City & Guilds MATHS & ENGLISH

STARTING POINTS FOR DELIVERY

LEVEL 3 CORE MATHS: USING AND APPLYING MATHEMATICS (3849)

About City & Guilds

As the UK's leading vocational education organisation, City & Guilds is leading the talent revolution by inspiring people to unlock their potential and develop their skills. We offer over 500 qualifications across 28 industries through 8,500 centres worldwide and award around two million certificates every year. City & Guilds is recognised and respected by employers across the world as a sign of quality and exceptional training.

City & Guilds Group

The City & Guilds Group operates from three major hubs: London (servicing Europe, the Caribbean and Americas), Johannesburg (servicing Africa), and Singapore (servicing Asia, Australia and New Zealand). The Group also includes the Institute of Leadership & Management (management and leadership qualifications), City & Guilds Licence to Practise (land-based qualifications), the Centre for Skills Development (CSD works to improve the policy and practice of vocational education and training worldwide) and Learning Assistant (an online e-portfolio).

If you have a complaint, or any suggestions for improvement about any of the services that we provide, email: **feedbackandcomplaints@cityandguilds.com**

City & Guilds
1 Giltspur Street
London EC1A 9DD
T +44 (0)844 543 0000
F +44 (0)20 7294 2413
www.cityandguilds.com

Contents

Introduction

This guidance document has been developed for teachers and centres planning to deliver City & Guilds' Level 3 Core Maths qualification, the *Certificate in Using and Applying Mathematics* (3849).

The qualification has been developed to support learners in a range of settings to develop the higher level mathematics skills needed for a variety of studies at university and for success in the full range of business, professional and technical careers.

This document provides **an introduction to the structure of the qualification and how this relates to real world activities**. Whilst the qualification is focused on three main areas of study, modelling, comprehension and communication, teaching should be structured around substantial tasks with scope to draw on and develop understanding and confidence in all three areas. As a way into this approach, we present three contexts as starting points each followed by task ideas, involving further exploration and development of the relevant mathematics.

This document is part of a package of materials being developed by City & Guilds to support our Level 3 Core Maths offer. This includes:

- **Guidance for Delivery**
  Substantial activities to form the basis of sequences of lessons. Samples can be found on the ***Using and Applying Mathematics* (3849) qualification webpage**.

  More of these will be made available by September 2015 to support centres delivering the qualification during 2015-16.

- **Scheme of work**
  To be published on the ***Using and Applying Mathematics* (3849) qualification webpage** in July 2015.

- **Sample assessment materials**
  Three sets of sample examination materials will be available to support implementation from September 2015.

  Worked examples, illustrating grading and material to support assessment of learners' progress will be added in autumn 2015.

About Core Maths

City & Guilds' Level 3 Certificate in Using and Applying Mathematics is a new qualification which has been developed particularly to align with the Government's Level 3 Core Mathematics Initiative for 16-18 learners in England.*

Level 3 Core Maths is a new category of mathematics qualification, designed especially to encourage continued study of maths up to the age of 18 by learners who:

- have already achieved GCSE Mathematics at grade C or above
- are not intending to study AS level or A level Mathematics
- will benefit from the ability to apply higher level maths skills in their studies at University and across a range of business, professional and technical careers.

* See **www.gov.uk/government/news/launch-of-new-high-quality-post-16-maths-qualifications**

Why is Core Maths important…

…in Higher education?

“too many students arrive in higher education without a realistic understanding either of the relevance of mathematics and statistics to their discipline or of the demands that will be put upon them”.

‘*Mathematical Transitions:* A report into the mathematical and statistical needs of students undertaking undergraduate studies in various disciplines’

The Higher Education Academy, 2015[†]

Mathematics and statistics play a fundamental role in the curricula of a wide range of the ‘non-maths’ subjects that your learners will go on to study at university.

Universities tell us that this content is a very often a challenge for learners and a barrier to being able to make the most of studies at higher levels.

[†] **www.heacademy.ac.uk/mathematical-transitions-report-mathematical-and-statistical-needs-students-undertaking** (p5)

Why is Core Maths important…

…in employment and careers?

"Higher level maths skills, like forecasting, interpreting statistics and using maths to inform decision-making, are essential… all too often potential employees lack these vital skills."

Head of Early Careers, Barclays Bank PLC

Employers express similar concerns and stress the importance of the ability to think mathematically, solve problems, work with 'big data' and communicate mathematically for success and progression in the modern workplace.

Both sets of stakeholders tell us that many entrants to HE and employment, even those with good grades for GCSE Maths, lack confidence in their mathematical abilities and are often unable to transfer skills to new or unfamiliar contexts and problems.

The time elapsed between GCSE Mathematics study and learners failing to see the relevance of maths to their higher education and career aspirations are seen as contributory factors.

Level 3 Core Maths qualifications have been introduced to address these concerns. The aim is to increase study of maths post-16, develop learners' confidence and for delivery to be relevant and engaging to the needs of learners future study and employment goals.

About City & Guilds' Core Maths qualification, *'Using and Applying Mathematics'*

City & Guilds is proud to be part of this initiative. In developing our qualification we have listened to and worked closely with Higher Education and employers to ensure that our qualification is relevant to their needs and an engaging and rewarding experience for learners.

Using and Applying Mathematics focuses on three main areas of study:

- **Mathematical modelling**
  learners will develop the ability to create and use mathematical modelling to solve complex and meaningful real world problems
  (eg *to identify trends, calculate interest and growth*).

- **Mathematical Comprehension**
  the ability to understand and critically engage with the way others have used and applied statistics and other mathematical information
  (*eg market/business intelligence, financial analysis; risk analysis, product development*).

