Level 3 Certificate in Using and Applying Mathematics (3849)

October 2017 Version 1.5 (June 2016)

Qualification Handbook

City & Guilds MATHS & ENGLISH

Qualification at a glance

Industry area	Maths and English
City & Guilds number	3849
Age group approved	16+
Entry requirements	All learners must have achieved GCSE Mathematics at Grades A*-C before starting this programme
Assessment	To gain this qualification, learners must successfully achieve the following assessment: • Externally set, externally marked written examination
Approvals	Fast track approval is not available for this qualification
Support materials	Centre handbook Sample assessments Teaching and learning resources
Registration and certification	Consult the Walled Garden/Online Catalogue for last dates

Title and level	Size (GLH)	TQT	City & Guilds number	Accreditation number
Level 3 Certificate in Using and Applying Mathematics	180	270	3849	601/4708/8

Version and date	Change detail	Section
1.4 June 2016	Addition of TQT	Introduction
1.5 October 2017	Assessment section updated	4

Contents

1 Introduction

City & Guilds Level 3 Certificate in Using and Applying Mathematics has been designed to meet the requirements set out by the Department for Education (DfE) in its publication, *Core maths qualifications: Technical guidance* (DfE, July 2014):

https://www.gov.uk/government/publications/core-maths-qualifications-technical-guidance

The qualification is designed to be relevant and engaging and to equip young people with the higher level maths skills that underpin a wide range of non-maths specialist Higher Education programmes and for success across a broad spectrum of technical, business and professional careers.

This document tells you what you need to do to deliver the qualification:

Area	Description
Who is this qualification for?	16-19 learners studying a range of technical / vocational and general programmes at level 3 who have achieved GCSE Mathematics at A*-C.
What does the qualification cover?	The central purpose of this qualification is for learners to • develop confidence in using and applying mathematics from a critical perspective to solve problems and make sense of complex situations that arise in their studies, in the world of work or in society more widely • develop understanding of how mathematical models can be used across a wide range of contexts to gain insight and solve problems • gain confidence in developing their own mathematical models and in making sense of such mathematical work produced by others • communicate effectively using a range of mathematical representations such as graphs and charts as well as paying particular attention to developing convincing mathematical explanations and arguments. There are three content areas: • Mathematical modelling • Mathematical comprehension • Communicating with mathematics. In working towards the three content areas of the qualification learners will become confident and fluent in using a wide range of mathematical techniques and mathematical reasoning.
What opportunities for progression are there?	Higher Education and employment.
Who did we develop the qualification with?	This qualification has been developed in collaboration with Higher Education, Further Education colleges, subject associations, employers and mathematics education specialists.

Structure

The content and assessment of **City & Guilds Level 3 Certificate in Using and Applying Mathematics** is organised into three content areas:

- **Mathematical modelling**
- **Mathematical comprehension**
- **Communicating with mathematics**

Assessment is linear and to be delivered at the end of the programme of study.

It is not the intention that these areas be taught separately. Guidance on the delivery of the qualification is given in Section 3.

Guided learning hours

The total guided learning hours for the qualification is 180.

Total qualification time (TQT)

Total Qualification Time (TQT) is the total amount of time, in hours, expected to be spent by a Learner to achieve a qualification. It includes both guided learning hours (which are listed separately) and hours spent in preparation, study and assessment.

Title and level	GLH	TQT
Level 3 Certificate in Using and Applying Mathematics	180	270

UCAS Points

The Level 3 Certificate in Using and Applying Mathematics aligns with the Government's 'Level 3 Core Maths' initiative for 16+ learners in England. As a recognised Core Maths qualification, it is included in school/college performance tables and attracts points on the UCAS tariff.

Grade	Tariff points
A	20
B	16
C	12
D	10
E	6

Scope of content

This section gives details of the scope of content to be covered in the teaching of the qualification to ensure that all the learning outcomes across the three content areas can be achieved.

The learning outcomes focus on important practices in using and applying mathematics across a wide range of contexts in relation to studying, working and living as a numerate citizen.

Mathematical techniques and reasoning developed at GCSE Higher level will be drawn on (excluding geometrical reasoning and construction, vectors and the use of sine and cosine rules). In addition, learners should be able to use and apply techniques and reasoning associated with:

- recurrence relations including the logistic equation;
- critical path analysis;
- normal distribution as a probability distribution used to model commonly and naturally occurring phenomena.

Further details are given in Section 6.

Mathematical practices

To be able to successfully achieve the learning outcomes required in each of the three content areas of the qualification. It is important that mathematical techniques are not learnt in abstract isolation. Rather, techniques and reasoning should be organised around the following mathematical practices that have application across a wide range of settings when using and applying mathematics to solve problems.

Working with data graphically

The focus is on exploring potential relationships between bi-variate data (including time series data), exploring ideas of correlation, and identifying how a function(s) can be used to model an underlying relationship. From the mathematical content identified in Section 6 this will draw in particular on the areas of:

- number (processing and scaling data and so on, and monitoring the suitability of developing models by estimating and approximating numerical values)
- algebra (finding functions that can be used to model relationships between data and using these to make predictions of quantities for which there is no data)
- statistics (using measures of location and spread to process data and consider the reliability and validity of functions as models).

Interpreting data critically

Critically inquiring into situations and contexts using both raw and processed data is the main focus. This will include:

- content usually associated with statistics and probability with the emphasis on critical interpretation of how the data is coupled with the reality from which it is drawn
- understanding probability in the context of risk and applications of this to decision making (for example, in relation to forensic science)
- working with large data sets (although working with such data sets will be excluded from assessment). In presenting data and statistical measures links with 'communicating with mathematical diagrams' and 'working with data graphically' will be made.

Communicating with mathematical diagrams

Mathematical diagrams are used to represent many situations that occur in everyday life. In the main these capture different forms of data in a range of ways. Diagrams to be considered include those:

- displaying standard statistical information
- based on either raw or derived data
- involving measures from an entire population or sample
- of a usual standard 'mathematical' form or more innovative design to convey information to a less mathematically informed audience
- conveying technical data relating to
  - buildings, construction or the natural world (focusing on 3-D to 2-D transformation)
  - the world of business or finance (tables, charts or graphs)
  - other workplaces (for example, relating to flow of traffic, water, electricity or other work processes).

Estimating and predicting

In many situations it is important to have a sense of, and be able to work with, number to make estimates and approximations to either check that information given is reasonable, or to be able to predict likely outcomes. In effect, one is being asked to work with a simple mathematical model. Having verified its likely validity one may be able to make the model a little more complex by either including more factors or introducing variability into one or more important factors. (This may lead to the use of a function that allows a range of predictions to be made.)

Costing and organising

In workplaces and everyday life one is often involved in 'costing and organising' a range of different quantities including time and money to ensure that problems are solved effectively and to budget. A simple example of this is cooking a meal to a recipe, scaling quantities appropriately for the size of the group being catered for and ensuring that different components of the meal are ready at the right time. This is analogous to the type of activity required to plan any discrete or continuous event in many situations. For example, when constructing a building complex, planning related to quantities of materials and use of time is required to ensure materials and workers are available at the right times and so that monetary costs are known. A range of mathematical diagrams, such as Gantt charts, might be helpful in such work.

The use of technology should be encouraged as, when applying mathematics in such ways in out-of-school/college settings, this is common-place. In particular, learners will be expected to develop expertise in using graph plotting capabilities using technology and all should be familiar with the use of spreadsheets wherever they are helpful; for example, when handling relatively large amounts of data and when working with recurrence relations.

2 Centre requirements

Approval

There is no fast track approval for this qualification; existing centres who wish to offer this qualification must follow the **standard** Qualification Approval Process.

To offer these qualifications, new centres will need to gain both centre and qualification approval. Please refer to the *Centre Manual - Supporting Customer Excellence* for further information.

