3849301051 8SD

3849-301 MAY18
Level 3 Certificate in Using and Applying Mathematics
Pre-release material

Dates of examination
Paper 1 – Mathematical Modelling
Wednesday 16 May 2018 (09:30 – 11:00)
Paper 2 – Mathematical Comprehension and Communicating with Mathematics
Wednesday 23 May 2018 (09:30 – 11:30)

This booklet will be issued to centres two months in advance of the date of examination.

Candidates will be issued with a clean copy of this booklet for each paper sitting. Copies will be issued at the start of the examination session and collected at the end of the session. Candidates must **not** take their own copies of this booklet into the examination.

Centres should ensure that candidates are familiarised with the contexts and information contained in this booklet in preparation for the examination.

All examination questions will be based on this material.

This booklet contains a set of four documents
1) Taxation
2) Half-life
3) Anthropometry
4) Smoking.

© The City and Guilds of London Institute 2018
ST00052725 F2 PO4500134150 10366353

1) Taxation

The two tables below in **Figure 1** and **Figure 2** show the Income Tax and National Insurance rates in the United Kingdom for the 2017-18 financial year. These rates apply to many people who work for an employer. You should be aware that there may be other factors to take into account when working out the tax that a particular person would pay. However, these are the rates that apply to many people.

Income Tax

Taxable income	Tax rate	
Personal Allowance	Up to £11,500	0%
Basic rate	£11,501 to £45,000	20%
Higher rate	£45,001 to £150,000	40%
Additional rate	over £150,000	45%

Figure 1. Income Tax rates for the 2017-18 financial year.

This system means that you can earn up to £11,500 without paying any Income Tax. This is sometimes known as your 'tax-free allowance'.
For any amount you earn from £11,501 up to, and including, £45,000 you pay 20%, from £45,001 to £150,000 you pay 40% and you pay 45% on anything you earn over £150,000.

For example, if you earn £30,000:
You earn £11,500 without paying any Income Tax.
You earn £18,500 above £11,500 so you pay 20% of this in Income Tax; that is, you pay £3,700. This is 12.3% of your earnings.

Such an Income Tax system is said to be progressive as the more you earn the more Income Tax you pay and also what you pay in tax becomes a greater percentage of your earnings.

For example, if you earn £40,000 per year you pay £5,700 in Income Tax which is 14.25% of your earnings.

National Insurance

Below £8,164 per year	0%
Above £8164 per year up to £45,000 per year	12%
Over £45,000 per year	2%

Figure 2. National Insurance rates for the 2017-18 financial year.

In addition to Income Tax you pay what is, in effect, a further tax which is known as "National Insurance". Again, you can earn a small amount of income, effectively £8,164 per year, without paying any National Insurance. If you earn more than £8,164 per year but less than £45,000 you pay 12% of this in National Insurance. For anything you earn above £45,000 per year you pay 2% of this in National Insurance.

In this case if you earn £30,000 per year you pay no National Insurance on £8,164 but on the remainder (that is, £21,836) you pay 12%. This means you pay £2,620.32 per year in National Insurance.

If you earn £40,000 per year you pay £3,820.32 per year in National Insurance.

So overall, someone who earns £30,000 per year pays a total of £3,700 + £2,620.32 in Income Tax and National Insurance, this is a total of £6,320.32, that is 21.1% of their earnings.

Someone who earns £40,000 per year pays a total of £5,700 + £3,820.32 in Income Tax and National Insurance, this is a total of £9,520.32, that is 23.8% of their earnings.

You can see from these sample calculations that the UK taxation system is indeed progressive.

2) Half-life

The term half-life can be used to describe how long it takes anything to decrease to half of its original value. It is used in particular in situations where exponential decay is involved. For example, it is used to describe the decay of radioactive substances.

One particular use of this phenomenon is to date the age of a material that used to contain a living organism. For example, it is possible to find the age of a wooden object or any object that contains wood as this was once alive. When alive all plants photosynthesise: a process that provides the plant with oxygen due to the decomposition of carbon dioxide. Plants consequently contain radioactive carbon dioxide and when they die this starts to decay. By measuring the radioactivity levels at a particular point in time it is possible to find how long ago the plant material died. This is done by calculating how many half-lives have passed. For example, if we know that a substance would naturally have a radioactive count of 40 units per minute when alive but now only has a radioactive count of 10 units per minute then 2 half-lives have passed and if the half-life of the substance is known to be 9 months then it is one and a half years since the substance died.

