UNIT 202

Principles of electrical science

Electrical principles are fundamental to understanding the behaviour of electricity. They represent major scientific breakthroughs, but are now second nature to those in the trade.

Throughout their career, a person will use many of these principles subconsciously, without giving them much thought. However, this knowledge is vital because it is the basis on which we can predict and safely control electrical systems.

LEARNING OUTCOMES

There are seven learning outcomes to this unit. The learner will:

1 know the principles of electricity
2 know the principles of basic electrical circuits
3 know the principles of electromagnetism
4 know the principles of basic mechanics
5 know the electrical quantities in star delta configurations
6 know the operating principles of a range of electrical equipment
7 know the principles of a.c. theory.

This unit will be assessed by:

- online multiple-choice assessment.

EQUIPMENT

You are strongly advised to use a standard scientific (non-programmable) calculator.

Calculators do not all work in the same way. Therefore, any calculator key sequence suggested in this unit is only an example of what would be required on a standard scientific (non-programmable) calculator. If the key sequence does not give the expected result on your calculator, either ask your tutor for advice or refer to the manual for your calculator.

You will also need drawing instruments for this unit such as protractors, rulers and compasses.

OUTCOME 1

Know the principles of electricity

SmartScreen Unit 202
PowerPoint 1 and Handout 1

The basic principles of electricity have been studied for centuries and what is now common electrical theory was once groundbreaking new discovery. The basic principles, including atomic composition, need to be understood in order to work safely with electricity, magnetism and electrochemical reactions, and to progress in the industry.

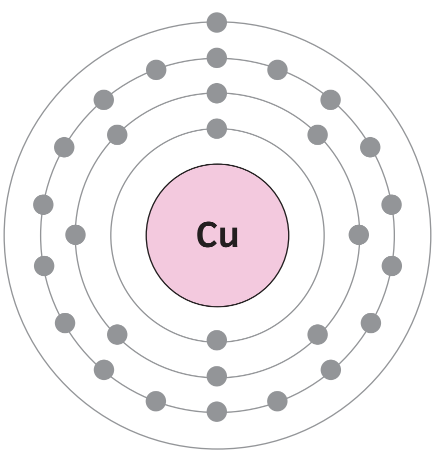
Assessment criteria

1.1 Describe the reaction of electrons when charged to form an electric current

ATOMIC THEORY

In order to understand where electricity comes from and what it is, it is necessary to understand a small amount of atomic theory.

Atoms are very small particles that are sometimes arranged as molecules. An atom is not solid but is made up of smaller particles, separated by space. The centre of an atom is the nucleus, which is made up from various particles including protons and neutrons. Protons are positively charged and neutrons have no charge. Orbiting the atom are the electrons, which are negatively charged.



A simplified copper atom structure with electrons

The atoms that make up different materials have different numbers of electrons. In the steady state an atom has equal numbers of protons and neutrons, and this leaves the atom electrically neutral.

Atoms in solids and liquids are more tightly bound together than those in gases. The diagram (left) shows the simplified structure of copper, which is often used to conduct electricity. The representation is two-dimensional when in fact the actual atom is three-dimensional. Where there are two or more electrons orbiting a nucleus, their orbiting paths are known as shells. The electron paths (shells) form an elliptical orbit.

KEY POINT

Atoms are bound by an electrical force, whereas molecules are bound by a chemical force.

Reaction of atoms

Different atoms have different numbers of electrons. Copper has 29 electrons and 29 protons. The outer shell is weakly held in orbit and can break free, causing random movement among other copper atoms. The loss of an electron causes an atom to become positively charged. This type of atom is known as a positive ion. Positive ions attract electrons, causing electron movement. Negative ions are protons that have additional electrons orbiting them.