Centre staff should familiarise themselves with the structure, content and assessment requirements of the qualifications before designing a course programme.

Resource requirements

Centre staffing

Centre staff should be secure in their own mathematical understanding and application and have a detailed understanding of this qualification's content and purpose. It is strongly recommended that staff involved in the delivery of the qualification should hold a teaching qualification and be qualified in mathematics at level 3 or above.

Continuing professional development (CPD)

Centres are expected to support their staff in ensuring that their knowledge and practice remains current. This includes currency within mathematics education and best practice in delivery, mentoring, training, assessment and quality assurance. Centres should also take account of any national, international policy and legislative developments.

Learner entry requirements

All learners must have achieved GCSE Mathematics at Grades A*-C before starting this programme.

Although these are not formal entry requirements the qualification assumes that learners:

- have reading, writing and communication skills at level 2 or above
- are confident users of digital technology.

Age restrictions

These qualifications are intended for learners over the age of 16.

3 Delivering the qualification

Initial assessment and induction

An initial assessment of each learner should be made before the start of their programme to identify:

- if they have any specific training needs
- support and guidance they may need when working towards this qualification.

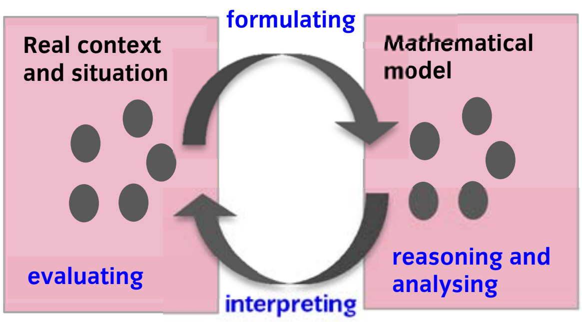
We recommend that centres provide an induction programme so the learner fully understands the requirements of the qualification, their responsibilities as a learner, and the responsibilities of the centre. This information can be recorded on a learning contract.

Guidance for delivery

Although this qualification is split into three content areas - *Mathematical modelling, Mathematical comprehension* and *Communicating with mathematics* - it is not the intention that these should be taught separately; nor is it the intention that the mathematical content that underpins the content areas is taught in isolation. Rather, it is preferable that the course is designed to engage learners in **substantial activities** (see next page). In these, mathematical models are central in ways that involve learners in understanding the mathematical work of others and in both communicating what they have done in arriving at solutions/conclusions to problems *and* the outcome of their work.

Mathematical models and modelling are central to the qualification and the mathematical activity in which learners engage.

Modelling



Modelling *example i)*	
Mathematical models provide a mathematical representation of a real context. They can help make sense of a situation or solve a problem. Often the reality is complex and before a mathematical model can be developed it has to be simplified.	To develop a model that will allow passengers to work out how much a taxi cab is likely to charge for a journey it might be assumed that there is a fixed charge of £2.50 and a charge of £1.25 per mile travelled. This neglects additional charges such as those for carrying suitcases and a charge for when the cab is stationary in traffic.
The mathematical model reflects the structure of this simplified reality and provides insight into this.	This can be represented mathematically: The charge for a journey, £C, can be found using the algebraic equation $C = 2.5 + 1.25m$ where m represents the number of miles travelled. Alternatively a graphical representation plotting C against m might be developed. This gives a straight line that intersects the C axis at (0, 2.5) and has gradient 1.25 (£ per mile). Key aspects of the situation (the fixed charge and the cost per mile travelled) are represented by significant features of each of these mathematical representations.
Modelling *example ii)*	
After a simplified model has been successfully developed this may be refined or modified to more accurately reflect the real situation.	The model might be developed to take account of the factors that were originally neglected, such as the additional charges for carrying suitcases and for when the cab is stationary in traffic.
Each iteration of the model needs to be evaluated in terms of the situation it is representing and its purpose. *Is the model fit for purpose?*	In this case it may be felt that a more complex model that takes account of the additional factors might make it too difficult for passengers to use to get a quick estimate of the fare. Although less accurate, the simplified model is good for its intended use.

In teaching this course learners should have opportunities to engage in activities that require them to develop mathematical models themselves across a range of different contexts. In doing so they should be involved in the following mathematical practices:

- working with data graphically
- interpreting data critically
- communicating with mathematical diagrams
- estimating and predicting
- costing and organising.

In addition to developing their own mathematical models, learners are also required to make sense of the mathematical models of others. They should learn to take a critical view of such modelling activity and be prepared to suggest ways in which it might be improved. This forms the basis of the *Mathematical comprehension* content area but, in teaching the course, comprehension of the work of others can be integrated within activities that also require learners to engage in mathematical modelling themselves.

Equally, learning how to communicate with mathematics, which forms the basis of the *Communicating with mathematics* content area, can be integrated into the activities which learners work on. It is required that learners learn to communicate their mathematical reasoning in a way that is clear and which takes account of the intended audience, as well as to be able to similarly communicate the outcomes of their work. Learning towards *Communicating with mathematics* can be focused at various points during the activities that learners will undertake when working on a substantial modelling task.

Centre staff are therefore advised to structure teaching around substantial tasks that support learners in learning towards each of the content areas of the qualification. The following examples illustrate such an approach, highlighting where learning towards the different content areas can be supported.

The previous examples both involve learners working within the mathematical practice of costing and organising, with *example ii* developing greater complexity than *example i*.

Example Activity 1

This sequence of lessons starts with a context and problem situation. Learners are asked to develop their own model from the outset (*Mathematical modelling*). In this activity they can develop mathematical content in relation to number, ratio and proportion, algebra, sequences and graphs. Their work allows them to engage with the mathematical practice of costing and organising.

Developing a sequence of lessons	Notes and learning outcomes
Starting point A company decides to develop a mobile phone app "Let's Go!" *Investigate a number of different pricing scenarios so that you can write a brief report to explain to them what you would recommend they do about pricing* ***and*** *why.*	This starting point allows learners to develop a range of models of different levels of sophistication. It requires an eventual product in the form of a communication (a brief report) which demands both a solution to the problem and reasoning to support this. The work generated by this starting point clearly lends itself to learning in relation to the content area *Mathematical Modelling* (*LO2*). Because of the demand for a communicative product it can also be used to emphasise learning required for the content area *Communicating with mathematics*.
First of all learners need to identify the different factors that may have an impact on the profit they might earn from selling an app. In this case factors that need to be considered include: • the price that they will charge for the app • any price they will charge for using the app (monthly, annually, …) • any cost involved in developing the app • the number of apps that they will sell and how this number may vary with price.	The first step in developing a mathematical model is to simplify the real situation: identify all of the factors that may have an impact and make decisions about what to do about each. This may include finding and using known values for some factors, approximating values for others, choosing to ignore others, and so on. The intention is to develop a simplified version of reality that can then be explored mathematically using the maths that the learners know.

Developing a sequence of lessons

For example, a learner might decide to investigate the profit that will be gained when selling the app for £1, £2, £3 and so on. At this stage it might be assumed that the number sold is not affected by the price but is due to other factors. A learner may produce a graph like the one below if they simply assume that each app can be sold for either £1, £2 or £3.

Questions that might be asked to prompt learners' thinking include:

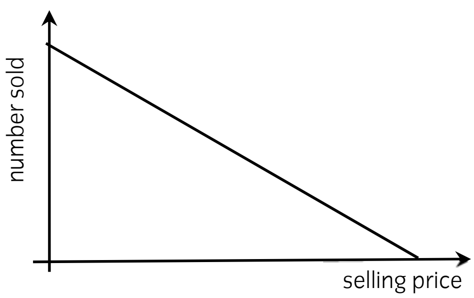
Why do you have a series of straight lines? What would make a straight line steeper? Why? Where does each line cross the axes? Why? Why might the lines cross the vertical axis at another point?

How can the profit be maximised?