In fact, the half-life of radioactive carbon is about 5730 years, so after 2 half-lives, that is 11,460 years, its radioactivity will have reduced to about one-quarter.

Mathematically exponential decay can be considered using the function

$$N = N_0 e^{-\lambda t},$$

where N represents the number of radioactive emissions per unit time, at time t years, N_0 represents the number of radioactive emissions at time $t = 0$ and λ is a constant which depends on the radioactive material that is decaying.

When a radioactive substance has decayed sufficiently so that only half of it remains, that is at $t = t_{\frac{1}{2}}$ it will only be emitting half the number of emissions as it was when $t = 0$. Consequently,

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

leading to $\lambda = \frac{0.693}{t_{\frac{1}{2}}}$.

This means that $N = N_0 e^{-\left(\frac{0.693}{t_{\frac{1}{2}}}\right)t}$.

In the case of radioactive carbon, where $t_{\frac{1}{2}} = 5730$,

$$N = N_0 e^{-0.0001209t}$$

and $\frac{N}{N_0} = e^{-0.0001209t}$

If a scientist tested a sample of material that was emitting 20% of the radioactive carbon of living material, we would immediately know that it was between about 11,000 and 17,000 years old. More accurate calculations would suggest that it was about 13,300 years old.

Radioactive carbon is not the only radioactive substance that is used to date materials. After only relatively few half-lives, say 10, the radioactive carbon has decayed so much that the release of radioactivity is so low that it is difficult to measure. The graph in **Figure 3** illustrates this.