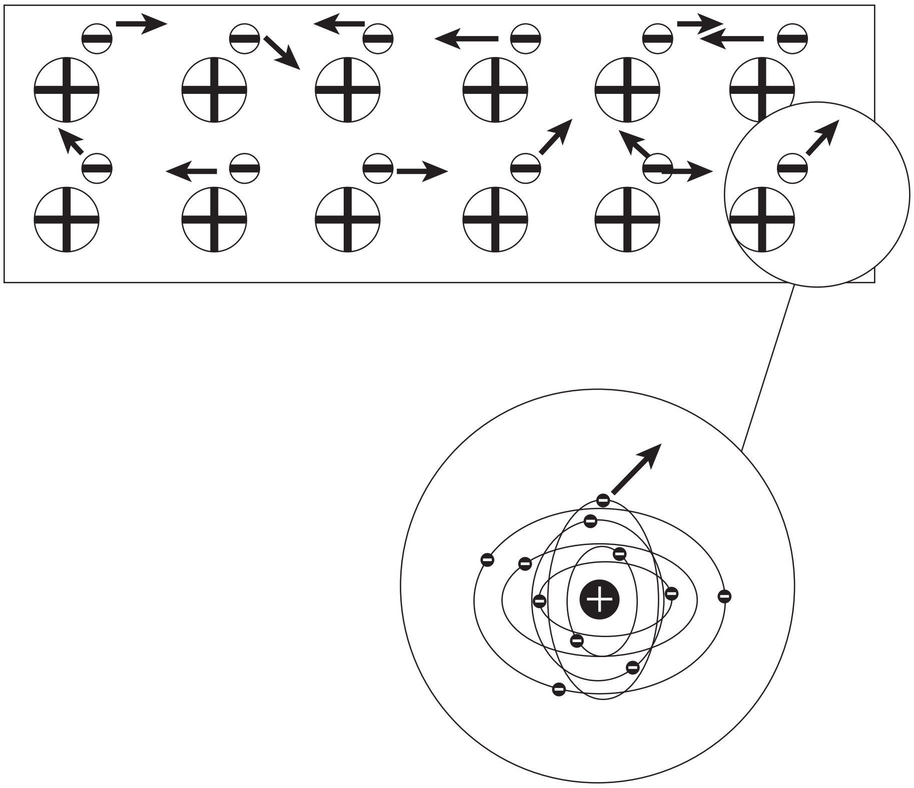
ACTIVITY

Copper has 29 electrons and protons. Using textbooks or the internet, find how many there are for:

a) carbon
b) aluminium
c) silicon
d) gold.

The movement of electrons throughout a material is random but, by the laws of electric charge, like charges repel and unlike charges attract.

The diagram below shows the random movement of electrons in a material. The inset shows the electrons orbiting the proton. Electrons on an outer shell are released, as the force of attraction by the proton is weak, and the electron moves to the next proton.



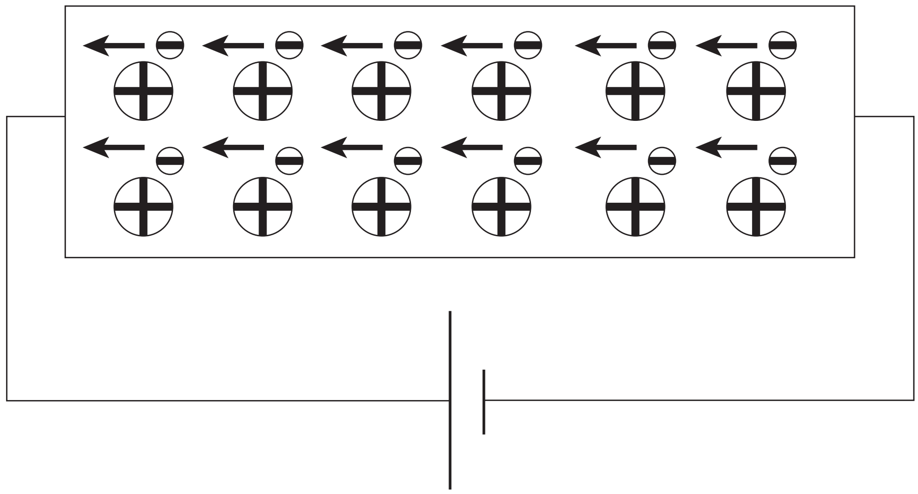
Random movement of electrons in a material

Flow of electrons

Random free electrons can be configured if the conducting material is connected to a battery. The free electrons are attracted to the positive plate and repelled by the negative plate. This causes the electrons to drift, in a conducting material, from the negative terminal of the battery to the positive terminal. As positive ions are unable to drift in solids, every time an electron leaves the negative terminal, one enters the positive terminal. This flow of electrons is electric current.

KEY POINT

Electron is the Greek word for amber. Amber is a material that is easily electrified by static. Rubbing wool on amber can charge the amber, which can then release an electric charge when held against another material or person.



Electrons are attracted in one direction when a source of energy is connected to the material.

Closed circuit

A complete circuit connected to a source of energy. If the circuit contains a switch and the switch is switched off, it becomes an open circuit.

In order for electric current to flow, there must be a **closed circuit**. Once the circuit is opened, the drift of electrons is immediately stopped and the current flow ceases.

Current direction

Electric current flows from a negative terminal to a positive terminal. However, before atoms and electrons were understood, scientists believed that electricity was a fluid and flowed from positive to negative. This is known as conventional current direction and, although it is now understood that electrons flow from negative to positive, we still refer to conventional current direction as being positive to negative.

