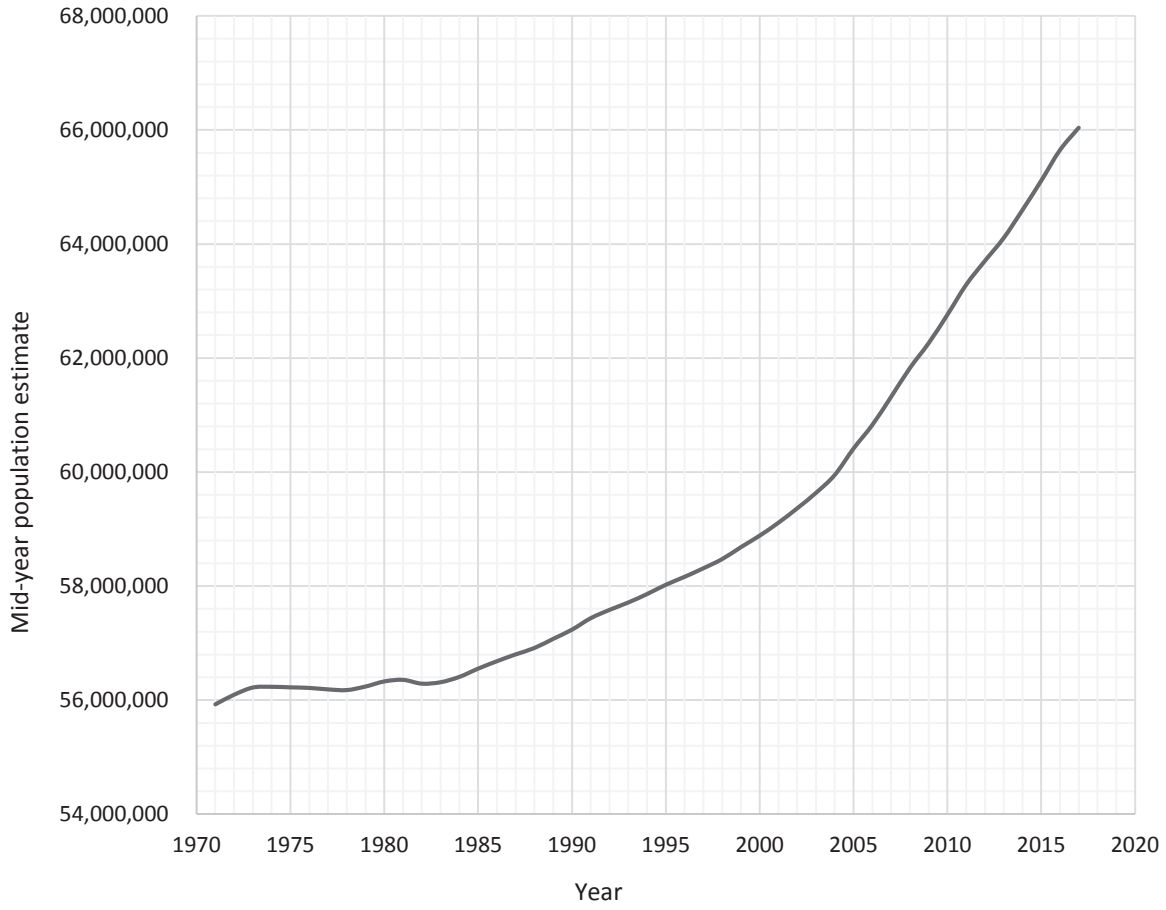


Figure 8. Graph showing how the mid-year estimated population of the UK has changed between 1971 and 2017.

The 2017 ONS report included the following comment:
"This is the lowest annual population growth since 2004 due to a fall in net migration, fewer births and more deaths than previously seen. The effect is most pronounced in London and other areas that have seen high levels of immigration in recent years. Nevertheless, the population is still growing faster than at any time since the post war 'baby boom' and the expansion of the EU in 2004."

The population pyramid in **Figure 9**, gives further insight into how we might be able to draw conclusions such as these.

