Level 3 Certificate in Using and Applying Mathematics (3849)

Teacher Guidance

City & Guilds
MATHS & ENGLISH

Delivering the qualification

Initial assessment and induction

An initial assessment of each learner should be made before the start of their programme to identify:

- if they have any specific training needs
- support and guidance they may need when working towards this qualification.

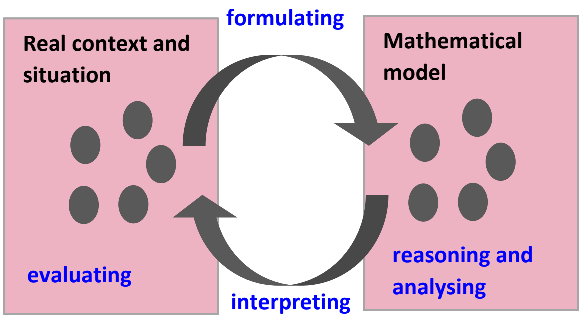
We recommend that centres provide an induction programme so the learner fully understands the requirements of the qualification, their responsibilities as a learner, and the responsibilities of the centre. This information can be recorded on a learning contract.

Guidance for delivery

Although this qualification is split into three content areas - *Mathematical modelling, Mathematical comprehension* and *Communicating with mathematics* - it is not the intention that these should be taught separately; nor is it the intention that the mathematical content that underpins the content areas is taught in isolation. Rather, it is preferable that the course is designed to engage learners in **substantial activities** (see next page). In these, mathematical models are central in ways that involve learners in understanding the mathematical work of others and in both communicating what they have done in arriving at solutions/conclusions to problems *and* the outcome of their work.

Mathematical models and modelling are central to the qualification and the mathematical activity in which learners engage.

Modelling



Modelling *example i)*	
Mathematical models provide a mathematical representation of a real context. They can help make sense of a situation or solve a problem. Often the reality is complex and before a mathematical model can be developed it has to be simplified.	To develop a model that will allow passengers to work out how much a taxi cab is likely to charge for a journey it might be assumed that there is a fixed charge of £2.50 and a charge of £1.25 per mile travelled. This neglects additional charges such as those for carrying suitcases and a charge for when the cab is stationary in traffic.
The mathematical model reflects the structure of this simplified reality and provides insight into this.	This can be represented mathematically: The charge for a journey, £C, can be found using the algebraic equation $C = 2.5 + 1.25m$ where m represents the number of miles travelled. Alternatively a graphical representation plotting C against m might be developed. This gives a straight line that intersects the C axis at (0, 2.5) and has gradient 1.25 (£ per mile). Key aspects of the situation (the fixed charge and the cost per mile travelled) are represented by significant features of each of these mathematical representations.

Modelling *example ii)*	
After a simplified model has been successfully developed this may be refined or modified to more accurately reflect the real situation.	The model might be developed to take account of the factors that were originally neglected, such as the additional charges for carrying suitcases and for when the cab is stationary in traffic.
Each iteration of the model needs to be evaluated in terms of the situation it is representing and its purpose. *Is the model fit for purpose?*	In this case it may be felt that a more complex model that takes account of the additional factors might make it too difficult for passengers to use to get a quick estimate of the fare. Although less accurate, the simplified model is good for its intended use.

In teaching this course learners should have opportunities to engage in activities that require them to develop mathematical models themselves across a range of different contexts. In doing so they should be involved in the following mathematical practices:

- working with data graphically
- interpreting data critically
- communicating with mathematical diagrams
- estimating and predicting
- costing and organising.

In addition to developing their own mathematical models, learners are also required to make sense of the mathematical models of others. They should learn to take a critical view of such modelling activity and be prepared to suggest ways in which it might be improved. This forms the basis of the *Mathematical comprehension* content area but, in teaching the course, comprehension of the work of others can be integrated within activities that also require learners to engage in mathematical modelling themselves.