KEY POINT

Conventional current direction is actually opposite to actual current direction. Current flows from negative to positive but, in our industry, we still refer to conventional current flow.

KEY POINT

The flow of current in one direction is called direct current (d.c.), which is the main current form referred to in this unit.

How many electrons make one ampere?

The flow of electrons in one direction is known as charge, which is measured in coulombs (C).

As electron flow is electrical current and current is measured in amperes (A), how is one converted to the other?

French physicist Charles Coulomb (1736–1806) determined that 1 coulomb of charge is equal to 6.7×10^{18} electrons. That is equal to 6 700 000 000 000 000 000 electrons! So if that many electrons flowed through a material such as copper and past an electron counter, that would be equal to 1 coulomb. He also determined that if the drift of electrons was at a rate of 1 coulomb per second, the resulting current would be 1 ampere.

Therefore a current of 1 ampere flowing indicates a charge of 1 coulomb per second, giving:

$$Q = It \text{ or } I = \frac{Q}{t}$$

where:

Q = charge transferred in coulombs (C)
I = current in amperes (A)
t = time in seconds (s).

Example

If a total charge to be transferred is 750 C in one minute, the current flow is calculated as follows.

Using the formula $Q = It$, calculate the current:

$$I = \frac{Q}{t}$$

Since 1 minute = 60 seconds (s): $\frac{750}{60} = 12.5$ A

The current flow is therefore 12.5 A for 60 seconds to give the total charge of 750 C.

If a current of 25 A was to flow for 2 minutes in the above circuit arrangement, the total charge would be calculated like this.

Since 2 minutes = 120 s and $Q = It$:

$$Q = It = 25 \times 120 = 300 \text{ C}$$

Therefore the total charge would be 300 coulombs.

SOURCES OF ELECTROMOTIVE FORCE

When electric current flows, energy is dissipated because it cannot be created or destroyed. As energy cannot be created, electrical energy has to be converted from an existing form of energy. The form of energy converted may be chemical, as in a battery, it may be mechanical, as in a generator, or a combination of materials reacting to a source of energy such as a solar photo-voltaic (PV) cell reacting to sunlight.

In the early days of electrical research, electricity was believed to be a fluid, which circulated as a result of an applied force. The term 'electromotive force' (emf) (E) was, and still is, used.

ASSESSMENT GUIDANCE

These equations may look strange, but do not let them put you off. The letters stand for numbers. Simple calculations are all you will need to do.

KEY POINT

Where a formula shows two symbols together with no mathematical symbol, it means they must be multiplied. So $Q = It$ simply means $Q = I \times t$.

ACTIVITY

Remember that the SI unit of time is the second, not the minute, hour or day. Use the internet to find the definition of the second.

Assessment criteria

1.2 Identify sources of an electromotive force

Joule

The unit of measurement for energy (W), defined as the capacity to do work over a period of time.

In determining units of electricity, emf is defined as the number of **joules** (J) of work required to move 1 C of electricity around a circuit. This unit of joules per coulomb is referred to as a volt (V).

$$1 \text{ volt (V)} = \frac{1 \text{ joule (J)}}{1 \text{ coulomb (C)}}$$

Example

If a battery of 12 V gives a current of 5 A for 10 minutes, the amount of energy provided over the 10-minute period is calculated as follows.

To find total energy: $W = Q \times V$

Total charge transferred:

$$Q = It = 5 \times (10 \times 60) \text{ C}$$
$$Q = 3000 \text{ C}$$
$$W = Q \times V = 3000 \times 12$$
$$W = 36\,000 \text{ J or } 3.6 \text{ kJ}$$

Electromotive force can be produced through:

- a chemical source
- heat
- electromagnetic induction (see page 97).

Chemical sources

Electrolyte

A chemical solution that contains many ions. Examples include salty water and lemon juice. In major battery production, these may be alkaline, sulphuric acid or zinc/carbon.

When two different metals are placed in an **electrolyte**, ions are drawn towards one metal and electrons to the other. This is called a cell and produces electricity. Many cells joined together are called batteries.

Heat

Simply wrapping a copper wire around a nail and heating one side with a flame can produce electricity, although in very small amounts. This is known as thermoelectric generation. Because the two metals react to the differences in temperature on the heated side and the cool side, a magnetic effect occurs (see magnetism, later in this unit), which creates a current and emf. This process is sometimes called the Seebeck effect. This principle is used in thermocouples, which are used to sense temperature. The amount of electricity generated is in proportion to the temperature.

In some waste disposal plants where waste is burnt, this effect is used to generate electricity.