- **Communicating with mathematics:**
  collection and interpretation of data and statistics effective reporting and communication of mathematical information
  (eg *in written reports and communication of key findings through presentations to colleagues, suppliers, customers to make recommendation*).

Technology as a mathematical tool

In addition to the strong focus on mathematical communication. City & Guilds recognises the importance of technology as a tool for mathematical modelling, processing and communicating.

We strongly encourage learners to have an attit ude of critical exploration and use technology throughout their work.

In line with this approach **we are the only awarding organisation to allow learners the option of using technology in our Level Core Maths end-assessment**.

*"Software packages are now very widely used in industry. The availability of computers has not just changed how work is done, it has changed **what** work is done. This change requires staff to be more mathematically competent. Although employees may not be required to undertake routine calculations, they are required to undertake higher cognitive tasks such as interpreting the meaning of the computer-generated results of calculations."*[‡]

Advisory Committee on Mathematics Education (ACME)
"Mathematics in the workplace and in Higher Education", 2011)

Using and Applying Mathematics promotes the use of technology throughout, including in assessment to help ensure learners are well-prepared for Higher Education and work.

[‡] **www.acme-uk.org/media/7624/acme_theme_a_final%20(2).pdf** (p17)

Modelling, comprehension and communication in Higher Education and employment.

Mathematical models are used throughout Higher Education courses and in many different workplaces. They are central to both workers' and HE students' engagement with mathematics. In these contexts when undergraduates and employees encounter mathematics it is almost always the use of mathematical models that is most important.

Some HE courses will involve learning new and more sophisticated mathematical techniques and reasoning, but in almost all instances, other than on mathematics based courses, it is application that is central and skills in modelling, comprehending and communicating that are required.

In relation to workplaces, the Advisory Committee on Mathematics Education (ACME) identified that learners will need to be well-prepared to engage with mathematical models[§]:

> *"The development and application of mathematical models occurs across a range of industries. It was common to find individuals who used a model that was developed elsewhere in the company or a software package that was essentially a mathematical model."*

It is not just in specialist and highly-paid professions where mathematical models are found. They are often used to give a simplified measure of an aspect or outcome of human activity and therefore have a major influence on how we see the world that we live in. Statistical models are often used to inform us about what is happening in terms of production in the workplace. Such measures are not only used to control the production process but also to inform employers and employees themselves about worker effectiveness –something we are all interested in when it is applied to our jobs!

Many learners studying 'Using and Applying Mathematics' may already have come across such measures, for example through part-time employment. A simple measure, for example, is to count the number of items produced or sold in a given time. Since the introduction of barcode scanners at supermarket checkouts, the rate at which checkout assistants scan goods and process the shopping of customers can be tracked and monitored. Comparisons between workers can be made based on statistics such as averages and measures of spread like standard variation.

Mathematical models underpin many aspects of study in Higher Education, and not just in courses where this is immediately obvious. Many social science courses will require an understanding of mathematics from a modelling perspective. The current lack of preparation for this has been recognised by the Nuffield Foundation, ESRC, HEFCE, the British Academy, the Royal Statistical Society (RSS) and others.

The UK has a shortage of social scientists trained in quantitative methods and consequently is unable to meet the demand from employers across all sectors – academia, government, charities and business – for staff who can apply such methods to evaluating evidence and analysing data."[**]

---

§ **www.acme-uk.org/media/7624/acme_theme_a_final%20(2).pdf** (p17)

** **www.nuffieldfoundation.org/sites/default/files/files/QM%20Programme%20Background_v_FINAL.pdf** (p2).

In response to such needs identified in HE, *Using and Applying Mathematics* provides an opportunity for post-GCSE learners to engage in meaningful mathematics. It caters not only for those destined to work in the social sciences but for the whole range of learners who might need to work with mathematical models across the full range of HE courses including those in technology, biological and agricultural sciences, business and economics and social sciences including psychology

"Effective communication is central to almost all high-skill activities and essential for those looking to take on management and leadership functions in the future"[††]

It is not only important that learners are equipped with the ability to apply mathematics and develop mathematical models to solve problems it is also important that they can communicate the outcomes of their work in meaningful ways to a range of different audiences. Indeed, they need to be able to develop arguments in which mathematics is central and informative to decision making.

In *Using and Applying Mathematics* learners will have opportunities to learn how to do this by working in two different content areas: Comprehension and Communication. In working with a range of sources, they should have opportunities to consider them from a critical stance.

Learners should consider not only the validity of the mathematics and the mathematical reasoning developed itself (*comprehension*) but they should also consider the approaches taken to the mathematical model that is used and how all aspects might be improved. Such considerations should provide insight into how they might develop effective communications of their own when engaged in substantial tasks involving mathematical models and application. Some starting points that might be used in this way are suggested in the next section.

With its focus on modelling, comprehension and communication, *Using and Applying Mathematics* leads to development of courses that not only allow exploration of everyday issues from a mathematical perspective but also to tailor courses that meet the needs and aspirations of young people as they make their way towards HE and the world of work.

The following sections provide a more detailed introduction to the three main areas of study and a closer look at ways in which technology can be used as a tool to support mathematical problem solving and communication.

[††] Gateway to growth – CBI/Pearson Education and skills survey 2014
www.cbi.org.uk/media/2807987/gateway-to-growth.pdf (p70)

Getting started with teaching

The qualification is focused on three main areas of study (modelling, comprehension and communication), drawing on a range of defined mathematical techniques and reasoning.

It is recommended that teaching is structured around substantial tasks, so as to ensure that using and applying mathematics is seen as an integrated whole with teachers ensuring that learning is supported across all three areas but by careful design focuses learners' development more in one particular area than the others to meet their emerging needs.