Equally, learning how to communicate with mathematics, which forms the basis of the *Communicating with mathematics* content area, can be integrated into the activities which learners work on. It is required that learners learn to communicate their mathematical reasoning in a way that is clear and which takes account of the intended audience, as well as to be able to similarly communicate the outcomes of their work. Learning towards *Communicating with mathematics* can be focused at various points during the activities that learners will undertake when working on a substantial modelling task.

Centre staff are therefore advised to structure teaching around substantial tasks that support learners in learning towards each of the content areas of the qualification. The following examples illustrate such an approach, highlighting where learning towards the different content areas can be supported.

The previous examples both involve learners working within the mathematical practice of costing and organising, with *example ii* developing greater complexity than *example i*.

Example Activity 1

This sequence of lessons starts with a context and problem situation. Learners are asked to develop their own model from the outset (*Mathematical modelling*). In this activity they can develop mathematical content in relation to number, ratio and proportion, algebra, sequences and graphs. Their work allows them to engage with the mathematical practice of costing and organising.

Developing a sequence of lessons	Notes and learning outcomes
Starting point A company decides to develop a mobile phone app “Let’s Go!” *Investigate a number of different pricing scenarios so that you can write a brief report to explain to them what you would recommend they do about pricing* ***and*** *why.*	This starting point allows learners to develop a range of models of different levels of sophistication. It requires an eventual product in the form of a communication (a brief report) which demands both a solution to the problem and reasoning to support this. The work generated by this starting point clearly lends itself to learning in relation to the content area *Mathematical Modelling* (*LO2*). Because of the demand for a communicative product it can also be used to emphasise learning required for the content area *Communicating with mathematics*.
First of all learners need to identify the different factors that may have an impact on the profit they might earn from selling an app. In this case factors that need to be considered include: • the price that they will charge for the app • any price they will charge for using the app (monthly, annually,…) • any cost involved in developing the app • the number of apps that they will sell and how this number may vary with price.	The first step in developing a mathematical model is to simplify the real situation: identify all of the factors that may have an impact and make decisions about what to do about each. This may include finding and using known values for some factors, approximating values for others, choosing to ignore others, and so on. The intention is to develop a simplified version of reality that can then be explored mathematically using the maths that the learners know.

Developing a sequence of lessons

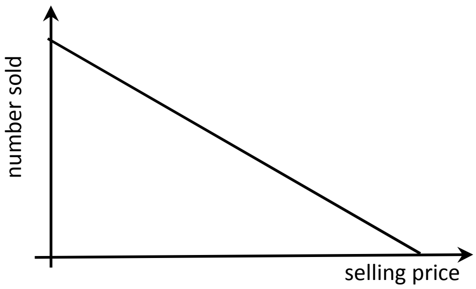
For example, a learner might decide to investigate the profit that will be gained when selling the app for £1, £2, £3 and so on. At this stage it might be assumed that the number sold is not affected by the price but is due to other factors. A learner may produce a graph like the one below if they simply assume that each app can be sold for either £1, £2 or £3.

Questions that might be asked to prompt learners' thinking include:
Why do you have a series of straight lines? What would make a straight line steeper? Why? Where does each line cross the axes? Why? Why might the lines cross the vertical axis at another point? How can the profit be maximised?

In this model it was assumed that the number of sales was effectively independent of price whereas, in reality, if the price is set too high very few may be sold and if the price is set low (or there is even no charge) then a large number may be sold.

At this point you may ask learners to sketch what they think a graph of number sold v selling price might look like.

You might also introduce some alternatives for learners to interpret such as:



Notes and learning outcomes

Having arrived at an understanding of a simplified version of the situation the next step is to develop a mathematical model of the situation ….

…. and then to interpret this in terms of the situation.

At this stage it is important to consider how features of the mathematics relate to the situation and context (*Mathematical modelling LO1 Topic 1.2*). It is helpful to ask a series of questions that might start with, "*What happens if…?*" and "*why….?*"

Learners could be asked to write a brief explanation of this relationship addressing *Communicating with mathematics, LO2, Topics 2.1 and 2.2.*

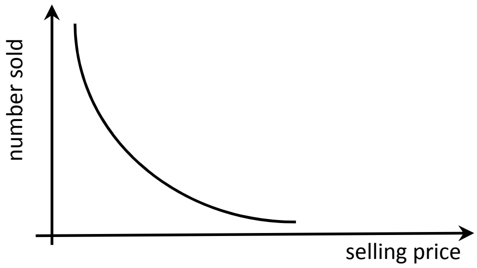
Having developed a mathematical model it is necessary to interpret and communicate how the model relates to the real situation.