ACTIVITY

Find a lemon, a zinc-coated nail and a piece of copper. Place the nail and copper into the lemon at opposite ends, ensuring there is a good gap between them inside the lemon. Use a sensitive voltmeter to measure the voltage between the two metals. You can see that a cell has been produced.

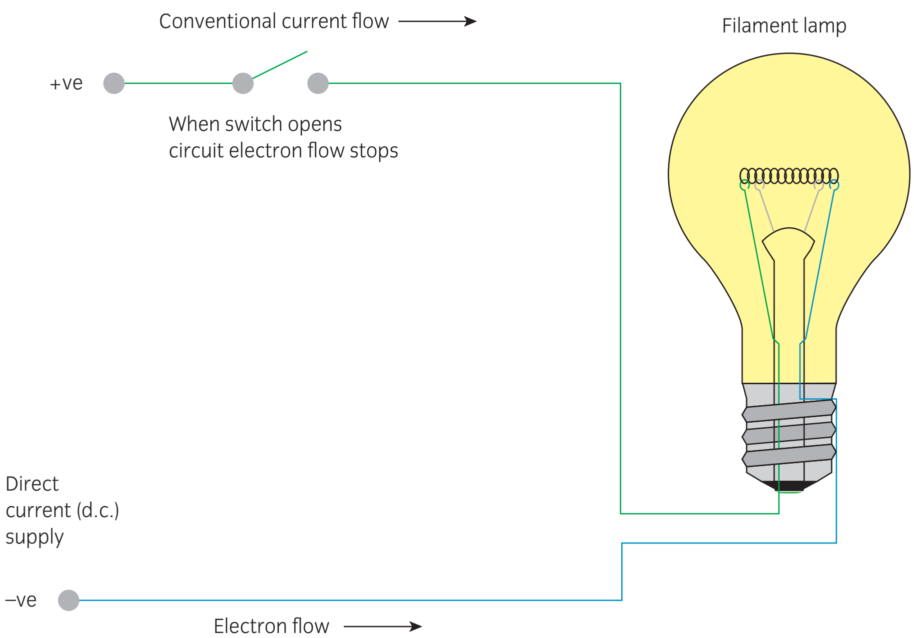
EFFECTS OF ELECTRIC CURRENT

The three main effects of electrical current, similar to sources of electricity, are:

- thermal (heating)
- chemical
- magnetism.

Heating

When current flows in a wire, apart from the flow of electrons, there is a thermal effect; the wire starts to heat up. The amount it heats up depends on factors such as the cross-sectional area of the wire, the amount of current flowing and the material that the wire is made of. The heating effect of electricity is used in electric fires. Variations of this heat effect are used to make light from light-bulb (lamp) filaments, which give off large amounts of light as they glow white hot as a result of the current passing through the thin filament.



An electric light circuit using a d.c. source such as a battery

The full effect of current passing through a wire and producing heat is one of the main considerations when designing electrical installations and will be covered at length during your course. Current that produces heat can be used as an advantage in electrical installations, for example in:

- electric heating
- lighting
- cooking

Assessment criteria

1.3 Describe the effects of an electric current

ACTIVITY

Early incandescent lamps used carbon filaments that were quite fragile. Unfortunately, carbon has a negative coefficient of resistance. This means that, as it gets hotter, the resistance goes down and so it can carry more current and become hotter still. This continues until the filament burns out. For this reason, carbon filaments were limited to small power ratings. What other material has a negative coefficient of resistance?

ASSESSMENT GUIDANCE

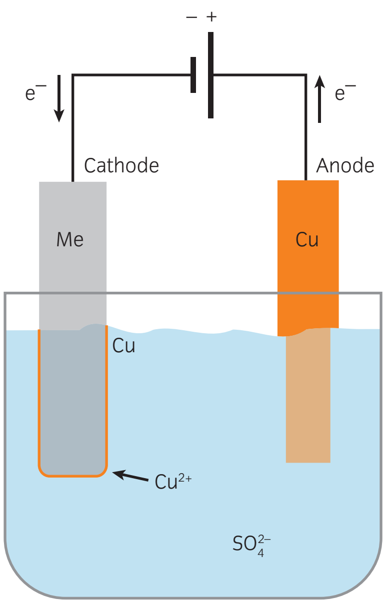
Larger cables have a lower power loss than smaller cables carrying the same current. Although they save on power loss, larger cables cost more to buy and install.

- circuit or equipment protection devices such as fuses or circuit breakers
- monitoring equipment.

There are also disadvantages, which include:

- circuit cables heating up, causing failure
- equipment getting too hot, causing danger
- energy loss.

A cable is designed to carry electricity from one place to another and is not supposed to heat up by any large amount. If it does heat up, it is using energy to do so which means less energy is available where it is required.