The Qualification Handbook gives three example teaching sequences. You can find more on the ***Using and Applying Mathematics* (3849) qualification webpage**.

In the following sections we show how authentic situations where mathematical models have been used can be used to start to plan activities to use with learners. The suggestions here are starting points that might be used to introduce ways of working and thinking that are important to a *Using and Applying Mathematics* course.

As you will see, each starting point has multiple different ways in which you might direct learning to covers the important mathematical practices involved in modelling, comprehension and communication as well as the mathematical techniques and reasoning that underpin the course.

The three starting points here provide a brief scenario for learners to engage with.

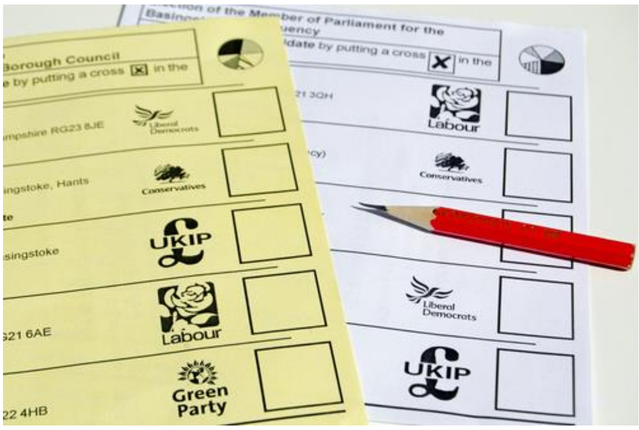
These are different from those that learners might usually meet in traditional mathematics lessons. Their length and use of language might not be familiar to maths teachers.

However, they reflect the nature of such material in the public domain and learners will be need to engage with similar materials in other areas of their studies.

Your learners may need some support and encouragement when meeting unfamiliar contexts for the first time. This is all part of developing problem-solvers being well-prepared to meet the needs of Higher Education, employment and to become critical citizens in the 21st Century.

Modelling activity: 2015 UK General Election:

An example of modelling that raised public debate was the polls used to predict likely voting in the General Election of 2015. In the run up to the election, opinion polls in almost all cases predicted that the two main parties, Conservative and Labour, would receive roughly the same percentage of votes cast.



At the moment that voting was completed an 'exit poll' based on data collected on the day of the election suggested that this would not be the case and that a greater percentage of voters would vote for Conservative candidates. In the following hours this prediction was found to be considerably more accurate than the earlier opinion polls.

There followed considerable discussion about the different models that the polling organisations used and how these were not as valid as they might be.

This newsworthy focus on mathematical models is used to illustrate possible approaches to teaching towards *Using and Applying Mathematics* on the following pages.

It is not often that discussion about mathematical modelling hits the news as their use doesn't often have such immediate impact. However, mathematical models underpin many aspects of everyday life - although they are often hidden and quite complex, drawing on specialist and complex mathematics. For example, the everyday weather forecast is based on highly complex models that require powerful computers to run them. Similarly in business and industry mathematical models are used to determine where to locate distribution centres for goods to supermarkets and parcels to homes and businesses and in the emergency services. Airlines use mathematical models to decide the pricing of seats on aircraft on a day-by-day basis and how they can optimise flight times by reducing the time of passengers boarding the plane and the time spent refuelling.

Most of the models used to solve such problems in reality use mathematics beyond the scope of this course. However, mathematical models that are 'good enough' and lead to solutions that are workable might well be developed by students working towards the *Using and Applying Mathematics* qualification. Indeed simple, or simplified, models can be very useful when communicating with members of the general public.

Starting point: making your vote count

The headline prediction for the May 2015 election was not accurate. The final prediction was for a hung parliament with Labour/SNP able to form the largest bloc. The actual result was a small Conservative majority.

In numerical terms, the prediction and the outcome were:

Party	2010 Vote share	2010 Seats won	2015 Pred Vote Share	2015 Pred Seats Won	Actual 2015 Vote Share	Actual 2015 Seats Won	Vote Error	Seat Error
CON	37.0%	307	33.5%	280	37.8%	331	+4.3%	+51
LAB	29.7%	258	31.2%	274	31.2%	232	0.0%	-42
LIB	23.6%	57	11.0%	21	8.1%	8	-2.9%	-13
UKIP	3.2%	0	13.0%	1	12.9%	1	-0.1%	-0
Green	1.0%	1	5.1%	1	3.8%	1	-1.3%	0
SNP	1.7%	6	4.1%	52	4.9%	56	+0.8%	+4
Plaid	0.6%	3	0.6%	3	0.6%	3	0.0%	0
MIN	3.4%	18	1.5%	18	0.8%	18	-0.7%	0

Adapted from the website 'Electoral Calculus'

Conservative support was significantly underestimated, which caused the number of Conservative seats to also be underestimated. Although the Labour support figure was quite accurate, the error in the Conservatives caused the predicted number.

As the BBC website reported,
"UKIP's vote share was up by 10 percentage points to a total of 3.9 million.
Still, the party won just one constituency under the UK's first-past-the-post voting system.
The Greens' ambitions were similarly thwarted: they won more than a million votes but just one seat."

The Electoral Reform Society, a campaign group, has explored how different proportional voting systems would be fairer in terms of the relationship between votes cast and seats won . For example, it has looked at the **D'Hondt method** of converting votes to seats.

This is the system used to elect Britain's representatives in the European parliament. Again the *BBC website* explains,

"Seats in the European Parliament representing England, Scotland and Wales are distributed according to the D'Hondt system, a type of proportional representation. The nations are divided into 11 electoral regions: nine in England, plus Scotland and Wales. Parties vying for election submit a list of candidates to the electorate in each region for their approval.