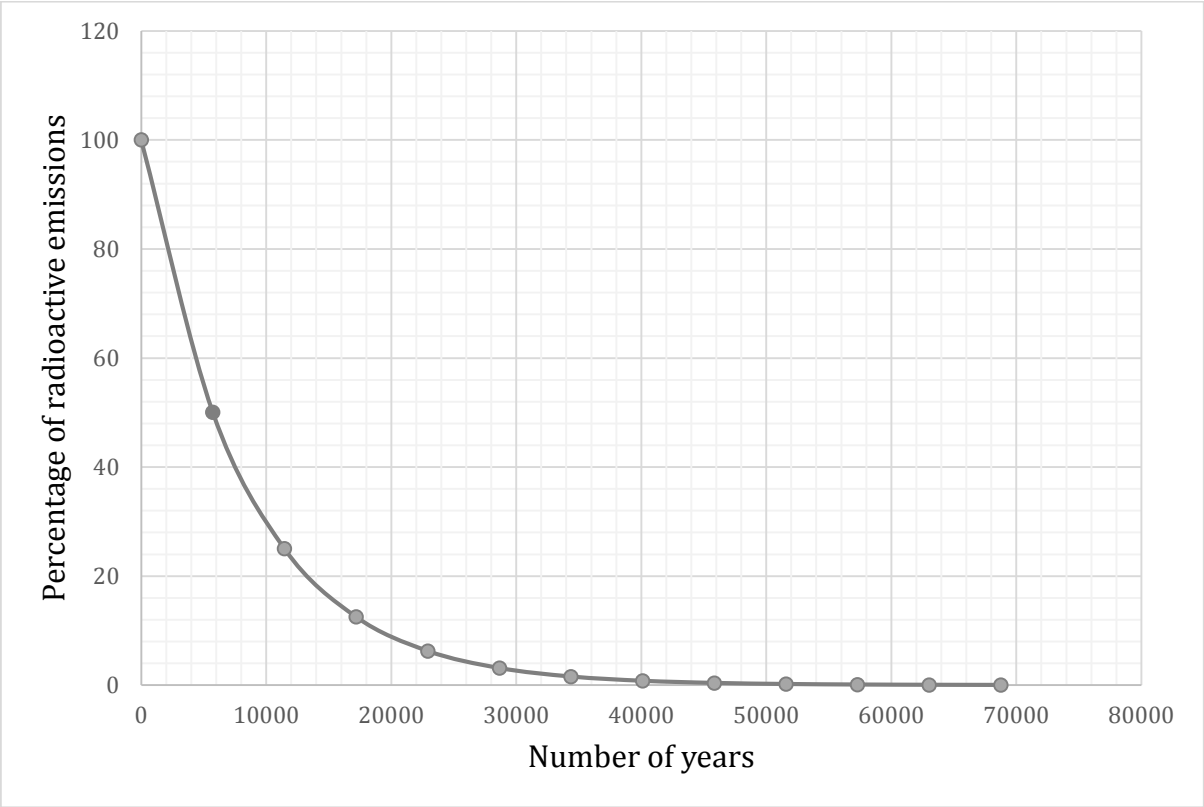

Figure 3. Graph showing the percentage of radioactive carbon remaining in a material plotted against number of years.

If we want to date, for example, the age of rock that contains a fossil, a substance other than radioactive carbon is used that has a much longer half-life. Radioactive Potassium-40 is often used in such cases. This has a half-life of 1.25 billion years, that is, 1,250,000,000 years.

3) Anthropometry

Anthropometry is the word used to refer to the application of mathematics that uses the data of measurement of the human body. It is used in the design of furniture, clothing, architecture etc. It may be used, for example, in deciding on the height of a doorway by door manufacturers, the height of tables and chairs, and so on by furniture designers, and by fashion designers when designing clothing. It is important to have data relating to human dimensions to design all of these products, and countless others, if they are to be comfortable for use by the general population.

Figure 4 shows some data referring to the height of a small sample of 18 year olds. Data are given for both males and females. You may see straight away that there are some noticeable differences between these two groups. For example, the female 18 year olds are 'on average' a little shorter. This of course, has implications for the design of chairs that students might use to sit a computer work-station, for example. Designers, therefore, make sure it is often possible to adjust the height of office chairs so that everyone is able to sit so that they have a good view of the computer display screen.

Designers carry out calculations to ensure that an appropriate percentage of users of their products can use them comfortably. When, for example, designing a new doorway for a building, an architect will want to ensure that the vast majority of people can get through the doorway without having any danger of hitting their head. In such circumstances it is clearly a good idea to design a doorway that is as high as possible, but a good height is not always possible because of height restrictions in the building.

To help with this sort of design designers draw on the mathematics associated with the normal distribution. Like many naturally occurring features the heights of people are normally distributed. The heights of males and females in **Figure 4** have been grouped into intervals, 1.40 – 1.45, 1.45 – 1.50, 1.50 – 1.55 and so on. The data for males organised in this way is shown plotted in the frequency diagram of **Figure 5**. From this you can see that it seems as though the data might be considered to be normally distributed.

We can see this better in the graph of Figure 7.

First, the graph of **Figure 6** shows the graph of **Figure 5** redrawn so that the total area of the bars is 1. To obtain this frequency each interval is divided by 2.3.

Figure 7 shows **Figure 6** redrawn with a normal distribution superimposed for the case where the mean and standard deviation are 1.84 and 0.0661 respectively. You can see that this gives a reasonably good approximation.

Height (m)	No. (Males)	No. (Females)
1.40-1.45	0	0
1.45-1.50	0	1
1.50-1.55	0	3
1.55-1.60	0	6
1.60-1.65	0	5
1.65-1.70	1	5
1.70-1.75	4	3
1.75-1.80	7	0
1.80-1.85	15	0
1.85-1.90	10	0
1.90-1.95	8	0
1.95-2.00	1	0
2.00-2.05	0	0

Figure 4. Table of grouped frequency data for heights of 18 year olds.