The chemical process of electroplating

Chemical

When electric current is passed through a chemical solution, this causes basic chemical changes as ions are allowed to move to the positive plate, creating the process of electroplating or electrolysis. This process is used to coat material for example, as in copper cladding on steel. If a copper-based solution were used, the steel would become coated with copper.

Magnetism

When current passes through a cable, a magnetic field created by the current surrounds the cable. This is described in greater detail in Learning outcome 3.

This magnetic effect can be used extensively in electrical installations and equipment, and is one of the most important principles that has shaped the modern world. Without it, much of the technology used today would not exist, from telecommunications right down to supplying electricity to your house or other buildings.

With careful design, equipment can be produced to create and enhance this magnetic field to great effect. Equipment such as electromagnets, transformers, mobile phone chargers, bells, energy-efficient lighting and electric motors all rely on this magnetic effect.

ACTIVITY

Look around your home. Identify:

- three items that use electromagnetism
- two items that use chemical effects
- three items that use heating effects.

Summary of current effects

Electricity has changed our world more than any other discovery. Before we learnt how to use electricity, the world advanced very little in terms of technology. Widespread use of electricity has only come about in the last 100 years and, in that time, technology has progressed faster than in any other period of history.

A few examples of the application of electrical current

Magnetic effect	Heating effect	Chemical effect
Ammeter	Circuit breaker	Cell/battery
Bell	Electric heater	Electroplating
Circuit breaker	Filament lamp	Fuel cell
Contactor	Furnace	
Data recorder	Fuse	
Discharge lighting	Generation	
Generator	Monitoring/measurement	
Mobile charger		
Motor		
Transformer		

SI UNITS

As the world of science, and electrical science in particular, developed, it became necessary to agree on some form of standardisation so that scientists could understand one another's work and share their ideas.

The system of SI units (short for *Système International d'Unités*) is internationally recognised and based on the metric system. The alternative, based on the imperial system of measurement and still used in the USA (US imperial), is far more complex and less elegant, as the non-electrical comparisons below between SI and imperial demonstrate.

Imperial

1 ton = 20 cwt (hundred-weight) = 2240 lb (pounds weight)
= 35 840 oz (ounces)

Metric

1 tonne = 1000 kg (kilograms) = 1 000 000 g (grams)

There are seven base SI units, which then generate many derived units.

Assessment criteria

1.4 Identify SI units for various electrical quantities

Base SI units

Quantity	Quantity symbol	Unit name	Unit symbol
Current	I	ampere	A
Length	l	metre	m
Luminous intensity	I	candela	cd
Mass	m	kilogram	kg
Temperature	T	kelvin	K
Time	t	second	s

KEY POINT

There are seven base SI units in total. The seventh unit is the measure of substance which is the mole. This unit is mainly used in chemistry to measure chemical substances. It is not really relevant to Electrotech.

Most commonly used electrical science SI units

Quantity	Quantity symbol	Unit name	Unit symbol
Area	A	square metre	m^2
Capacitance	C	farad	F
Charge	Q	coulomb	C
Energy (work)	W	joule	J
Force	F	newton	N
Frequency	f	hertz	Hz
Impedance	Z	ohm	Ω
Inductance	L	henry	H
Magnetic flux	Φ	weber	Wb
Magnetic flux density	B	tesla	T
Potential difference	V	volt	V
Power	P	watts	W
Reactance	X	ohm	Ω
Resistance	R	ohm	Ω
Resistivity	ρ	ohm-metre	Ωm

KEY POINT

In other units, you will see a U used as the symbol for voltage. This is because U is used in BS 7671 Requirements for Electrical Installations (the IET Wiring Regulations) and other European standards as the voltage between two parts of an electrical system.

ACTIVITY

Look in Part 2 of BS 7671 in the section called 'Symbols Used in the Regulations'. What is represented by:

a) U
b) U_0
c) U_{oc}.

Indices

Indices are used to replace repetitive multiplications. For example, 10 × 10 × 10 = 1000, so the calculation could be written easily by saying 10^3, which means ten multiplied by itself twice, or three lots of ten multiplied together.

Where indices are negative, the value becomes a fraction. For example:

$$5^{-1} = \frac{1}{5}$$

$$\text{or } 5^{-2} = \frac{1}{25}$$

$$\text{or } 5^{-3} = \frac{1}{125}$$

Most calculators will have a (x^2) button to square a number and scientific calculators also have a button (x^y), which allows a number to be raised to any power or index. For example, to calculate 5^5, use buttons 5 x^y 5 = 3125. This is much easier than keying 5 × 5 × 5 × 5 × 5.