A system devised by Victor D'Hondt, a Belgian lawyer and mathematician active in the 19th Century, then dictates the results:
In the first round of counting the party with the most votes wins a seat for the candidate at the top of its list.

In the second round the winning party's vote is divided by two, and whichever party comes out on top in the re-ordered results wins a seat for their top candidate.

*The process repeats itself, with the **original vote** of the winning party in each round being divided by one plus their running total of MEPs, until all the seats for the region have been taken."*

This article goes on to give an example of how votes resulted in European members of parliament MEPs based on 2009 voting figures for the West Midlands: "which had a total of six seats in the European Parliament up for grabs". For simplicity, only the five largest parties by vote share are included.

Round 1
The Conservatives win the largest number of votes, so the candidate at the top of their list is elected.

Party	Votes	MEPs elected in Round 1
Conservative	396,847	1
UK Independence Party	300,471	0
Labour	240,201	0
Liberal Democrats	170,246	0
British National Party	121,967	0

In the **second round**, the Conservative Party's vote is reduced to 198,424 so the highest placed party is now UKIP – so the candidate at the top of their list is elected. In the third round the UKIP vote is reduced to 150,236 leading to the election of a Labour candidate. The Conservative's second candidate is then the highest placed in the fourth round, then the Liberal Democrats' first candidate in the fifth round and finally UKIP's second candidate.

After **all six** rounds have been completed the region has 2 Conservatives, 2 UKIP and 1 Labour and 1 Liberal Democrat.

Task ideas: making your vote count

Comprehension

This starting point, as in all the cases here, presents aspects of mathematical modelling that have been developed and communicated by someone else. Before students can work with the article or other starting point effectively they have to understand it themselves.

You may, therefore, want to start with a series of questions that engage students in making sense of the data and mathematical reasoning and techniques that underpin what is being communicated. These should focus on how the techniques and reasoning are connected with the reality that is central to the material.

Example questions that you might use:

- *In 2010 the Conservatives received 37.0% of the votes and gained 307 seats (Members of Parliament), the Labour Party received 29.7% of the votes and gained 258 seats.*
  - *If the voting system had been based on direct proportion using the Conservative votes as a base how many seats would the Labour Party have gained? How many seats would UKIP have gained?*
  - *What if the Labour Party votes had been used as a base? How many seats would the Conservatives have gained? What about UKIP?*
  - *What would have happened in 2015?*

- *In 2015 the error in votes for the Conservative party was +4.3%. What percentage of the votes were they predicted to get? What about all the other parties?*

- *Approximately how many people voted altogether?*

- *In the West Midlands in 2009 if the 6 MEPs had been elected using a system of direct* proportion how many MEPS would each political party have gained? How does this compare *with the number they got using the D'Hondt system?*

There are many more questions that you could ask that would ensure that students work substantially with the data and have opportunities to fully understand the underpinning mathematics before they move on to developing mathematics of their own.

Modelling

This starting point emphasises that the democratic process of voting for an MP for parliament is dependent on a mathematical model. The method we use, known as "first past the post" is reliant on a simple mathematical model. The entire population of the country is divided by the total number of MPs we wish to have in parliament. Then parliamentary constituency boundaries are drawn up so as to try to have the same number of voters in each. Of course this only works approximately!

The article introduces an alternative voting system based on a different mathematical model. There are in fact many other models used to allocate the number of representatives to legislatures around the world.

You might start a piece of work using this starting point by drawing attention to the issue of mathematical models underpinning the voting process. Questions you might ask learners to think about and work on are suggested below. You may want to encourage them to find additional data so that they can explore the models introduced here and other models they may find by researching themselves.

- Is the current 'first past the post' voting system fair?
- Can you devise a numerical measure of fairness for the 'first past the post' system?
- Investigate the D'Hondt system by using the data in the article to illustrate how it would work with the data given for the West Midlands. Investigate this system further for cases of the votes being distributed differently – try extreme/special cases such as when the votes are spread unequally or when they are spread very evenly. Can your measure of fairness be applied in these different cases? What do they suggest?
- Find at least one other model for distributing votes. Investigate how this works in terms of your measure of fairness.

After looking at such voting systems/models you may be able to use the ideas to suggest that learners explore voting for an issue that is meaningful for them. If possible they may collect data based on a vote that they organise and they can be asked to see if they can use different models to arrive at different results.

Communication

This might be covered in two distinct ways using this starting point. Learners might be asked to:

a) critique the starting point article, or
b) develop a communication of their own that explains an aspect of a voting system.

Whichever of these two different approaches is adopted it is important that learners have a sense of the audience they are being asked to communicate with. This will often mean doing *less* rather than *more*.

For example, you might ask learners to:

- Compare the way the 2015 General Election polls were reported for different audiences (eg tabloid/ broadsheet newspapers)
- Rewrite the explanation of the D'Hondt system in simple terms so that it could be used in a tabloid newspaper
  (drawing attention to a tabloid newspaper is to ensure that language and the use of mathematics is simplified)
- Design three PowerPoint slides that contrast the fairness of two different voting systems
    - alternatively develop a brief video as might be used in the TV news.

Working with technology

This starting point provides opportunities for learners to work with the data using a spreadsheet. Not only does this allow them to quickly work out how all political parties are affected by a suggested change to the model used but it also allows them to develop their understanding of algebraic structure in an informal way.

For example when working on the types of comprehension questions suggested above, if trying out a method of allocating votes using direct proportion their attention can be directed to how they are setting this up using 'spreadsheet algebra'.