In doing so, thought should turn to how effective the model is, and how it might be improved. This will help give insight into how effective the model is at reflecting the reality it is meant to represent.

Almost inevitably, because the first step is almost always the development of a simple model, there will be an opportunity to develop a more sophisticated model.

To help with this it is useful at this stage to return to the initial factors that were considered and the assumptions that were made at that point. How could these have been more realistic to ensure a more effective model? (*Mathematical modelling, LO1 Topic 1.3*). This approach additionally provides potential to cover learning for *Mathematical comprehension, LO1*.

Developing a sequence of lessons	Notes and learning outcomes



Learners could be asked to write a brief explanation of their interpretation of each graph and make a critique of the approach that has been taken.

Learners should now develop their revised and, in this case, more sophisticated, model. The simplest way to do this is to assume that the number of apps sold decreases linearly as the price increases. Values will have to be assumed for how many will be 'sold' when the price is nothing and what price will first lead to no apps being sold.

In this phase it is helpful to emphasise learning towards *Mathematical modelling*, *LO3*.

Again, learners could be asked to write a brief explanation of the outcome of their new model to engage them in learning towards *Mathematical modelling*, *LO3*.

An effective way to work would be to use a spreadsheet to calculate how the number of apps sold varies with price and therefore how the total profit also varies.

In addition this will also provide opportunities to learn towards *Communicating with mathematics*, *LO2*.

This model may be made increasingly sophisticated by considering, for example, production costs and commission charged by the app retailer (possibly a small percentage of the price might be added).

Now that learners have developed an approach that produces a useful model, this can be made increasingly sophisticated.

Example Activity 2

This sequence of lessons starts by introducing a context and problem. It allows learners to develop mathematical content in relation to number, ratio and proportion, algebra, sequences, graphs and probability. Their work allows them to engage with the mathematical practice of costing and organising. Before learners are asked to develop their own model (*Mathematical modelling*) they are asked to make sense of approaches that have been taken by others allowing them at the start of their activities to engage in learning towards *Mathematical comprehension*.

Developing a sequence of lessons	Notes and learning outcomes
Starting point Investors pay a set amount at the beginning of a period of time with the amount each pays depending on how likely they are to die. If an investor dies during the period the fund pays their beneficiary a sum of money. At the end of the period any survivors share the amount of money left in the fund.	This is a potentially complex situation and learners might find starting from scratch quite daunting. One way to assist learners is to present them with some sample work. This will allow them insight into how they might get started.
Develop a spreadsheet to model the situation with instructions so that a member of the public can use it to calculate what they are likely to experience if they join the scheme. The instructions should also explain the scheme as clearly as possible.	Taking this approach to starting learners off working will allow aspects of *Mathematical comprehension* tackling topics relating to each of *LO1*, *LO2* and *LO3*.
Learners could be asked to make sense of simple spreadsheet models that others have developed. They could be required to write brief instructions for a member of the public to follow that tells them how to use these spreadsheets and include an explanation as to why each works in the way that it does. (To do this effectively learners will have to consider the formulae in the spreadsheet cells).	Starting in this way will also support learners in their work towards the product that is eventually required assisting them in learning towards each of the LOs of *Communicating with mathematics*.
Examples of the printouts of typical spreadsheets of simple models are shown on page 17.	
The simple models that learners are asked to investigate require them to consider growth due to the compounding of interest. If they have any difficulties with this you may have to prompt them to develop a spreadsheet that allows them to investigate this without worrying about the more complex issue of making any pay-outs to people who die (much as in the case of Table 1 on page 17).	

Developing a sequence of lessons

It may not be obvious to learners which factors are more important than others in terms of having an impact on outcomes. Again it may be useful to generate discussion about this by asking learners to work collaboratively with only some of them investigating the impact of each of the factors.