Generally, in electrical science and principles, large values are used, such as thousands of watts or millions of ohms. Other aspects of electrical work deal with tiny amounts, such as millionths of an ampere or thousandths of an ohm. This can become a problem in calculations, as errors may occur if the correct amount of zeros is not entered into the calculator. Instead of inserting the actual number with lots of zeros, we use 'to the power of ten'.

The 'power of' numbers are given names that are explained in the table below. There is less chance making an error using this method.

Numbers expressed as indices (to the power of 10)

Actual number	Number shown to the power of 10	Prefix used
1 000 000 000 000	10^{12}	tera (T)
1 000 000 000	10^{9}	giga (G)
1 000 000	10^{6}	mega (M)
1000	10^{3}	kilo (k)
100	10^{2}	hecto (h)
10	10^{1}	deka (da)
0.1	10^{-1}	deci (d)
0.01	10^{-2}	centi (c)
0.001	10^{-3}	milli (m)
0.000 001	10^{-6}	micro (μ)
0.000 000 001	10^{-9}	nano (n)
0.000 000 000 001	10^{-12}	pico (p)

To perform this on a calculator, use the button marked **EXP** (it may also be marked **×10**) then insert the index (the 'to the power of' number).

To perform a complex calculation such as 325 giga × 5 micro ÷ 12 mega, you would need to insert a lot of zeros before and after the decimal point, like this:

$$\frac{325\,000\,000\,000 \times 0.000\,005}{12\,000\,000}$$

To make it easier to perform on a calculator, use the indices and the **EXP** button, so the formula becomes:

$$\frac{325 \times 10^{9} \times 5 \times 10^{-6}}{12 \times 10^{6}}$$

So, on a calculator **325** **EXP** **9** **×** **5** **EXP** **–** **6** **÷** **12** **EXP** **6** **=** will give the result 0.14.

If you try calculating $25 \times 10^{6} \times 2 \times 10^{3}$, depending on your calculator, the result obtained may be 5 with a 10 in the right of the screen. If you press the button marked **ENG** or **SHIFT ENG**, you will see the 'to the power of' number change to 50 to the power of 9, which is equal to 50 G, or, if you keep pressing the **ENG** button, you will eventually get 50 000 000 000. All the numbers displayed have the same values, just represented in different ways.

ASSESSMENT GUIDANCE

Energy is normally measured in joules (J), but for many purposes this unit is too small so the kilowatt hour (kWh) is used. A 1 kW heater running for 1 hour will use 1 kWh (the units recorded by a household electricity meter).

KEY POINT

Calculators do not all work in the same way. Therefore, any calculator key sequence suggested is only an example of what would be required on a standard scientific (non-programmable) calculator. If the key sequence does not give the expected result on your calculator, either ask your tutor for advice or refer to the manual for your calculator.

SmartScreen Unit 202

Worksheet 1

KEY POINT

Be wary as sometimes the result on a calculator can be expressed as a 'to the power of', so keep an eye on any numbers to the right of the result in the calculator screen.

Assessment criteria

1.5 Transpose basic formulae

SmartScreen Unit 202

PowerPoint 2 and Handout 2

Algebra

The branch of mathematics that uses letters and symbols to represent numbers, to express rules and formulae in general terms.

Transposition

Rearranging a formula to make the unknown you need to find the subject of the formula.

TRANSPOSING BASIC FORMULAE

In electrical science we use **algebra** all the time when dealing with formulae. For example, Ohm's law states that a voltage can be determined by multiplying current by resistance. Instead of writing it out in full, we use letters and symbols to represent the unknown values. Therefore $V = I \times R$ is a basic use of algebra. Algebra can be used to show relationships between different quantities.

If, for example, you want to find the total cost, b, of four bars of chocolate, and the price of one bar is a, the formula would be:

$$b = 4a$$

Many formulae are used in electrical science. It is much easier to remember one particular formula in one particular way. For example:

$$R = \frac{\rho l}{A}$$

where:
R = resistance in ohms (Ω)
ρ = resistivity value of a particular material (Ωm)
l = length of a cable conductor
A = cross-sectional area of the conductor.
(This formula is explained in more detail on page 87.)

The above formula could be rearranged, to determine the formula for A (see page 76). This is called **transposition**.

To learn the rules of transposition, you need to consider three types of formula:

- those that use addition and subtraction
- those that use multiplication and division
- those involving both (mixed formulae).

The following methods follow simplified mathematical rules and will give you the ability to transpose any formulae you come across at Levels 2 and 3.

Transposing formulae involving addition and subtraction

The rules of transposition for addition and subtraction

1. The unknown you want to find must be on its own on one side of the equals sign.
2. The unknown should not have a minus sign in front of it.