Working with technology in this way raises other issues of mathematical content. For example, when working with the data given in the article and a model of direct proportion to allocate parliamentary seats, as shown here, there are issues of rounding to consider (the total number of seats as calculated by the spreadsheet is one more than actually available).

	A	B	C	D
1	Party	2010 Votes	2010 Seats	direct proportion
2	CON	37.00%	307	241
3	LAB	29.70%	258	193
4	LIB	23.60%	57	153
5	UKIP	3.20%	0	21
6	Green	1.00%	1	7
7	SNP	1.70%	6	11
8	Plaid	0.60%	3	4
9	MIN	3.40%	18	22
10	Total		650	651
11				

Mathematical Comprehension

This content area engages learners in understanding how to analyse and work critically with mathematical information, arguments and interpretations developed by others. They may on occasions have read a mathematics text book when learning or revising mathematics as part of previous courses and may have tried to read and comprehend mathematical arguments developed in such texts.

In *Using and Applying Mathematics* it is expected that students will engage in such activity much more substantially.

They are expected not only to be able to follow and make sense of mathematical reasoning (that is making sense of someone else's written account of 'doing the mathematics') but also to consider, and take a critical view of the mathematical modelling of someone else.

This will involve considering:

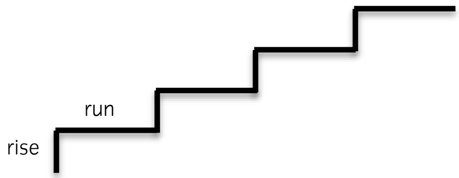
- how someone has set up a model by simplifying a real context and making the real problem into a mathematical one
- how well the mathematics and the mathematical solution fit with the reality
- how the author presents the mathematical outcomes of their work and how they present the implications of these for the reality being modelled
- the effect of decisions they took in setting up their mathematical model.

Throughout their considerations it is important that students take a critical stance questioning the validity of what has been done.

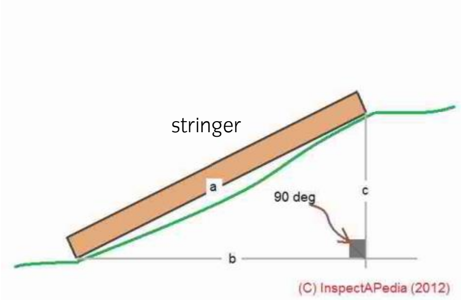
Starting point: building stairs

Sometimes an outdoor staircase is provided when a footpath needs to climb over uneven ground and no natural path seems possible.

When building such a staircase it is important to consider the geometry of the location as well as the stair rise and run (shown in this diagram) and what is comfortable for people to walk up and down.



The article below shows an approach to planning how to build such a staircase. The approach uses mathematics and it is edited from a longer article at **http://inspectapedia.com/Stairs/Stair_Calculations.php**.



$a^2 = b^2 + c^2$ - the square of the length of the hypotenuse (a) equals the squares of the lengths of the opposite sides of a right triangle (b) and (c).

When we refer to the angle ab we mean the angle formed at the intersection of lines a and b, or in this case the lower left angle in our sketch.

And so the three angles formed by the triangle's sides are ab, bc, and ac.

Knowing that the triangle abc will always include a 90 degree right angle (i.e. this is always a "right triangle"- angle bc in our sketch) allows use of sine and cosine to obtain the lengths of the two unknown sides b and c in the sketch.

sin of angle ab = (length of the opposite side c / the triangle's hypotenuse a) = c / a

cos of angle ab = (length of the adjacent side b / the triangle's hypotenuse a) = b / a

We need to know at least two of the angles in this triangle. We know bc is always 90 degrees.

On the ground we'd have to measure either angle ab or angle ac. You can do this using a transit, a protractor or other angle measuring tool either by placing your angular measuring tool on the sloped surface of the stringer side a, or by actually measuring the angle formed between side a and a horizontal or level surface.

Example: if our stairs are to run from the two extreme ends of the stringer in the diagram above right, and if that length (after cutting to 'fit' the hill and desired stair run) measured 100 inches exactly, using just that known length of the hypotenuse of the triangle we would obtain the key measurements of the total *stair rise c* and total *stairway run b* by first converting all measurements to inches and then using sine and cosine. Suppose we measure *angle ab* (sketch above) at 30 degrees.

sin of angle ab = (length of the opposite side c / the triangle's hypotenuse a) = c / a = c / 100

cos of angle ab = (length of the adjacent side b / the triangle's hypotenuse a) = b / a = b / 100

We find that side c (rise) = 50″ and that side b (run) = 86.6″

Calculating the Total Number of Steps up or Risers Necessary to Achieve a Total Vertical Rise

For a total rise of 78"

If we used a minimum rise/step of 4″ then 78 / 4 = 19.5 - we'd have to use 19 steps (since we can't build a fraction of a step and since we're not using 20 steps which would have a slightly short rise of 3.9″ though the local building inspector might accept that)

- 78″ total rise / 19 steps = 4.1″ riser height

Really this means we need 19 steps *up* or 19 *risers* to make up the *vertical rise* - the total change in height.

Task ideas: building stairs

This starting point can, of course, be used to engage students in all aspects of *Using and Applying Mathematics*. In designing a sequence of lessons however, you may wish to ensure students focus on developing their comprehension skills. This requires that they should be able to follow both the mathematical reasoning and techniques developed and any diagrammatic representations of situations being modelled. In developing a lesson sequence that emphasises learning of such skills there are, of course, opportunities to also allow students to consider the modelling aspect of the mathematics presented here as well as taking a critical view of how this is communicated.