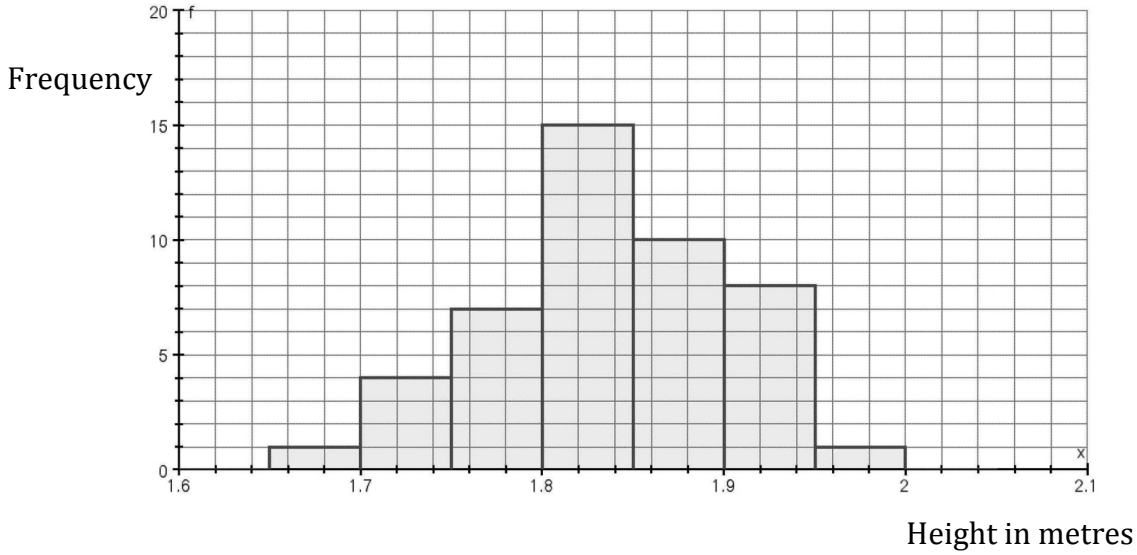

Figure 5. Frequency diagram for the heights of male 18 year olds.

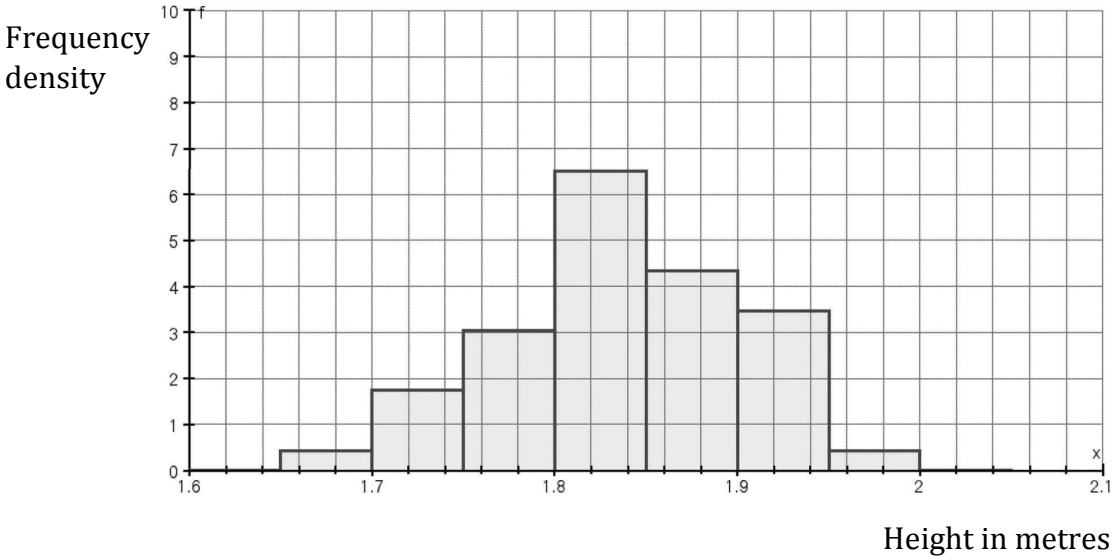

Figure 6. Frequency density diagram for the heights of male 18 year olds where the total area represented by the bars equals 1.