3 Any unknown that moves over the equals sign has the sign in front of it changed from addition to subtraction or from subtraction to addition.

4 Any unknown that does not have a sign (positive or negative) in front of it is assumed to be positive.

Consider how to transpose this formula to find c.

$$a + b + c - d = e$$

The unknown c has a plus sign (+) in front of it so it stays where it is and the other unknowns around it need to move. The unknowns a and e have no sign in front of them so we assume they are not being subtracted from anything.

So, to transpose it:

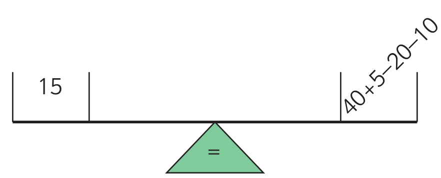
$a + b + c - d = e$	move d over the equals sign
$a + b + c = e + d$	$-d$ changes to $+d$ when moved
$a + c = e + d - b$	move b, changing the + to –
$c = e + d - b - a$	move a (changing the sign) to leave c alone
$c = e + d - b - a$	the finished, transposed formula.

At all times, the equal sign acts as the centre of a set of scales and the formula remains balanced. If numbers are substituted for the letters, this might give:

10 + 20 + 15 – 5 = 40	move numbers away from 15 over the equals sign
10 + 20 +15 = 40 + 5	see that the formula is balanced as it equals 45 on each side
10 + 15 = 40 + 5 – 20	keep the balance and move the 20
15 = 40 + 5 – 20 – 10	move 10 to leave 15 alone
15 = 40 + 5 – 20 – 10	the finished, transposed formula.

ACTIVITY

There are many formulae that will have to be transposed as you progress through the course. Get as much practice as you can. Start with all the formulae you can think of with three items, then four and so on.



Formulae involving addition and subtraction remain balanced over the equals sign.

Transposing formulae involving multiplication and division

Formulae involving division often include parts that are made up of letters and numbers written above and below a line, as fractions. The number above the line is called the numerator, and the number below is the denominator. A denominator is the number of parts a whole is divided into, a numerator is the number of these parts that we are dealing with. So, thinking of ¾ of a cake, the cake is divided into four equal parts, we have three of them.

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{number of parts we have}}{\text{number of equal parts in the whole}}$$

The rules of transposition for multiplication and division

1. The unknown you want to find must be on its own on one side of the equals sign.
2. The unknown to be found must be on its own, not part of a fraction.
3. Any unknown moved over the equal sign changes from top to bottom or bottom to top.
4. Any unknown or number can be written as a fraction by writing it over a denominator of 1, for example, $4 = {}^{4}\!/_{1}$.

KEY POINT

Where a formula shows two unknowns together with no symbol in between, they should be multiplied, so ρl means $\rho \times l$.

The following formula can be transposed, to find a formula for A:

$$R = \frac{\rho l}{A}$$

The A is at the bottom and it needs to be at the top. This is done by moving it over the equals sign. The letter R can be written as a fraction, as ${}^{R}\!/_{1}$. So moving A over the equals sign gives:

$$\frac{RA}{1} = \frac{\rho l}{1}$$

Now all the unknowns are at the top. Remember that anything divided or multiplied by 1 remains the same, but writing the 1s in helps us to remember that there is a top and bottom. Now divide both sides by R.

$$\frac{A}{1} = \frac{\rho l}{R}$$

R has now been moved over the equals sign to leave A alone, so:

$$A = \frac{\rho l}{R}$$

This can once again be demonstrated by using numbers to see how balance is maintained:

$$50 = \frac{20 \times 5}{2}$$

Insert a 1 to show 50 as a fraction:

$$\frac{50}{1} = \frac{20 \times 5}{2}$$

Move the 2 over the equals sign from bottom to top:

$$\frac{50 \times 2}{1} = \frac{20 \times 5}{1}$$

Keep the balance when moving the 50:

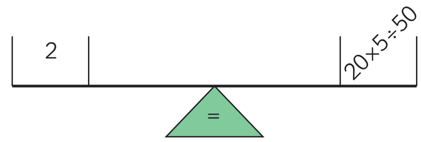
$$\frac{2}{1} = \frac{20 \times 5}{50}$$

Then to finish off, remove the 1 from the left-hand side to get:

$$2 = \frac{20 \times 5}{50}$$

Job done!

With practice, this routine becomes second nature. You just need to remember the rules.