In this model it was assumed that the number of sales was effectively independent of price whereas, in reality, if the price is set too high very few may be sold and if the price is set low (or there is even no charge) then a large number may be sold.

At this point you may ask learners to sketch what they think a graph of number sold v selling price might look like.

You might also introduce some alternatives for learners to interpret such as:



Notes and learning outcomes

Having arrived at an understanding of a simplified version of the situation the next step is to develop a mathematical model of the situation ….

…. and then to interpret this in terms of the situation.

At this stage it is important to consider how features of the mathematics relate to the situation and context (*Mathematical modelling LO1 Topic 1.2*). It is helpful to ask a series of questions that might start with, "*What happens if…?*" and "*why….?*"

Learners could be asked to write a brief explanation of this relationship addressing *Communicating with mathematics, LO2, Topics 2.1 and 2.2.*

Having developed a mathematical model it is necessary to interpret and communicate how the model relates to the real situation.

In doing so, thought should turn to how effective the model is, and how it might be improved. This will help give insight into how effective the model is at reflecting the reality it is meant to represent.

Almost inevitably, because the first step is almost always the development of a simple model, there will be an opportunity to develop a more sophisticated model.

To help with this it is useful at this stage to return to the initial factors that were considered and the assumptions that were made at that point. How could these have been more realistic to ensure a more effective model? (*Mathematical modelling, LO1 Topic 1.3*). This approach additionally provides potential to cover learning for *Mathematical comprehension, LO1*.

Developing a sequence of lessons	Notes and learning outcomes

Learners could be asked to write a brief explanation of their interpretation of each graph and make a critique of the approach that has been taken.

Learners should now develop their revised and, in this case, more sophisticated, model. The simplest way to do this is to assume that the number of apps sold decreases linearly as the price increases. Values will have to be assumed for how many will be 'sold' when the price is nothing and what price will first lead to no apps being sold.

In this phase it is helpful to emphasise learning towards *Mathematical modelling*, *LO3*.

Again, learners could be asked to write a brief explanation of the outcome of their new model to engage them in learning towards *Mathematical modelling*, *LO3*.

An effective way to work would be to use a spreadsheet to calculate how the number of apps sold varies with price and therefore how the total profit also varies.

In addition this will also provide opportunities to learn towards *Communicating with mathematics*, *LO2*.

This model may be made increasingly sophisticated by considering, for example, production costs and commission charged by the app retailer (possibly a small percentage of the price might be added).

Now that learners have developed an approach that produces a useful model, this can be made increasingly sophisticated.

Example Activity 2

This sequence of lessons starts by introducing a context and problem. It allows learners to develop mathematical content in relation to number, ratio and proportion, algebra, sequences, graphs and probability. Their work allows them to engage with the mathematical practice of costing and organising. Before learners are asked to develop their own model (*Mathematical modelling*) they are asked to make sense of approaches that have been taken by others allowing them at the start of their activities to engage in learning towards *Mathematical comprehension*.

Developing a sequence of lessons	Notes and learning outcomes
Starting point Investors pay a set amount at the beginning of a period of time with the amount each pays depending on how likely they are to die. If an investor dies during the period the fund pays their beneficiary a sum of money. At the end of the period any survivors share the amount of money left in the fund.	This is a potentially complex situation and learners might find starting from scratch quite daunting. One way to assist learners is to present them with some sample work. This will allow them insight into how they might get started.
Develop a spreadsheet to model the situation with instructions so that a member of the public can use it to calculate what they are likely to experience if they join the scheme. The instructions should also explain the scheme as clearly as possible.	Taking this approach to starting learners off working will allow aspects of *Mathematical comprehension* tackling topics relating to each of *LO1*, *LO2* and *LO3*.
Learners could be asked to make sense of simple spreadsheet models that others have developed. They could be required to write brief instructions for a member of the public to follow that tells them how to use these spreadsheets and include an explanation as to why each works in the way that it does. (To do this effectively learners will have to consider the formulae in the spreadsheet cells). Examples of the printouts of typical spreadsheets of simple models are shown on page 17. The simple models that learners are asked to investigate require them to consider growth due to the compounding of interest. If they have any difficulties with this you may have to prompt them to develop a spreadsheet that allows them to investigate this without worrying about the more complex issue of making any pay-outs to people who die (much as in the case of Table 1 on page 17).	Starting in this way will also support learners in their work towards the product that is eventually required assisting them in learning towards each of the LOs of *Communicating with mathematics*.

Developing a sequence of lessons	Notes and learning outcomes
It may not be obvious to learners which factors are more important than others in terms of having an impact on outcomes. Again it may be useful to generate discussion about this by asking learners to work collaboratively with only some of them investigating the impact of each of the factors. For example, groups of learners could explore the impact of: • compounding interest annually or more frequently • varying the interest rate • varying the amount of investment. It is important to include discussion about assumptions that are implicit in the approach taken so far. For example, it has probably been assumed that the risk of death is independent of factors such as age and gender. Examination of selected data from the Office for National Statistics (ONS) will soon confirm that this is not the case. http://www.statistics.gov.uk/hub/index.html However, to ensure that initial models are simple enough to work with it is wise to take such an approach. This can be addressed in later, more sophisticated models.	Having understood the context and considered how they might develop a very simple model of the situation a next step is to try a next iteration of the model that adds in some complexity. A useful way to proceed is to consider factors that are likely to have the most impact on outcomes rather than factors that have much less impact. In developing their own models of increased complexity learners will have opportunities for learning towards *LO1 and LO2* of *Mathematical modelling*.
Learners can be asked to prepare their results in the form of an explanation for others to follow. By sharing their outcomes with each other they will have an opportunity for peer assessment, allowing them to develop their skills in communication and comprehension. As learners work towards their final product they should be encouraged to ensure that they do this in a way that results in an increasingly complex model. They should not attempt to alter too many aspects of an existing model at each iteration. You may need to prompt learners to search for data that informs their model development – or you may provide this to speed up the process. For example, ONS tables of life expectancy by age and gender might be usefully supplied. Other factors that may be varied include: • the proportion of males and females contributing to the fund • the length of time for which each investor invests their money • the rate of interest (changing each year or more frequently).	Working in a collaborative way as a whole group to investigate how various factors impact on outcomes provides opportunities for engagement with the LOs of *Mathematical comprehension* and *Communicating with mathematics*. Depending on the teaching approach taken, different aspects of these content areas can be emphasised. Taking such an approach can stimulate group discussion about how best to proceed to work towards more elaborate models ensuring learning towards *Mathematical modelling*, *LO3*. Learners should be aware of how to develop more sophisticated models even if they do not achieve this themselves. Learners should draw attention to any limitations there are in their models when working on their final communicative product.

Developing a sequence of lessons

Peer review of the final spreadsheets and instructions that are produced provides learners with opportunities to critically consider the work of others. Learners should be encouraged to provide written feedback that focuses not only on the final product but also on explanations of reasoning. This could be carried out in two stages with learners redrafting their work to arrive at a final product.

Notes and learning outcomes

Critical exploration of the work of peers can be used to emphasise learning towards *LO2* of *Mathematical comprehension*.

The final product of their work can be used to consider many aspects of *Communicating with mathematics*.