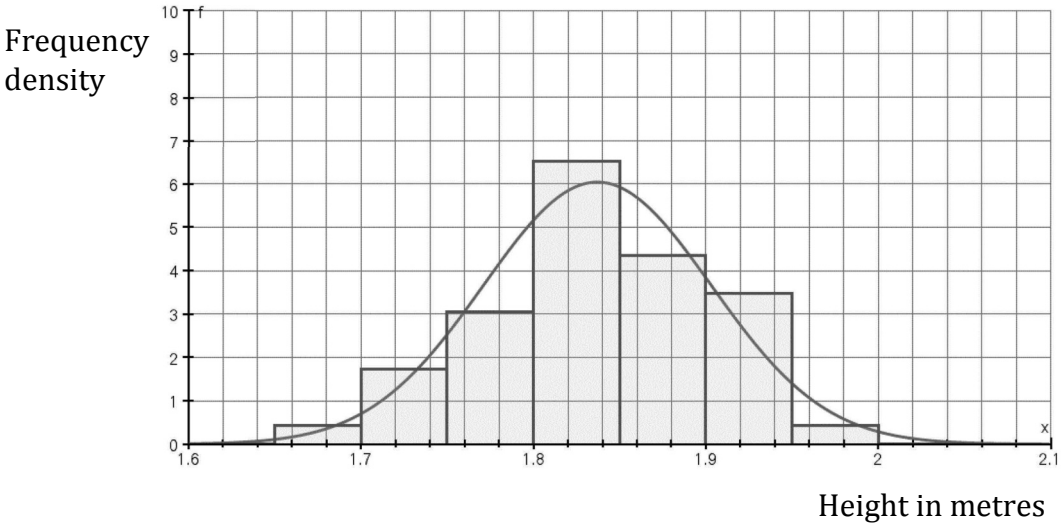

Figure 7. Frequency density diagram for the heights of male 18 year olds where the total area represented by the bars equals 1 with a normal distribution superimposed.

Figure 8 shows a similar diagram for 18 year old females where the total area of the bars is 1.

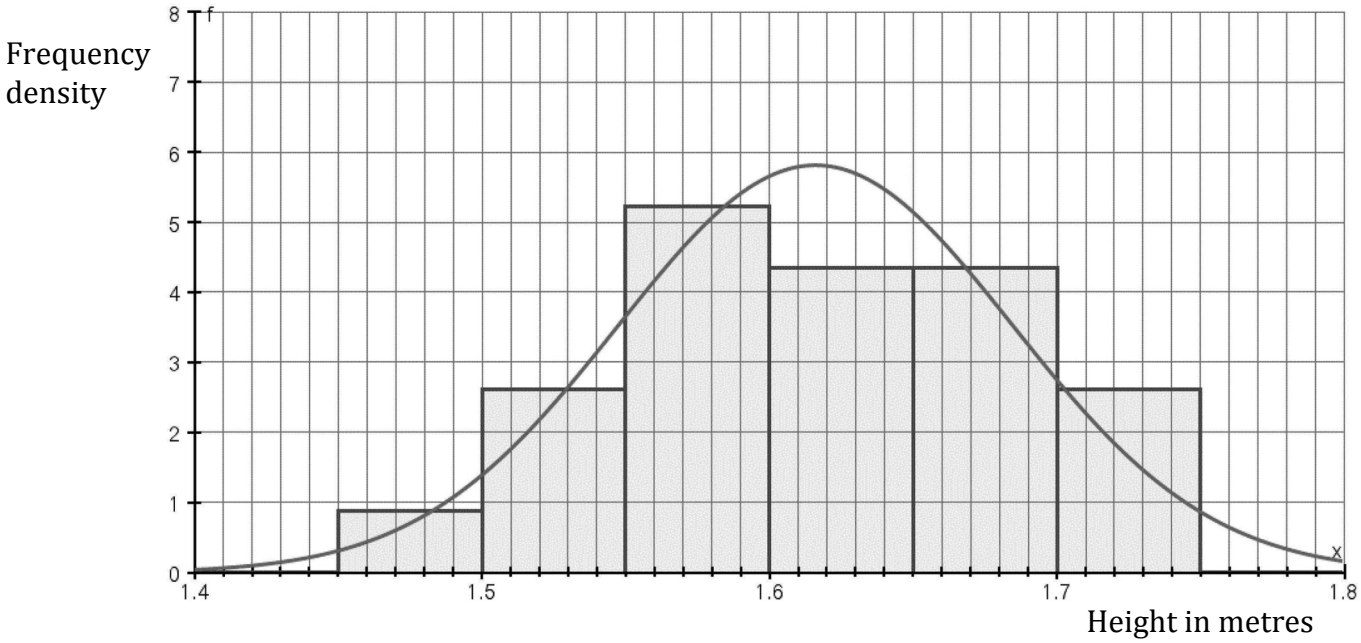

Figure 8. Frequency density diagram for the heights of female 18-year-olds where the total area represented by the bars equal 1 with a normal distribution superimposed.