For example, groups of learners could explore the impact of:

- compounding interest annually or more frequently
- varying the interest rate
- varying the amount of investment.

It is important to include discussion about assumptions that are implicit in the approach taken so far. For example, it has probably been assumed that the risk of death is independent of factors such as age and gender. Examination of selected data from the Office for National Statistics (ONS) will soon confirm that this is not the case.
http://www.statistics.gov.uk/hub/index.html
However, to ensure that initial models are simple enough to work with it is wise to take such an approach. This can be addressed in later, more sophisticated models.

Notes and learning outcomes

Having understood the context and considered how they might develop a very simple model of the situation a next step is to try a next iteration of the model that adds in some complexity. A useful way to proceed is to consider factors that are likely to have the most impact on outcomes rather than factors that have much less impact.

In developing their own models of increased complexity learners will have opportunities for learning towards *LO1 and LO2* of *Mathematical modelling*.

Learners can be asked to prepare their results in the form of an explanation for others to follow. By sharing their outcomes with each other they will have an opportunity for peer assessment, allowing them to develop their skills in communication and comprehension.

As learners work towards their final product they should be encouraged to ensure that they do this in a way that results in an increasingly complex model. They should not attempt to alter too many aspects of an existing model at each iteration. You may need to prompt learners to search for data that informs their model development – or you may provide this to speed up the process. For example, ONS tables of life expectancy by age and gender might be usefully supplied.

Other factors that may be varied include:

- the proportion of males and females contributing to the fund
- the length of time for which each investor invests their money
- the rate of interest (changing each year or more frequently).

Working in a collaborative way as a whole group to investigate how various factors impact on outcomes provides opportunities for engagement with the LOs of *Mathematical comprehension* and *Communicating with mathematics*. Depending on the teaching approach taken, different aspects of these content areas can be emphasised.

Taking such an approach can stimulate group discussion about how best to proceed to work towards more elaborate models ensuring learning towards *Mathematical modelling*, *LO3*. Learners should be aware of how to develop more sophisticated models even if they do not achieve this themselves. Learners should draw attention to any limitations there are in their models when working on their final communicative product.

Developing a sequence of lessons

Peer review of the final spreadsheets and instructions that are produced provides learners with opportunities to critically consider the work of others. Learners should be encouraged to provide written feedback that focuses not only on the final product but also on explanations of reasoning. This could be carried out in two stages with learners redrafting their work to arrive at a final product.

Notes and learning outcomes

Critical exploration of the work of peers can be used to emphasise learning towards *LO2* of *Mathematical comprehension*.

The final product of their work can be used to consider many aspects of *Communicating with mathematics*.

Table 1

	A	B	C	D
1		Amount in account		
2	Year	Start of year	Growth	End of year
3	1	£25,000,000.00	£500,000.00	£25,500,000.00
4	2	£25,500,000.00	£510,000.00	£26,010,000.00
5	3	£26,010,000.00	£520,200.00	£26,530,200.00
6	4	£26,530,200.00	£530,604.00	£27,060,804.00
7	5	£27,060,804.00	£541,216.08	£27,602,020.08
8	6	£27,602,020.08	£552,040.40	£28,154,060.48
9	7	£28,154,060.48	£563,081.21	£28,717,141.69
10	8	£28,717,141.69	£574,342.83	£29,291,484.53
11	9	£29,291,484.53	£585,829.69	£29,877,314.22
12	10	£29,877,314.22	£597,546.28	£30,474,860.50
13	11	£30,474,860.50	£609,497.21	£31,084,357.71
14	12	£31,084,357.71	£621,687.15	£31,706,044.86
15	13	£31,706,044.86	£634,120.90	£32,340,165.76
16	14	£32,340,165.76	£646,803.32	£32,986,969.08
17	15	£32,986,969.08	£659,739.38	£33,646,708.46
18	16	£33,646,708.46	£672,934.17	£34,319,642.63
19	17	£34,319,642.63	£686,392.85	£35,006,035.48
20	18	£35,006,035.48	£700,120.71	£35,706,156.19
21	19	£35,706,156.19	£714,123.12	£36,420,279.31
22	20	£36,420,279.31	£728,405.59	£37,148,684.90