Table 1

	A	B	C	D
1		Amount in account		
2	Year	Start of year	Growth	End of year
3	1	£25,000,000.00	£500,000.00	£25,500,000.00
4	2	£25,500,000.00	£510,000.00	£26,010,000.00
5	3	£26,010,000.00	£520,200.00	£26,530,200.00
6	4	£26,530,200.00	£530,604.00	£27,060,804.00
7	5	£27,060,804.00	£541,216.08	£27,602,020.08
8	6	£27,602,020.08	£552,040.40	£28,154,060.48
9	7	£28,154,060.48	£563,081.21	£28,717,141.69
10	8	£28,717,141.69	£574,342.83	£29,291,484.53
11	9	£29,291,484.53	£585,829.69	£29,877,314.22
12	10	£29,877,314.22	£597,546.28	£30,474,860.50
13	11	£30,474,860.50	£609,497.21	£31,084,357.71
14	12	£31,084,357.71	£621,687.15	£31,706,044.86
15	13	£31,706,044.86	£634,120.90	£32,340,165.76
16	14	£32,340,165.76	£646,803.32	£32,986,969.08
17	15	£32,986,969.08	£659,739.38	£33,646,708.46
18	16	£33,646,708.46	£672,934.17	£34,319,642.63
19	17	£34,319,642.63	£686,392.85	£35,006,035.48
20	18	£35,006,035.48	£700,120.71	£35,706,156.19
21	19	£35,706,156.19	£714,123.12	£36,420,279.31
22	20	£36,420,279.31	£728,405.59	£37,148,684.90

Table 2

	A	B	C	D	E
1		Amount in account			
2	Year	Start of year	Growth	Payouts	End of year
3	1	£2,000,000.00	£100,000.00	£25,000.00	£2,075,000.00
4	2	£2,075,000.00	£103,750.00	£25,000.00	£2,153,750.00
5	3	£2,153,750.00	£107,687.50	£25,000.00	£2,236,437.50
6	4	£2,236,437.50	£111,821.88	£25,000.00	£2,323,259.38
7	5	£2,323,259.38	£116,162.97	£25,000.00	£2,414,422.34
8	6	£2,414,422.34	£120,721.12	£25,000.00	£2,510,143.46
9	7	£2,510,143.46	£125,507.17	£25,000.00	£2,610,650.63
10	8	£2,610,650.63	£130,532.53	£25,000.00	£2,716,183.17
11	9	£2,716,183.17	£135,809.16	£25,000.00	£2,826,992.32
12	10	£2,826,992.32	£141,349.62	£25,000.00	£2,943,341.94
13	11	£2,943,341.94	£147,167.10	£25,000.00	£3,065,509.04
14	12	£3,065,509.04	£153,275.45	£25,000.00	£3,193,784.49
15	13	£3,193,784.49	£159,689.22	£25,000.00	£3,328,473.71
16	14	£3,328,473.71	£166,423.69	£25,000.00	£3,469,897.40
17	15	£3,469,897.40	£173,494.87	£25,000.00	£3,618,392.27
18	16	£3,618,392.27	£180,919.61	£25,000.00	£3,774,311.88
19	17	£3,774,311.88	£188,715.59	£25,000.00	£3,938,027.48
20	18	£3,938,027.48	£196,901.37	£25,000.00	£4,109,928.85
21	19	£4,109,928.85	£205,496.44	£25,000.00	£4,290,425.29
22	20	£4,290,425.29	£214,521.26	£25,000.00	£4,479,946.56

Example Activity 3

This sequence of lessons prioritises providing an opportunity to develop the mathematical practice of working with data graphically whilst also providing opportunities to engage in learning that highlights mathematical modelling and developing skills in communication. The lessons provide opportunities to work with content in the areas of number, ratio and proportion, algebra, sequences and graphs.

Developing a sequence of lessons	Notes and learning outcomes
Starting point Many bio-chemical processes require the continuous flow of liquid through a fermentation vessel. The liquid flowing in is a mixture of biomass and substrate. Biomass is the substance that the biochemical engineer is interested in growing and the substrate is the 'food' which is necessary for growth to occur. There is a continuous flow of liquid into and out of the vessel. The proportion of biomass in the outflow is much increased from that flowing in whilst the proportion of substrate in the outflow is decreased from that in the in-flow. It is crucial therefore that something is known about the way in which liquids flow in and out of vessels. *Investigate how the flow rate of a liquid varies as the level of liquid falls in a container.* This can be done by using a large 'cylindrical' plastic bottle such as those used to contain water or lemonade. Take measurements of height of water in the bottle and time so that you can calculate the volume of liquid in the vessel as the time varies. *Use your data to find a mathematical model or models that describe how the volume of liquid in the vessel varies with time.* *Use your model(s) to develop a single slide for a presentation that advises a laboratory technician about what they should expect and how they might predict flow rates using mathematics in such a situation.*	This starting point sets the scene indicating a purposeful activity within a science context. The proposed approach engages learners in a practical experiment that will generate data with which they can work. A final communicative product is required: a single presentation slide that should allow learners to demonstrate their ability to explain their understanding of the outcomes of their investigations. The work generated allows the development of learning in relation to *Mathematical Modelling*. Because of the requirement for a final communicative product, learning towards *Mathematical comprehension* and *Communicating with mathematics* with can also be emphasised.

Developing a sequence of lessons	Notes and learning outcomes
Learners initially need to organise their experimentation in a structured way that will allow them to capture the data they require. There are some practical issues to be considered as well as ensuring that the quality of data is good enough to allow appropriate mathematical analysis. For example, learners need to consider issues such as how they will need to: • zero scale readings • convert observed measurements into useful data in the correct content areas • average readings for repeated measurements • ensure data is recorded to an appropriate degree of accuracy. Having considered these issues learners may need advice about how their experiments might address them. Following this, learners need to formulate a mathematical model that allows them to gain insight into the situation. They should be encouraged to produce a graph of their data after it has been processed to give values for the volume of water in the bottle as time increases from some starting point. It will be useful to use a spreadsheet to record measured data values and then process these to provide the two variables they will plot a graph for (volume of water in the bottle and time elapsed). The next step is to consider how best to model the data. Learners' graphs at this stage should show a curve with the value of the gradient being negative but becoming less steep as time proceeds. It may be that if considering a restricted period of time the data might be modelled by a straight line (with perhaps one straight line being used in early stages of the flow and another being used in later stages of the flow). Learners may find equations of straight lines or curves using a range of methods including using the fitting trend line options of the spreadsheet software.	When working with a class there is an opportunity for learners to work in pairs or small groups, with each pair or group collecting data using bottles of different sizes. This will allow for later discussion of generality by drawing on findings from all pairs or groups. In this particular context a simplified model of a technical situation that one would meet in a science laboratory is designed in such a way that a mathematical model of water flow can be investigated. In doing this simplifying assumptions have been made, such as modelling a plastic bottle as a cylinder (although this is not often the case), focusing only on outflow rather than the case where there is both inflow and outflow, and so on. In this initial phase of work there are opportunities to focus on the simplification of the real context that is a first step in preparing to develop a mathematical model. This allows learning towards, *Mathematical modelling LO2, Topic 2.1*. Whenever modelling, a representation (such as a graph) will provide some insight into both the real situation and the mathematics that might be used to represent this. Indeed, the diagram itself can be considered as providing a mathematical model of the situation. A useful way to encourage thinking about further mathematical approaches is to ask learners to consider how features of the diagram relate to the real situation (for example, in the case of graphs, how intercepts with axes and gradients of lines/curves of best fit) relate to the situation. In developing mathematical models of a situation it is important to do this in a way that allows understanding of the situation to inform the development of the mathematics. Mathematical diagrams such as graphs can facilitate such thinking. This allows learning towards all LOs of *Mathematical modelling*. Teaching staff may wish to emphasise how interpretation of the initial representation of the situation can help provide insight into useful ways of further developing mathematical models (*LO3)*. To carry out such interpretation learners need to be aware of how mathematical measures such as gradients relate to the reality of the context/situation.

When learners have developed a mathematical model(s) of the flow of water they should consider how they can communicate the understanding of the situation that this can provide. They will need to write a brief and clear explanation of this to meet the requirement that they do this on a single presentation slide. They might be encouraged to consider if, and how, their explanation is appropriate for a laboratory technician: they might consider this by thinking about how they could alter their slide if asked to address it to a member of the general public.