Comprehension

The mathematical modelling carried out by carpenters when planning to build a staircase in the situation as described here relies heavily on the use of Pythagoras' theorem and the use of trigonometry. Students working towards the qualification will have learnt the principles associated with this content previously so the article here provides an opportunity for them to re-familiarise themselves with this. One way to start is to ask students to read through the article and ask them to identify any parts they don't understand. This can provide an opportunity to revisit the important mathematical principles that underpin the development of models such as that presented here.

Questions you might use to prompt thinking about mathematical principles:

Explain how angles ab and ac are related (that is if you know one of the angles how you can work out the other?).

- *What are the sine and cosine of angles ac and ab?*

- *In the first example in the article angle ab is taken to be 30 degrees?*
  In this case what is angle ac?
  What is sine ab? What is cosine ab?
  What is sine ac? What is cosine ac?
  What do you notice? Why is this the case?

- *An inch is a unit used to measure length. How many centimetres are there in one inch?*
  Convert the lengths of the total rise and total run to centimetres.
  Carpenters often quote lengths in millimetres. Convert the total rise and run lengths to millimetres.

- *In the first example in the article it is found that the total stair rise is 50" and total run is 86.6". Show calculations in full that explain how these results are arrived at.*

- *In the second example in the article which works with a total rise of 78" 19 steps are used. If the run of a step is 12" what will the total run length be?*
  To fit the total run length into a length of 200" what would be the length of a stair run?
  In this case what will be the angle of the staircase (angle ab)?

Modelling

Having spent some time ensuring that students understand the article and how the mathematics relates to the real situation of building a staircase that provides a footpath over uneven ground it is possible to use these mathematical principles to develop models for different situations.

In reality builders have many options when designing staircases for individual houses although, of course, they often want to minimise the amount of space a staircase takes up so as to maximise other living space.

One way to provide a task which prompts students to consider important factors in the real situation and explore how varying these can lead to different solutions is to ask them to develop a range of options to be considered.

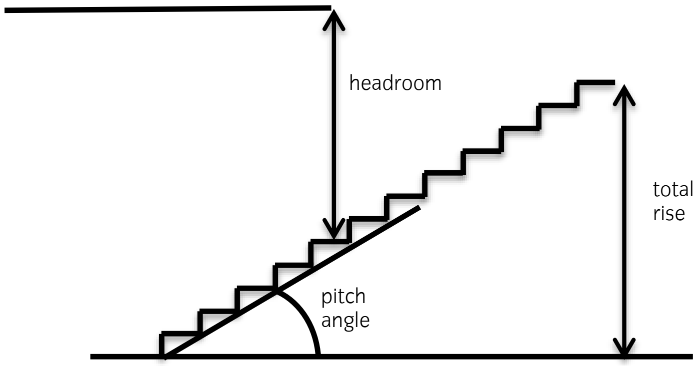
An example task that does this:

Provide three different designs for a staircase that fits in a house where the total rise has to be 2750mm.

Individual stairs can have a maximum rise of 220mm and a minimum run of length 200mm.

The maximum pitch angle of the stairs should be 42 degrees.

The stairs should cater for a person walking up the stairs by ensuring that at every point there is 2000mm headroom.



Communication

This starting point is useful in allowing learners to develop mathematical diagrams that assist them in communicating their modelling work. When working on the modelling task suggested above, or a similar task, the use of diagrams in communicating approaches and solutions is very important. You might start by considering how effective the diagrams in the text are in communicating the situation before attempting to develop their own responses to the task. The diagrams here are all two-dimensional representations of three-dimensional situations: you may wish to discuss the advantages and disadvantages of such diagrams and how the mathematical information that is required is best shown.

As always it is important that final products communicate well with the intended audience. For example, architects drawings and plans include much more detail than is often required by a person having alterations made to their home. That level of detail is not required here.

The three alternatives will require a different amount of space being taken from the area from where the stairs must rise:

- *how is it best to communicate that to a home owner?*
- *Would it be best to have three different diagrams or one single diagram that shows all three approaches?*
- *If three different diagrams should they be aligned in some way that emphasises the differences?*
- *Which units will it be best to use bearing in mind that builders' materials are often sold in a mix of both metric and imperial units?*
- *What will a homeowner understand?*
- *What will a builder need?*

Working with technology

This starting point provides opportunities for working with technology to provide drawings/plans of the situation being investigated. Students might be encouraged to explore possibilities using a spreadsheet in ways that might quickly provide insight into the situation.

Here a spreadsheet has been used to investigate how a range of different step heights would lead to different total run length for steps to fit the total rise of 2750 mm.

	A	B	C	D	E
1	number of steps			Total run	
2	step rise	calculated	rounded	mm	m
3	180.0	15.3	15	3000	3.00
4	185.0	14.9	15	3000	3.00
5	190.0	14.5	14	2800	2.80
6	195.0	14.1	14	2800	2.80
7	200.0	13.8	14	2800	2.80
8	205.0	13.4	13	2600	2.60
9	210.0	13.1	13	2600	2.60
10	215.0	12.8	13	2600	2.60
11	220.0	12.5	13	2600	2.60

There are still some requirements to check whether or not any of these suggestions is viable for the given situation. The spreadsheet can be quickly altered to try some new possibilities if this does not give suitable outcomes.

Communicating with Mathematics

The purpose of this content area is for learners to be able to make sense of, and effectively communicate, complex mathematical information with a strong awareness of purpose and the needs of their audience, including those who are non-specialists in maths.

It involves:

- understanding mathematical communications as part of their own working process and as finished products which they will use to present outcomes of their work to other audiences
- using mathematical reasoning and conventions to develop mathematical arguments and solutions to problems
- considering how best to communicate mathematics for a range of different audiences.