Table 2

	A	B	C	D	E
1		Amount in account			
2	Year	Start of year	Growth	Payouts	End of year
3	1	£2,000,000.00	£100,000.00	£25,000.00	£2,075,000.00
4	2	£2,075,000.00	£103,750.00	£25,000.00	£2,153,750.00
5	3	£2,153,750.00	£107,687.50	£25,000.00	£2,236,437.50
6	4	£2,236,437.50	£111,821.88	£25,000.00	£2,323,259.38
7	5	£2,323,259.38	£116,162.97	£25,000.00	£2,414,422.34
8	6	£2,414,422.34	£120,721.12	£25,000.00	£2,510,143.46
9	7	£2,510,143.46	£125,507.17	£25,000.00	£2,610,650.63
10	8	£2,610,650.63	£130,532.53	£25,000.00	£2,716,183.17
11	9	£2,716,183.17	£135,809.16	£25,000.00	£2,826,992.32
12	10	£2,826,992.32	£141,349.62	£25,000.00	£2,943,341.94
13	11	£2,943,341.94	£147,167.10	£25,000.00	£3,065,509.04
14	12	£3,065,509.04	£153,275.45	£25,000.00	£3,193,784.49
15	13	£3,193,784.49	£159,689.22	£25,000.00	£3,328,473.71
16	14	£3,328,473.71	£166,423.69	£25,000.00	£3,469,897.40
17	15	£3,469,897.40	£173,494.87	£25,000.00	£3,618,392.27
18	16	£3,618,392.27	£180,919.61	£25,000.00	£3,774,311.88
19	17	£3,774,311.88	£188,715.59	£25,000.00	£3,938,027.48
20	18	£3,938,027.48	£196,901.37	£25,000.00	£4,109,928.85
21	19	£4,109,928.85	£205,496.44	£25,000.00	£4,290,425.29
22	20	£4,290,425.29	£214,521.26	£25,000.00	£4,479,946.56

Example Activity 3

This sequence of lessons prioritises providing an opportunity to develop the mathematical practice of working with data graphically whilst also providing opportunities to engage in learning that highlights mathematical modelling and developing skills in communication. The lessons provide opportunities to work with content in the areas of number, ratio and proportion, algebra, sequences and graphs.

Developing a sequence of lessons	Notes and learning outcomes
Starting point Many bio-chemical processes require the continuous flow of liquid through a fermentation vessel. The liquid flowing in is a mixture of biomass and substrate. Biomass is the substance that the biochemical engineer is interested in growing and the substrate is the 'food' which is necessary for growth to occur. There is a continuous flow of liquid into and out of the vessel. The proportion of biomass in the outflow is much increased from that flowing in whilst the proportion of substrate in the outflow is decreased from that in the in-flow. It is crucial therefore that something is known about the way in which liquids flow in and out of vessels. *Investigate how the flow rate of a liquid varies as the level of liquid falls in a container.* This can be done by using a large 'cylindrical' plastic bottle such as those used to contain water or lemonade. Take measurements of height of water in the bottle and time so that you can calculate the volume of liquid in the vessel as the time varies. *Use your data to find a mathematical model or models that describe how the volume of liquid in the vessel varies with time.* *Use your model(s) to develop a single slide for a presentation that advises a laboratory technician about what they should expect and how they might predict flow rates using mathematics in such a situation.*	This starting point sets the scene indicating a purposeful activity within a science context. The proposed approach engages learners in a practical experiment that will generate data with which they can work. A final communicative product is required: a single presentation slide that should allow learners to demonstrate their ability to explain their understanding of the outcomes of their investigations. The work generated allows the development of learning in relation to *Mathematical Modelling*. Because of the requirement for a final communicative product, learning towards *Mathematical comprehension* and *Communicating with mathematics* with can also be emphasised.

Developing a sequence of lessons

Learners initially need to organise their experimentation in a structured way that will allow them to capture the data they require.