A carpenter may be making a door in a place where the height of the door has to be restricted to just 1.94 metres. In such a case all 18 year old females would be able to pass through and it is possible to use the normal distribution to calculate what percentage of males can pass through. In this case we need to find where the area enclosed by the normal distribution and the axis represents 10% of the total area (one).

In other words, we need to solve $\Phi\left(\frac{x-1.84}{0.0661}\right) = 0.9$.

Doing this gives x, that is the height of door needed is 1.920 metres or 1920 millimetres.

4) Smoking

The data below show some facts about smoking and smokers in recent years and includes information about admissions to hospital in relation to smoking. This has been drawn from publications by the Office for National Statistics (ONS) and the National Health Service (NHS).

	England	Wales	Scotland	Northern Ireland	United Kingdom
2010	19.9	23.3	24.7	18.8	20.1
2011	19.8	22.3	23.4	18.9	20.2
2012	19.3	21	21.7	19.2	19.6
2013	18.4	20.2	21.5	18.5	18.8
2014	17.8	19.4	20.3	18	18.1
2015	16.9	18.1	19.1	19	17.2
2016	15.5	16.9	17.7	18.1	15.8

Figure 9. Table showing percentage of population of countries in the U.K. who smoke.

Age	18 to 24	25 to 34	35 to 44	45 to 54	55 to 64	65 and over
2010	25.8	25.9	22.4	20.6	18.3	11.0
2011	25.7	25.8	23.3	21.6	18.5	10.2
2012	25.0	25.0	22.5	21.0	18.0	10.1
2013	23.5	24.6	21.2	20.3	17.1	9.8
2014	23.5	24.0	20.1	19.6	16.7	9.3
2015	20.7	23.0	19.5	19.0	16.0	8.8
2016	19.3	20.8	18.1	17.3	15.1	8.3

Figure 10. Table showing percentage of age groups who smoke in the U.K. Note that population is not distributed uniformly across the different age groups.

Year	Current smokers	Smokers who have quit
2010	20.3	54.9
2011	19.8	55.0
2012	20.4	50.2
2013	19.2	54.3
2014	18.8	54.6
2015	17.8	56.7
2016	16.1	60.6

Figure 11. Table showing percentage of population who smoke and percentage who have quit in the U.K. Note, the proportion of cigarette smokers who have quit is the proportion of all those who said that they have smoked cigarettes regularly, who do not currently smoke.

	All admissions		All admissions which can be caused by smoking		Cancers which can be caused by smoking		Respiratory diseases which can be caused by smoking	
Year	**Admissions**	**Attributable number**	**Admissions**	**Attributable number**	**Admissions**	**Attributable number**	**Admissions**	**Attributable number**
2010/11	10,012,763	459,900	1,534,176	459,900	327,451	160,300	260,819	126,200
2011/12	10,164,003	459,700	1,562,569	459,700	337,291	163,700	259,286	121,300
2012/13	10,281,507	460,900	1,600,202	460,900	323,287	156,600	290,916	131,000
2013/14	10,602,739	454,700	1,634,266	454,700	330,280	158,800	290,283	125,600
2014/15	11,011,882	474,600	1,713,330	474,600	331,988	158,500	336,586	141,500
2015/16	11,246,239	474,300	1,767,663	474,300	339,153	159,200	349,740	142,500

Figure 12. Table for England showing hospital admissions, admissions with a primary diagnosis of diseases which can be caused by smoking, and those estimated to be attributable to smoking, for adults aged 35 and over, by year.

	England	Scotland	Wales	Northern Ireland	UK
Population (millions)	55.3	5.4	3.1	1.9	65.6

Figure 13. Table showing population of constituent nations of the United Kingdom.