Working towards a final product of the form required here provides opportunities for both formative and summative peer assessment at various points during the process.

If time permits you may wish to encourage learners to extend their modelling of the situation to include flow into the container as well as flow out. Learners may like to predict what the effect would be before experimenting.

Learners should be aware of how mathematical models not only need to accurately represent a real context / situation, they also need to be appropriate for their purpose. This means that for effective use by a particular group, or in certain situations, a model may be better in a simple rather than more complex form. Discussions about the communication of mathematical models allows learning towards *Communicating with mathematics*.

Consideration of findings in relation to particular flow situations across the group can facilitate a more general understanding. For example, one might ask how changing the initial conditions will affect the mathematical model(s).

Staff delivering the qualification are encouraged to seek contexts and situations that relate to learners main programmes of study and their Higher Education and career goals.

To support delivery of the qualification, City & Guilds will be making available a full programme of support for teaching and learning. We are working with employers and maths specialists to make this relevant, engaging and current.

4 Assessment

Assessment model

Assessment of **City & Guilds Level 3 Certificate in Using and Applying Mathematics** is linear and delivered at the end of learners' programme of study.

The qualification is **entirely** assessed by an externally set, externally marked examination.

The qualification is designed to be delivered as a two year programme of study of 180 guided learning hours. There are no formal requirements that the learner must meet before being entered for the examination. All candidates who pass the examination will be awarded the qualification at the appropriate grade.

The examination will be based on pre-release material which will be made available to centres **two months in advance** of the date of the examination. The pre-release material will be available to download from the **Level 3 Certificate in Using and Applying Mathematics** page of the City & Guilds website.

The examination has three sections, relating to the three content areas of the qualification. The examination will be delivered **over two separate sessions.** These will be scheduled **one week apart.**

Session 1:

- Paper 1 (1½ hours)
    - Mathematical modelling: 45 marks

Session 2:

- Paper 2: (2 hours)
    - Part 1: Mathematical comprehension: 30 marks
    - Part 2: Communicating with mathematics: 30 marks

Total marks: 105
Total time: three and a half hours.

The two papers will **not** be separately graded. Both papers will be marked against a single mark scheme and graded on the basis of **total marks achieved for all three sections** of the examination.

Synoptic assessment

The assessment is synoptic, requiring candidates to identify and use effectively in an integrated way an appropriate selection of skills, techniques, understanding and knowledge from across the course.

Each of the three sections of the assessment contributes marks to all three assessment objectives. Weighting is used to ensure that marks are allocated to the assessment objectives accordingly.

The balance of marks relating to the three assessment objectives is shown below:

Assessment objectives

Assessment objective		Marks available
AO1	Select, use and apply mathematical techniques effectively when solving problems and communicating with mathematics.	**20-30**
AO2	Represent authentic situations mathematically and analyse these using mathematics to provide insight and solve associated problems.	**35-45**
AO3	Critically make sense of mathematical reasoning, and solutions to problems and communicate mathematical working effectively and with clarity.	**35-45**

By 'authentic situations,' we mean situations that arise in contexts that candidates may meet in relation to an issue arising in everyday life, work or society more generally and for which they might meaningfully employ mathematics to provide them with insight and a solution to the problem.

Grading

The **City & Guilds Level 3 Certificate in Using and Applying Mathematics** will be reported on a five point grading scale, A-E.

Learners who fail to reach the minimum standard for grade E will be recorded as U (Unclassified) and will not receive a qualification certificate.

Grade descriptors and awarding of overall grade

The grades will be awarded in the following way. Grades A, C and E will be determined judgementally through an awarding process. The remaining grades will be determined arithmetically, being set at the mid-point between the judgemental grades.

The following grade descriptors indicate the level of attainment characteristics of the A, C and E grades.

Grade	Descriptor
At Grade A:	Candidates will be able to formulate and work with mathematical models demonstrating • their understanding of how to deal with factors in the situation being modelled so that they might be dealt with effectively using mathematics • that they can identify and select data judiciously • evidence of understanding of how to modify/adapt/improve their model formulation so that it can be analysed using mathematics to develop a thoughtfully reasoned case • an ability to select data and methods of analysis with awareness of the intended audience of the work and its purpose • an ability to use appropriate mathematics effectively • that they can work almost entirely without error demonstrating efficiency and rigour • correct use of standard conventions and notation • evidence of having considered the validity of their model and how this should inform development of their model

- clear understanding of how outcomes of their mathematical work relate to real situations and awareness of the implications for the situations/contexts
- an ability to communicate their work in ways that show sensitivity to its intended audience and purpose
- their ability to comprehend and build on the mathematical thinking of others.

At Grade C: Candidates will be able to formulate and work with mathematical models based on at least one simplification of the situation being modelled demonstrating

- that they can deal adequately with important factors in developing a model
- that they can make reasonable assumptions allowing mathematical analysis to proceed
- an ability to select appropriate data that can be analysed using mathematics to develop a well-reasoned case
- that their selection of data, methods of analysis, interpretation and arguments take account of the intended audience of their work and its purpose
- an ability to work mathematically using appropriate mathematics and making only minor errors
- that their mathematical reasoning, statements and diagrams can, with few exceptions, be clearly followed
- only few slips in use of standard conventions and notation
- evidence of having considered some aspects of the validity of their model and some awareness of the implications of their mathematical work in terms of the situation it represents
- an ability to communicate their work in ways that is appropriate for its intended audience and purpose
- some ability to understand and work with the mathematical thinking of others.

At Grade E: Candidates will be able to formulate a simplification of a real situation/context and a mathematical model demonstrating

- that they can take account of some of the most important aspects in ways that may or may not allow mathematical work to proceed
- some limitations in their identification and selection of data that in most cases is appropriate for analysis allowing the development of a reasoned argument
- an ability to develop their model so that they can deal with it using limited mathematics
- that their mathematical work is in general correct using appropriate mathematics but may contain some errors in reasoning and use of standard conventions and notation
- an ability to produce mathematical reasoning, statements and diagrams that have some clarity and can in the main can be followed easily
- limited evidence of checking of the validity of their model based on some interpretation of their mathematics, arguments and conclusions that is in the main appropriate and correct but which may contain errors
- an ability to communicate their work in ways that show some awareness of its intended audience and purpose
- some but limited ability to understand and work with the mathematical thinking of others.

Each examination series will be subject to a results determination process to establish grade boundaries. As guidance, the following are given as indicative grade boundaries:

Total Marks	A	B	C	D	E	U
105	90	75	60	45	30	0-29

Conducting the examination

The examination will be delivered over **two** sessions. Both sessions will be timed and invigilated.

The examination materials for **both** sessions will be dispatched together in advance of session 1. Centres will receive two packs of examination materials. One pack contains the materials for session 1 and the other contains the materials for session 2 (to be held one week later).

At the end of the first session **all** scripts and copies of pre-release material **must** be collected in and stored securely in a locked cupboard until session 2 the following week.

Centres **must** keep the materials for session 2 locked away securely until the 2nd session. All examination materials must be packed up and returned together following session 2.

During the examination candidates may write their responses in the spaces provided within the question papers, use additional sheets, or can type up their responses using a word processor (see pg 25). Please note any additional sheets (including graph paper) should be labelled clearly with the candidate's name and enrolment number and attached to the question paper before returning to City & Guilds.

Candidates must **not** bring annotated copies of the pre-release material into the exam. Clean copies of the pre-release material will be issued at the start of each examination session.

The examination is to be conducted under timed, invigilated conditions in line with JCQ's Instructions for Conducting Examinations:
http://www.jcq.org.uk/exams-office/ice---instructions-for-conducting-examinations.

Access to tools and software

Candidates may use a scientific or graphical calculator.