In all aspects of our lives we are provided with information that is based on quantitative data and which has often been processed using a mathematical model. On any one day in newspapers, on TV and through other forms of media we come into contact with a lot of such information.

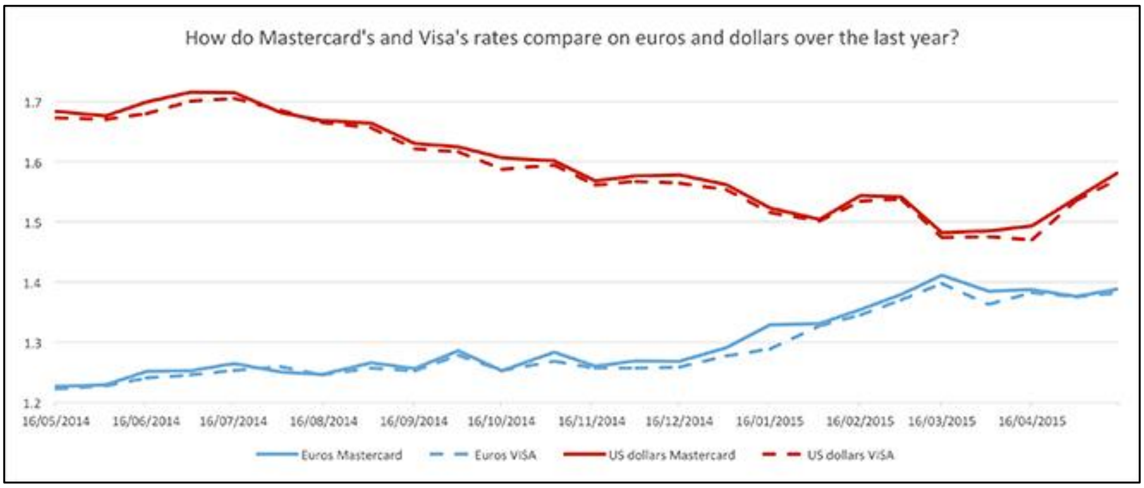
As part of this content area learners are expected to take a critical view of such communications and consider how they might use mathematics and mathematical models to develop such communications themselves always taking into account their intended audience. In doing so they will need to draw on what they have learnt in the other two content areas of modelling and comprehension.

In learning to communicate with mathematics, unusually as part of a mathematics course, learners are required to draft and redraft their work to ensure clarity of their mathematical reasoning and outcomes for different audiences. For example, they may originally write a piece of information for their peers working in a particular vocational area and then rework this for a more general audience. In doing so, the likely level of technical and mathematical understanding of the different readers needs to be considered and the complexity of the writing adjusted accordingly.

Starting point: using your credit card when travelling

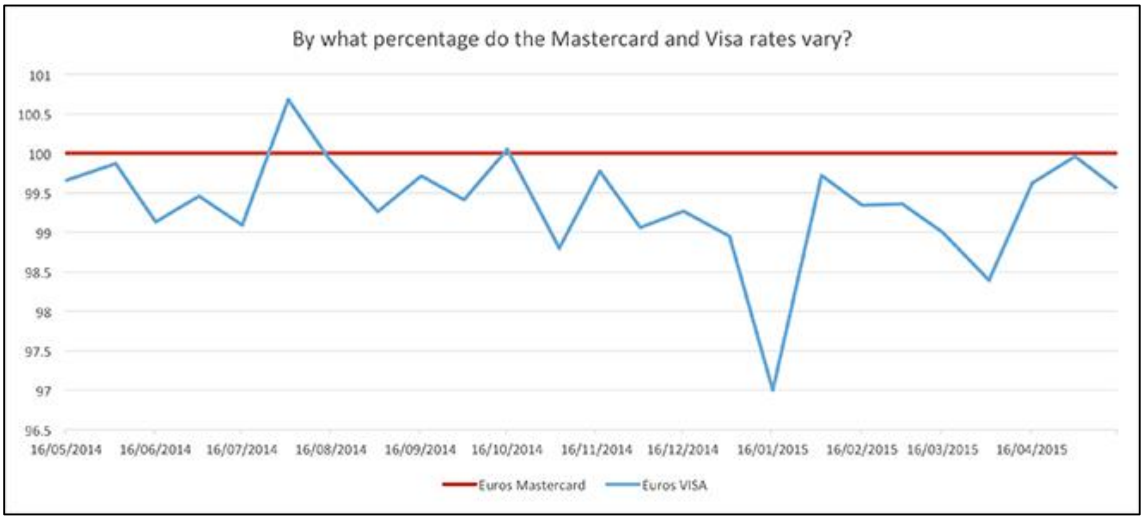
Consider two graphs taken from an article from the website **www.moneysavingexpert.com**.

The article[‡‡] looks at which sort of credit card it is best to use when on holiday in Europe or the USA by comparing the exchange rates that each uses when converting money between Sterling and Euros or Dollars. The first graph used shows the "the published exchange rates for MasterCard's wholesale rate and Visa's wholesale rate for euros and dollars at two different dates in each month for the last year"



In discussion the article illustrates what the difference might mean when someone is on holiday: "For example, in the middle of January 2015, for the euro, there was a variance of €0.04, which is a huge gap for one euro being exchanged. Scale that up and you could have got €132.88 spending £100 worth on a MasterCard, compared with €128.89 on the Visa – a big €4 difference, given the low values involved."

The article goes on to present a second graph explaining, "We've analysed the spread by 'indexing' the MasterCard rate for euros over a year (meaning it's always 100 on this graph) and then allowing the Visa euro exchange rate to vary around it."