There are some practical issues to be considered as well as ensuring that the quality of data is good enough to allow appropriate mathematical analysis. For example, learners need to consider issues such as how they will need to:

- zero scale readings
- convert observed measurements into useful data in the correct content areas
- average readings for repeated measurements
- ensure data is recorded to an appropriate degree of accuracy.

Having considered these issues learners may need advice about how their experiments might address them.

Following this, learners need to formulate a mathematical model that allows them to gain insight into the situation. They should be encouraged to produce a graph of their data after it has been processed to give values for the volume of water in the bottle as time increases from some starting point. It will be useful to use a spreadsheet to record measured data values and then process these to provide the two variables they will plot a graph for (volume of water in the bottle and time elapsed).

The next step is to consider how best to model the data. Learners' graphs at this stage should show a curve with the value of the gradient being negative but becoming less steep as time proceeds. It may be that if considering a restricted period of time the data might be modelled by a straight line (with perhaps one straight line being used in early stages of the flow and another being used in later stages of the flow).

Learners may find equations of straight lines or curves using a range of methods including using the fitting trend line options of the spreadsheet software.

Notes and learning outcomes

When working with a class there is an opportunity for learners to work in pairs or small groups, with each pair or group collecting data using bottles of different sizes. This will allow for later discussion of generality by drawing on findings from all pairs or groups.

In this particular context a simplified model of a technical situation that one would meet in a science laboratory is designed in such a way that a mathematical model of water flow can be investigated. In doing this simplifying assumptions have been made, such as modelling a plastic bottle as a cylinder (although this is not often the case), focusing only on outflow rather than the case where there is both inflow and outflow, and so on.

In this initial phase of work there are opportunities to focus on the simplification of the real context that is a first step in preparing to develop a mathematical model. This allows learning towards, *Mathematical modelling LO2, Topic 2.1*. Whenever modelling, a representation (such as a graph) will provide some insight into both the real situation and the mathematics that might be used to represent this. Indeed, the diagram itself can be considered as providing a mathematical model of the situation.

A useful way to encourage thinking about further mathematical approaches is to ask learners to consider how features of the diagram relate to the real situation (for example, in the case of graphs, how intercepts with axes and gradients of lines/curves of best fit) relate to the situation.
In developing mathematical models of a situation it is important to do this in a way that allows understanding of the situation to inform the development of the mathematics. Mathematical diagrams such as graphs can facilitate such thinking. This allows learning towards all LOs of *Mathematical modelling*. Teaching staff may wish to emphasise how interpretation of the initial representation of the situation can help provide insight into useful ways of further developing mathematical models (*LO3)*. To carry out such interpretation learners need to be aware of how mathematical measures such as gradients relate to the reality of the context/situation.

When learners have developed a mathematical model(s) of the flow of water they should consider how they can communicate the understanding of the situation that this can provide. They will need to write a brief and clear explanation of this to meet the requirement that they do this on a single presentation slide. They might be encouraged to consider if, and how, their explanation is appropriate for a laboratory technician: they might consider this by thinking about how they could alter their slide if asked to address it to a member of the general public.

Working towards a final product of the form required here provides opportunities for both formative and summative peer assessment at various points during the process.

If time permits you may wish to encourage learners to extend their modelling of the situation to include flow into the container as well as flow out. Learners may like to predict what the effect would be before experimenting.

Learners should be aware of how mathematical models not only need to accurately represent a real context / situation, they also need to be appropriate for their purpose. This means that for effective use by a particular group, or in certain situations, a model may be better in a simple rather than more complex form. Discussions about the communication of mathematical models allows learning towards *Communicating with mathematics*.

Consideration of findings in relation to particular flow situations across the group can facilitate a more general understanding. For example, one might ask how changing the initial conditions will affect the mathematical model(s).

Staff delivering the qualification are encouraged to seek contexts and situations that relate to learners main programmes of study and their Higher Education and career goals.

To support delivery of the qualification, City & Guilds will be making available a full programme of support for teaching and learning. We are working with employers and maths specialists to make this relevant, engaging and current.