City & Guilds Level 3 Certificate in Using and Applying Mathematics is designed to equip candidates with the higher level maths modelling, comprehension and communication skills needed to be effective in higher study and 21st Century workplaces. For this reason, in completing the examination, candidates are permitted access to a computer (PC or Mac) with standard office application software, including:

- word processor
- spreadsheet
- database
- graphics and presentation packages.

Use of such packages will not be explicitly assessed by the examination, but in delivering the qualification, centres are encouraged to consider how these can contribute to a candidate's effectiveness in meeting the qualification's overall objectives.

City & Guilds will keep the range of permitted tools and software under review and, as required, will issue periodic updates to this.

During the examination candidates may **not** have access to:

- internet
- e-mail
- mobile phones
- any pre-prepared files held on internal or external systems.

Access arrangements and special consideration

We have taken note of the provisions of equalities legislation in developing and administering this specification.

Arrangements for this qualification follow the guidelines published for general qualifications in the Joint Council for Qualifications (JCQ) document: *Access Arrangements and Reasonable Adjustments:*

http://www.jcq.org.uk/exams-office/access-arrangements-and-special-consideration/regulations-and-guidance/access-arrangements-and-reasonable-adjustments-2013-2014-standard-pdf-version

Access arrangements

We can make arrangements so that learners with disabilities, special educational needs and temporary injuries can access the assessment. These arrangements must be made before the examination. For example, we can produce a Braille paper for a learner with visual impairment.

Special consideration

We can give special consideration to learners who have had a temporary illness, injury or indisposition at the time of the examination. Where we do this, it is given after the examination.

Applications for either access arrangements or special consideration should be submitted to City & Guilds by the Examinations Officer at the centre.

Language of examinations

We will provide this in English only.

Avoidance of bias

City & Guilds has taken great care in the preparation of this specification to avoid bias of any kind.

Quality assurance

External quality assurance

City & Guilds requires the Head of Centre to

- facilitate any inspection of the centre which is undertaken on behalf of City & Guilds
- make secure arrangements to receive, check and keep examination material secure at all times, maintain the security of City & Guilds confidential material from receipt to the time

when it is no longer confidential and keep scripts secure from the time they are collected from the candidates to their dispatch to City & Guilds.

5 Content areas

Structure of content areas

The qualification content is organised into three content areas:

- Mathematical modelling
- Mathematical comprehension
- Communicating with mathematics

Each content area is divided into numbered learning outcomes. Each learning outcome is organised into topics.

Mathematical modelling

What is this content area about?

The purpose of this content area is for learners to be able to develop mathematical models of real and complex situations. They should learn to solve meaningful problems by

- considering how to simplify the situation in ways that allow them to formulate a mathematical model and a mathematical problem
- formulating an appropriate mathematical model
- using mathematical techniques to provide insight into the situation and mathematical solutions to the problem
- interpreting the outcomes of using and applying mathematical techniques and reasoning in their model
- evaluating the effectiveness and efficiency of their mathematical model in providing solutions to the problem posed in the real world situation
- revising and refining their mathematical model if appropriate.

Learning outcomes

Learners will be able to

1. Understand how mathematical models can provide insight into a simplified model of a complex situation in the real world
2. Develop and use mathematical models in a wide range of situations drawing on appropriate mathematical understanding and techniques in response to authentic problems
3. Interpret mathematical solutions to problems in terms of the situation and evaluate how mathematical models might be improved

Scope of content

This section gives details of the scope of content to be covered in the teaching of the content area to ensure that all the learning outcomes can be achieved.

Learners should be confident and fluent in using the mathematical techniques and reasoning identified in Section 6. This content should be developed as learners engage with the following mathematical practices:

- working with data graphically
- interpreting data critically
- communicating with mathematical diagrams
- estimating and predicting
- costing and organising.

Learning outcome 1: Understand how mathematical models can provide insight into a simplified model of a complex situation in the real world

Topic 1.1 explore mathematical models to gain insight into, and solve problems in, a wide range of situations in real world contexts

Topic 1.2 understand how important factors in real situations are related to mathematical constants and variables in mathematical models

Topic 1.3 consider how mathematical models might be improved to better reflect the reality they have been designed to represent.

Topic 1.1

- Consider the use of general models such as linear functions, exponential growth using recurrence relations etc. across a range of different problems arising in a variety of contexts.
- Consider models developed in response to specific problems arising in complex situations with potentially many variables.
- Critique, compare and contrast a number of models developed in response to the same problem. For example, consider the benefits and disadvantages of a simple model and complex model in response to the same problem.

Topic 1.2

- Understand how assumptions made when simplifying situations impact on parameters in the mathematical model. (For example how varying the interest rate impacts on a parameter in a financial calculation and the effect this may have on savings).
- Consider the effect of introducing variables when modelling: either to take account of variability of a factor in the real situation or to provide insight into potential different cases.
- Understand important factors as being those factors that have most impact on outcomes whereas other factors may have negligible influence (for example, when calculating the cost of a meeting in a central location the cost of refreshments might be negligible compared to travel and accommodation).

Topic 1.3

- Consider how a more complex model can be developed from an initial simple model in response to the same problem and context (for example, introducing variability of a factor that was originally held constant may allow the use of functions and graphs to consider a range of possible outcomes rather than arriving at a single numerical solution).
- Consider the limitations of models in relation to the reality they represent; for example, consider the length of time for which a model is potentially valid.

Learning outcome 2: Develop and use mathematical models in a wide range of situations drawing on appropriate mathematical understanding and techniques in response to authentic problems

Topic 2.1 analyse situations to inform their simplification so that they can be modelled by mathematics

Topic 2.2 formulate and use mathematical models appropriate to a wide range of different contexts and situations.

Topic 2.1

- Consider a wide range of contexts/situations in which the mathematical practices of working with data graphically, interpreting data critically, communicating with mathematical diagrams, estimating and predicting, costing and organising arise. Major areas of mathematical content as identified in Section 6 should be drawn upon as appropriate.
- Developing understanding that as a first step in mathematical modelling the real situation/context needs to be simplified in a way which will allow mathematics to be used.

Topic 2.2

- Develop a range of mathematical models of important types. That is, models that involve:
  - numerical estimates in complex scenarios
  - the use of functions and graphs
  - the use of statistical measures
  - recurrence relations
  - ideas of probability including the normal distribution.
- Use estimation and approximations to inform the validity of mathematical working.
- Work to appropriate degrees of accuracy and with usual mathematical conventions and standards of notation.

Learning outcome 3: Interpret mathematical solutions to problems in terms of the situation and evaluate how mathematical models might be improved

Topic 3.1 explain the outcomes of mathematical modelling by considering the implications that arise from the structure of the model for the reality it represents

Topic 3.2 understand and explain how a change in the reality of the context would require a change in its mathematical model

Topic 3.3 analyse relationships between structures of simplified real world contexts and mathematical models to inform development and refinement of mathematical models.

Topic 3.1

- Consider the use of general models such as linear functions, exponential growth using recurrence relations etc. and understand the generality and structure of the different contexts they can be used to represent (for example, understand that linear models have applicability in situations across different contexts when the rate of change of one variable with another is constant).
- Critique, compare and contrast a number of models developed in response to the same problem.
- Understand how features of the mathematical model represent factors in the context of the real situation they represent (such as how gradients of functions relate to rates of change).
- Critically assess the validity of mathematical solutions in terms of the real problem/situation.

Topic 3.2

- Understand how factors in the context of a real situation are related to factors in, and the structure of, a mathematical model (for example, understand how when modelling the growth of a population of bacteria using an exponential function, it's size, P, at a starting time taken as $t = 0$ corresponds to $P0$ in the mathematical equation P_0e^{kt} where P is the size of the population at time t and k is a factor that depends on the rate of growth).
- Consider how a change in a factor in the real context/situation will affect the structure of a mathematical model and mathematical solutions.
- Explore features of mathematical models and consider what they represent in the real situation/context they represent (for example, understand the implications for exponential growth of varying the parameter, k, in the exponential function P_0e^{kt}).