‡‡ **www.moneysavingexpert.com/news/cards/2015/05/mastercard-vs-visa-for-using-abroad-which-wins**

Task ideas: using your credit card while travelling

This brief starting point provides some mathematical information already presented as a “communication”: part of a website.

Typically this requires the reader to draw on not only their reading of the article but also on their understanding of additional information and mathematical understanding.

Underpinning part of the article is the idea of the ‘index’ as a mathematical model. Such indices are prevalent in the financial world, and indeed in many other aspects of the world of work. Common examples of such indices are the retail price index (RPI) and consumer price index (CPI) which are often in the news when discussing inflation in the economy and other related issues such as pay rises and increases in benefits such as pensions.

The tasks below centre on developing an understanding of such indices but again attempt to ensure that learning emphasises aspects of each of the content areas of the qualification.

Comprehension

Because of the use of an article to present the context of tasks it is useful to start by ensuring learners have a good detailed understanding of this.

Consequently as well as asking learners to read the article carefully it is also helpful have some questions prepared to prompt them to look at some particularly important aspects of how mathematics and context interact.

For example:
In the period referred to in the article:

- *When was the pound/dollar exchange rate at its peak?*
- *When was the pound/euro exchange rate at its peak?*
- *When was the pound/dollar exchange rate at its lowest?*
- *When was the pound/euro exchange rate at its lowest?*
- *When was there least difference between the pound/dollar and pound/euro exchange rates?*
- *What has been the percentage variation in the pound/dollar exchange rate?*
- *What has been the percentage variation in the pound/dollar exchange rate?*

Show calculations that confirm the statement in the article that: “For example, in the middle of January 2015, for the euro, there was a variance of €0.04, which is a huge gap for one euro being exchanged. Scale that up and you could have got €132.88 spending £100 worth on a MasterCard, compared with €128.89 on the Visa – a big €4 difference, given the low values involved.”
Show calculations that confirm that in the middle of January the index used in the article has a value of 97 as shown on the graph.

According to the graph at the beginning of May 2015 the index value for the euro was 100. What does this tell you about MasterCard and VISA exchange rates?
According to the graph at the beginning of January 2015 the index value for the euro was 99. . What does this tell you about MasterCard and VISA exchange rates?

What does a positive value of the index value for the euro mean? What does a negative value mean?

If the MasterCard euro exchange rate is 1.30 and the index used in the article has a value of 98 what will the Visa card euro exchange rate be?
If the MasterCard euro exchange rate stays the same but the index value increases to 99 what will happen to the Visa exchange rate?

Modelling and Communication

Fundamental to the use and application of mathematics in the second part of the article is the use of an index to illustrate the difference in the two exchange rates under discussion. This way of modelling the situation is not explained in the article and this could provide the starting point for students to communicate their understanding of such an index. An index is of course a measure used to communicate some sense of how quantitative data, often financial, changes over time.

However, it is open to misinterpretation, and this is particularly dangerous if it is not clear what is being measured.

The starting point can be used in a number of different ways and at different levels of complexity.

Two possible variations that will require very different levels of engagement and time to complete:

- *Write a brief paragraph that explains the ideas of "indexing" being used in the article. Use a diagram or diagrams and/or an example or examples if these will help in your explanation.*

- *Write a single page that could be used by the website moneysavingexpert.com that explains the idea of "indices" such as that used in this article. Illustrate your article with diagrams and examples that help clarify your explanation.*

If learners have answered the questions suggested to engage them in comprehending the article before they set out to develop their own writing you might direct some discussion towards considering the article itself as a means of communicating mathematical ideas asking questions such as:

- *Who is the intended audience of the article? Does it address this audience effectively? How might the article be improved when writing for this audience?*

- *What ideas does the article expect the reader to know? Which does it explain? How could any explanations be improved?*

- *Are the graphs clear and easy to understand? How might they be improved?*

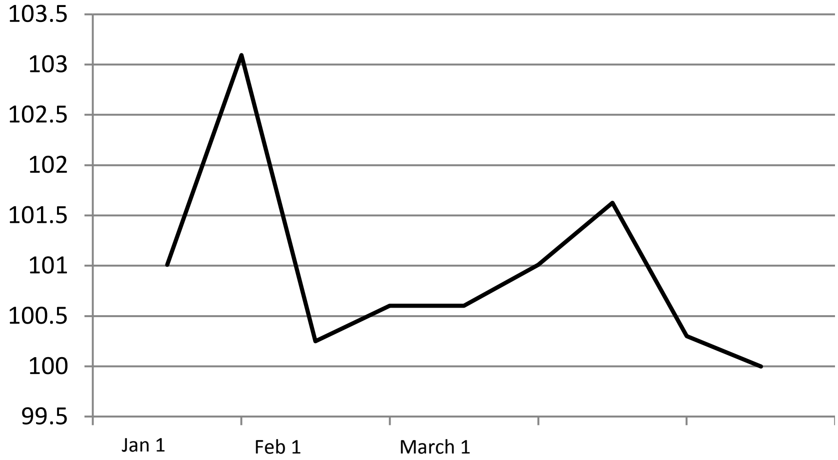
- *Can and should the diagrams be summarised in words – maybe a sentence or two?*

Working with technology

Learners should expect to work with technology in developing any graphs and diagrams they produce.

When working on the task suggested here or any other task that has been developed from this starting point.

Here the index has been recalculated for the first few months of 2015 using the Visa rate as the base



Published by City & Guilds
1 Giltspur Street
London
EC1A 9DD
T +44 (0)844 543 0000
F +44 (0)20 7294 2413
www.cityandguilds.com