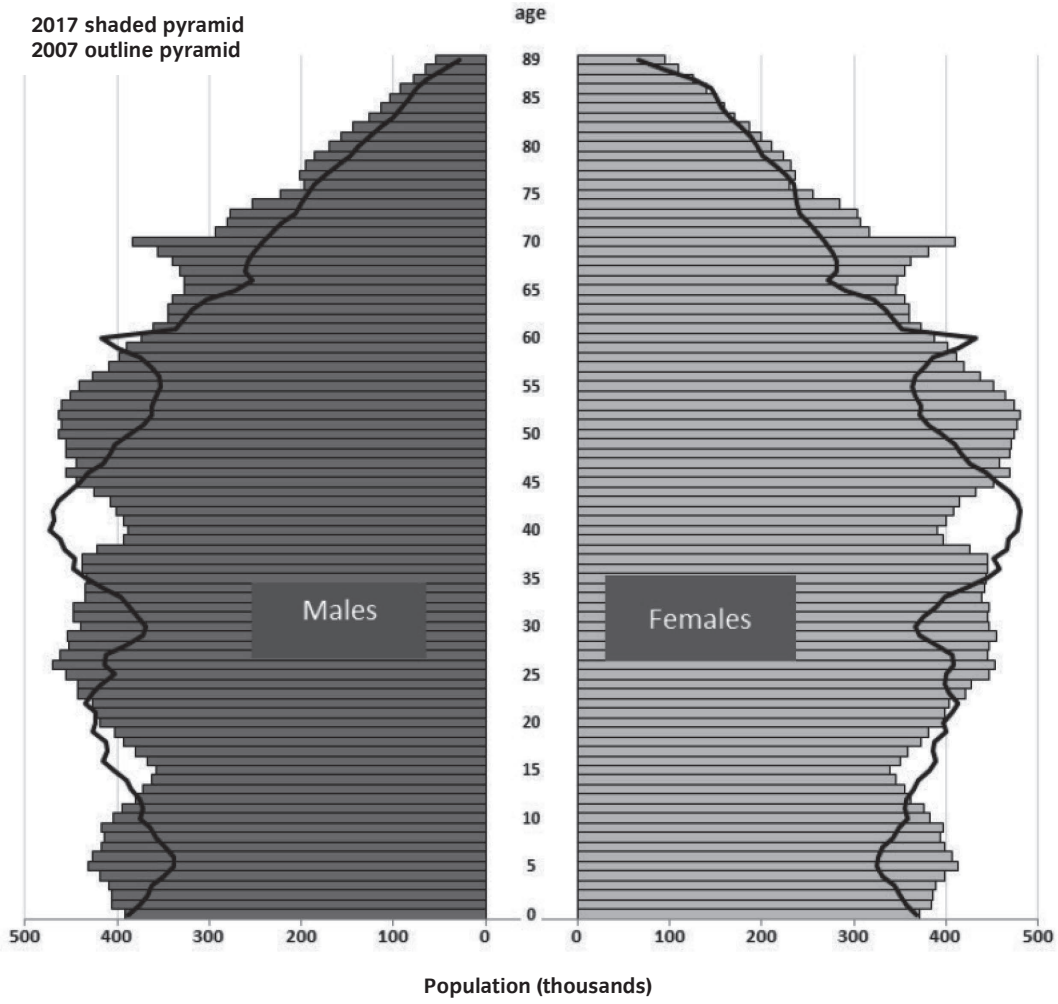


Figure 9. Population pyramid showing the UK population profile of males and females in 2017 compared to the profile in 2007.

Further data released later in 2017 gave the following information:
In 2017, there were 755,042 births in the UK, a decrease of 2.6% from 774,835 in 2016 and the lowest number of births since 2006.
There were 607,172 deaths registered in the UK in 2017, an increase of 1.7% from 597,206 in 2016 and the highest number registered annually since 2003.

4) Payday loans

The term payday loan is often used for a short-term loan that can be secured quickly to cover a person's need to have access to cash – often to see the person through to their next pay day. Hence the term, "payday loan".

Although such loans can be relatively easy to obtain, there is a price to pay; you often pay back a great deal more than the cash advance. In other words, the interest rate is high.

This is because payday loan lenders, like other lenders, offer their loans according to many different plans. We often use a measure, APR (annual percentage rate), to allow us to make comparisons.

This measure provides a value that tells you what would be the equivalent if the loan was taken out according to the same conditions over an entire year. For example, it may be that someone borrows £100 to cover just half a month, say 15 days, and repays £125 at the end of that time.
The interest paid is £25, that is 0.25 as a decimal fraction of the loan amount.
The APR is given by APR = $0.25 \times \frac{365}{15}$ = 6.08, or 608%, where there are 365 days in a year and the loan is for 15 days.

This compares to an APR that could be between, say, 5% - 25% for arranging a loan through a bank (although such loans are typically for much greater values and for longer time periods).

So, although payday loans can provide immediate cash when needed, problems can arise when they aren't paid back as agreed when taking out the loan. In such cases, penalties can be particularly steep and interest rates can then become even more excessive.

Some typical payday loan offers are given below:

Quick Cash	£200 over 3 months: Total repayable amount £345.60
Lending Flash	£200 over 3 months: Total repayable amount £345.60
Safety First	£200 over 3 months: Total repayable amount £283.07
Quick Buck	£100 over 2 months: Total repayable amount £132.00
Pounds in your pocket	£100 over 2 months: Total repayable amount £149.00