Topic 3.3

- Following consideration of the validity of a solution to a problem in terms of the context/situation consider how this might be improved by
  - making different assumptions
  - using more complex mathematical analysis
  - exploring the impact of varying parameters in the model.

Mathematical comprehension

What is this content area about?

The purpose of this content area is for learners to be able to critically inquire into and understand the use and application of mathematics by others.

Learners will be expected to engage with writing and mathematical diagrams that set out in some detail the context in which a mathematical model has been developed using one of, or a combination of, the mathematical practices identified as applicable across all of the content areas of this qualification.

They will learn to draw on their own

- ability to simplify real and complex situations, by making assumptions to assist them with developing a mathematical model
- fluent understanding of, and ability to apply, mathematical methods and techniques
- ability to interpret the outcomes of mathematical reasoning, and application of mathematical techniques in terms of implications for the reality that a mathematical model represents
- understanding of how to evaluate the effectiveness of a mathematical model in making sense of a real situation
- ability to use mathematics effectively in communicating mathematical reasoning and outcomes of mathematical work

to make sense of mathematical communications of the use of others.

Learners will be required to communicate, with a sense of audience, their critical inquiry into the mathematical modelling, reasoning and reporting of others effectively and with clarity.

Learning outcomes

Learners will be able to

1. Understand and evaluate approaches to problem solving as communicated by others
2. Understand by following, and inquiring into, the mathematical communications of others, the detail of mathematical reasoning and application of mathematical techniques
3. Understand the effectiveness of the mathematical models of others and evaluate and explain how these might be improved

Scope of content

This section gives details of the scope of content to be covered in the teaching of the content area to ensure that all the learning outcomes can be achieved.

Learners should be confident and fluent in using the mathematical techniques and reasoning identified in Section 6. This content should be developed as learners engage with the following mathematical practices:

- working with data graphically
- interpreting data critically
- communicating with mathematical diagrams
- estimating and predicting
- costing and organising.

Learning outcome 1: Understand and evaluate approaches to problem solving as communicated by others

Topic 1.1 understand and explain how contextual factors in a situation have been dealt with so that the situation can be modelled

Topic 1.2 consider how factors in the real world have been related to the features of mathematical models.

Topic 1.1

- Consider whether or not all important factors in the situation have been accounted for.
- Consider whether important factors in the situation have been accounted for in a suitable manner.

Topic 1.2

- Understand how quantifiable factors in the situation have been dealt with mathematically, for example, by assuming them to have a constant or variable value.
- Understand the implications of assumptions made for the development of the mathematics that will follow.
- Consider models developed using
  - numerical estimates in complex scenarios
  - functions and graphs
  - statistical techniques and measures
  - recurrence relations
  - ideas of probability including the normal distribution.

Learning outcome 2: Understand by following, and inquiring into, the mathematical communications of others, the detail of mathematical reasoning and application of mathematical techniques

Topic 2.1 understand, follow, and critically question the appropriateness and accuracy of the application of mathematical techniques and reasoning

Topic 2.2 critically read and understand how mathematical diagrams are used to provide insight into situations and communicate the outcomes of mathematical working.

Topic 2.1

- Consider mathematical working critically from a point of view of accuracy and use of standard mathematical conventions and notation.
- Analyse carefully and understand chains of mathematical reasoning and the development of mathematical arguments.
- Identify potential modifications to mathematical working that will improve clarity in mathematical arguments.
- Understand how aspects of a context being modelled relate to the mathematics throughout its development.

Topic 2.2

- Understand how features of mathematical diagrams relate to factors in a real context that is being modelled.
- Understand limitations of mathematical diagrams as models that are mathematical representations of aspects of real contexts.
- Understand the impact on mathematical diagrams of varying factors in the real situation (for example understanding the impact of changing an assumed constant value in the real context on the shape of the graph that represents this).
- Consider how mathematical diagrams such as graphs can be used to provide insight into a situation and how a different, perhaps simplified graph, may be used to communicate a final solution to a problem.

Learning outcome 3: Understand the effectiveness of the mathematical models of others and evaluate and explain how these might be improved

Topic 3.1 evaluate the outcomes of mathematical solutions to problems in terms of the context and how factors of the situation have been dealt with mathematically

Topic 3.2 analyse the effectiveness of mathematical models in terms of how they represent the simplified complexity of real situations

Topic 3.3 explain how mathematical models might be modified, refined or adapted for use in different situations.

Topic 3.1

- Consider the feasibility of the solution(s) found by working with a mathematical model in light of the real situation – asking the question: *Is this a reasonable solution?*
- Consider the solution(s) in light of how important factors in the situation were accounted for and the resulting limitations in the mathematical model developed – asking the question: *What was the effect on this solution of making that assumption?*

Topic 3.2

- Understand how a mathematical model might be further developed to provide solution(s) that are more appropriate or effective after consideration of how important factors in the situation were accounted for – asking the question: *How can the situation be modelled in such a way that leads to a better solution(s)?*
- Consider if the model can be developed or adapted to have wider applicability.

Topic 3.3

- Understand how, with modification or refinement, a mathematical model could be used in a similar real situation).
- Understand how general mathematical models (e.g. linear, quadratic, exponential functions) can be used across a range of different situations because of their structural features.

Communicating with mathematics

What is this content area about?

The purpose of this content area is for learners to be able to consider:

- critically the role of mathematics when communicating mathematical reasoning in pursuit of solutions to problems arising in real contexts and additionally when communicating final outcomes of modelling
- the intended purpose and audience of communications that involve mathematics.

Learners should work with a wide range of forms of either primary and secondary data/information including in written and diagrammatic forms. They should take a critical stance and be able to select which data/information to work with and communicate limitations and the need for caution where and when necessary.

Learners will be expected to engage in writing and other forms of communication that includes both mathematical arguments and diagrams that work to usual conventions and standards of notation. They should take account of intended purpose and audience when planning and drafting communications.

Learning outcomes

Learners will be able to

1. Understand mathematical communication as:
   - an important part of the process of developing understanding and insight towards reaching a solution to a problem
   - a means of explaining and presenting the outcomes of mathematical reasoning or providing insights into complex real contexts
2. Communicate mathematical reasoning and solutions to problems clearly using standard mathematical conventions and notation
3. Communicate with mathematics for different audiences and for different purposes

Scope of content

This section gives details of the scope of content to be covered in the teaching of the content area to ensure that all the learning outcomes can be achieved.

Learners should be confident and fluent in using the mathematical techniques and reasoning identified in Section 6. This content should be developed as learners engage with the following mathematical practices:

- working with data graphically
- interpreting data critically
- communicating with mathematical diagrams
- estimating and predicting
- costing and organising.

Learning outcome 1: Understand mathematical communication as:
- an important part of the process of developing understanding and insight towards reaching a solution to a problem;
- a means of explaining and presenting the outcomes of mathematical reasoning or providing insights into complex real contexts

Topic 1.1 analyse the effectiveness of mathematical arguments and diagrams when supporting mathematical reasoning

Topic 1.2 analyse the effectiveness of mathematical arguments and diagrams when presenting outcomes of mathematical work to others.

Topic 1.1

- Critique mathematical writing as a means of providing insight into mathematical thinking.
- Critique mathematical diagrams as a means of providing insight into mathematical thinking.

Topic 1.2

- Critique mathematical writing as a means of presenting solutions and providing insight into the real context and situation.
- Critique mathematical diagrams as a means of presenting solutions and providing insight into the real context and situation.

Learning outcome 2: Communicate mathematical reasoning and solutions to problems clearly using standard mathematical conventions and notation

Topic 2.1 select data to allow the development and communication of mathematical reasoning

Topic 2.2 develop effective mathematical arguments and diagrams for communicating solutions to problems.

Topic 2.1

- Work with quantitative data presented in a wide range of formats (for example, within written text, tables, graphs, charts, diagrams) to develop understanding of the context/situation from which they arise.
- Select judiciously from data to allow presentation or re-presentation of mathematical reasoning so as to provide insight into real contexts/situations.

Topic 2.2

- Draft and redraft mathematical communications that include written text, mathematical arguments and mathematical diagrams so that they are clear and easy to follow.

Learning outcome 3: Communicate with mathematics for different audiences and for different purposes

Topic 3.1 develop mathematical writing and diagrams with varying degrees of complexity (as appropriate for purpose and audience)

Topic 3.2 develop mathematical writing and diagrams with varying degrees of complexity using appropriate writing styles.

Topic 3.1

- Explore and critique the mathematical work, including mathematical arguments and diagrams, of others from the point of view of intended audience and purpose.
- Plan communications that include mathematical arguments and diagrams that take account of their intended purpose and the likely mathematical literacy of the intended audience.

Topic 3.2

- Explore and critique a range of different types of mathematical communications (such as written texts, posters, charts and diagrams) from the point of view of their intended purpose and the potential mathematical literacy of the likely audience.
- Develop a range of different mathematical communications that take account of intended purpose and the likely mathematical literacy of the intended audience.

6 Mathematical techniques and reasoning

The mathematical techniques and reasoning developed at GCSE Higher level may be drawn upon throughout the three content areas of the qualification. These are indicated here with some expectations of how they should be developed with application as a priority, GCSE specifications give details of range and scope.

Note the following topics from GCSE are **excluded**: geometrical reasoning and construction, vectors and the use of sine and cosine rules.

Content that is additional to GCSE is indicated in *italics*.

Number, ratio and proportion	Confident and fluent use of ▪ numbers expressed in different forms including decimals, fractions and standard form ▪ very large and very small numbers ▪ standard conventions and notation when communicating calculations ▪ standard - units of mass, length, area, volume, time, money - compound units such as density, unit costs - rates of change including speed and acceleration ▪ approximating and estimating to check *validity* of calculations ▪ ideas of *dimensional analysis* to check *validity* of calculations ▪ ratio ▪ direct and inverse proportion (including ideas of scale factors) Understanding of ideas of ▪ accuracy, including upper and lower bounds ▪ *logarithms and logarithmic scales*

<table>
<tr><td>Algebra, sequences and graphs</td><td>Confident and fluent in:
using algebraic expressions and formulae (including substitution, simplification, factorisation, manipulation and rearranging)
working with inequalities
developing algebraic expressions and formulae expressed using
- standard conventions and notation
- spreadsheets
Understand ideas of
functions
inverse functions
dimensional analysis in simple formulae to check validity
Work with
the algebra and graphs of common functions:
- linear
- quadratic
- powers of x (1/2 and positive and negative integers)
- exponential (for growth and decay, including ideas of half-life, connecting with ideas and use of logarithms)
(note this requires algebraic treatment beyond that usually required at GCSE)
transformations of functions when working graphically
gradients (instantaneous and average) of graphs as rates of change
areas under graphs (determined using numerical approximation) where appropriate
Confident and fluent ability to work with
sequences defined by a position-to-term rule
sequences defined by recurrence relations (including the logistic equation)</td></tr>
<tr><td>Geometry & Measures</td><td>Familiar with properties of, and terms used when working with, special geometrical shapes (square, rectangle, parallelogram, trapezium, kite, rhombus, isosceles triangle, equilateral triangle, circle) and solids (cube, cuboid, prism, cylinder, pyramid, cone, sphere).

Work with
standard units of measures and compound measures such as those associated with length, mass, time, capacity, density, speed, acceleration.
scale drawings, maps and other diagrams representing spatial situations such as elevations and topological maps.
formulae for area and volume of special shapes and solids (including fractional parts such as the area of a sector of a circle)
Pythagoras’ theorem
trigonometric ratios to solve problems in two and three dimensions</td></tr>
</table>

Probability and risk	Understand ideas of, and be able to calculate (where appropriate), ▪ experimental probabilities ▪ theoretical probabilities ▪ conditional probabilities ▪ expectation ▪ *risk (absolute and relative)* ▪ randomness and bias Use ▪ diagrammatic representations to calculate and communicate probabilities ▪ standard conventions and notation ▪ *the normal distribution as a probability distribution used to model commonly and naturally occurring phenomena*
Statistics	Confident and fluent in working with: ▪ measures of central tendency and spread ▪ discrete and continuous data ▪ standard statistical diagrams and representations (including histograms, cumulative frequency diagrams, scatter graphs) Understand ideas of ▪ *visualising/communicating data using non-standard/innovative representations* ▪ populations and samples (including representative, stratified, random etc.) ▪ correlation when working with bivariate data and lines of best fit *determined by calculator or spreadsheet* ▪ distributions (including comparisons with probability distributions) ▪ *natural variation*
Critical paths	*Use cascade diagrams/Gantt charts to determine timings of critical activities in projects and use these to calculate costings in terms of time, quantities, finances.*

Appendix 1 Sources of general information

The following documents contain essential information for centres delivering City & Guilds qualifications. They should be referred to in conjunction with this handbook. To download the documents and to find other useful documents, go to the **Centres and Training Providers homepage** on **www.cityandguilds.com**.

Centre Manual - Supporting Customer Excellence contains detailed information about the processes which must be followed and requirements which must be met for a centre to achieve 'approved centre' status, or to offer a particular qualification, as well as updates and good practice exemplars for City & Guilds assessment and policy issues. Specifically, the document includes sections on:

- The centre and qualification approval process
- Examination roles at the centre
- Registration and certification of candidates
- Non-compliance
- Complaints and appeals
- Equal opportunities
- Data protection
- Management systems
- Maintaining records.

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Useful contacts

UK learners General qualification information	**E: learnersupport@cityandguilds.com**
International learners General qualification information	F: +44 (0)20 7294 2413 **E: intcg@cityandguilds.com**
Centres Exam entries, Certificates, Registrations/enrolment, Invoices, Missing or late exam materials, Nominal roll reports, Results	F: +44 (0)20 7294 2413 **E: centresupport@cityandguilds.com**
Single subject qualifications Exam entries, Results, Certification, Missing or late exam materials, Incorrect exam papers, Forms request (BB, results entry), Exam date and time change	F: +44 (0)20 7294 2413 F: +44 (0)20 7294 2404 (BB forms) **E: singlesubjects@cityandguilds.com**
International awards Results, Entries, Enrolments, Invoices, Missing or late exam materials, Nominal roll reports	F: +44 (0)20 7294 2413 **E: intops@cityandguilds.com**
Walled Garden Re-issue of password or username, Technical problems, Entries, Results, e-assessment, Navigation, User/menu option, Problems	F: +44 (0)20 7294 2413 **E: walledgarden@cityandguilds.com**
Employer Employer solutions, Mapping, Accreditation, Development Skills, Consultancy	T: +44 (0)121 503 8993 **E: business@cityandguilds.com**
Publications Logbooks, Centre documents, Forms, Free literature	F: +44 (0)20 7294 2413

If you have a complaint, or any suggestions for improvement about any of the services that we provide, email: **feedbackandcomplaints@cityandguilds.com**

About City & Guilds

As the UK's leading vocational education organisation, City & Guilds is leading the talent revolution by inspiring people to unlock their potential and develop their skills. We offer over 500 qualifications across 28 industries through 8500 centres worldwide and award around two million certificates every year. City & Guilds is recognised and respected by employers across the world as a sign of quality and exceptional training.

City & Guilds Group

The City & Guilds Group is a leader in global skills development. Our purpose is to help people and organisations to develop their skills for personal and economic growth. Made up of City & Guilds, City & Guilds Kineo, The Oxford Group and ILM, we work with education providers, businesses and governments in over 100 countries.

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