Formulae involving multiplication and division remain balanced over the equals sign

Transposing mixed formulae

This requires combining all the rules. Sometimes numbers or unknowns need to be combined, so they are grouped in brackets. This effectively makes each group act as if it is a single number or unknown. For example, to determine the unknown d from the following formula:

$$\frac{(a + b) \times c \times d}{e} = f$$

As d is at the top, it needs to be left where it is and the other unknowns are moved. As the $(a + b)$ is in brackets, the whole thing can be moved together and treated as a single unknown. Remember, f is also over 1.

$$\frac{(a + b) \times c \times d}{e} = f$$

Move $(a + b)$ over the equals sign, so:

$$\frac{c \times d}{e} = \frac{f}{(a + b)}$$

And:

$$\frac{d}{e} = \frac{f}{(a + b) \times c}$$

So finally:

$$\frac{d}{1} = \frac{f \times e}{(a + b) \times c}$$

Or:

$$d = \frac{f \times e}{(a + b) \times c}$$

Once again, to prove this with numbers:

$$\frac{(3 + 2) \times 4 \times 10}{25} = 8$$

The (3 + 2) is treated as a single number (ie 5), so:

$$\frac{4 \times 10}{25} = \frac{8}{(3 + 2)}$$

ASSESSMENT GUIDANCE

Make sure you practise using as many formulae as you can. Make up your own if you wish and test them out by substituting numbers.

KEY POINT

This method of transposition may be slightly different to the one that you learnt at school, although both methods will give you the same result. You may have learnt to carry out the same operation to both sides of the equation. Use the method that suits you best.

ACTIVITY

Find out what the following mathematical terms mean: BIDMAS and standard form.

SmartScreen Unit 202
Worksheet 2

ASSESSMENT GUIDANCE

Remember Pythagoras' theorem only applies to right-angled triangles.

Hypotenuse
The longest side of a right-angled triangle, which is opposite the right angle.

Then:

$$\frac{10}{25} = \frac{8}{(3 + 2) \times 4}$$

And finally:

$$10 = \frac{8 \times 25}{(3 + 2) \times 4}$$

There is one further rule to remember:

> Whatever you do to one side of the formula, you must keep the balance on the other.

To transpose or rearrange the formula below to make b the subject:

$$\sqrt{a^2 + b^2} = c$$

the square root must be eliminated because it locks in b. The opposite of taking a square root is to square, so:

$$a^2 + b^2 = c^2$$

Applying the rules gives:

$$+b^2 = c^2 - a^2$$

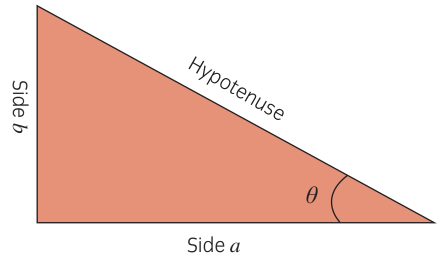
As the unknown needed is b, not b^2, take the square root on both sides:

$$b = \sqrt{c^2 - a^2}$$

Triangles and Pythagoras' theorem

Triangles are used to quantify electrical values. Later you will explore power triangles and phasor diagrams as a way of determining circuit values. To help with these, you need to understand basic principles of trigonometry (see page 79) and Pythagoras' theorem.

The triangle below is a right-angled triangle. Given the lengths of any two sides of a right-angled triangle, you can use Pythagoras' theorem to find the length of the third side. The **hypotenuse** of a right-angled triangle is always opposite the right angle.



A right-angled triangle

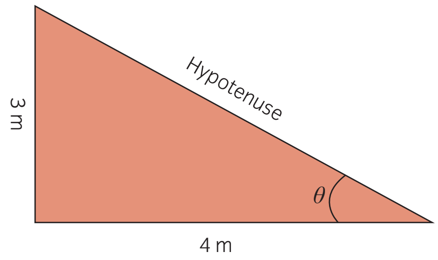
Pythagoras discovered that the square of the length of the hypotenuse is equal to the square of the length of side a added to the square of the length of side b. Or, to express this as a formula:

$$a^2 + b^2 = h^2$$

or:

$$\sqrt{a^2 + b^2} = h$$

The length of the hypotenuse for the triangle below can therefore be calculated by applying Pythagoras' theorem.



Applying Pythagoras' theorem

As:

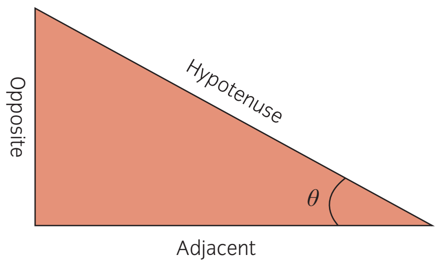
$$\sqrt{a^2 + b^2} = h$$

Then:

$$\sqrt{3^2 + 4^2} = 5 \text{ (the length of the hypotenuse)}$$

Trigonometry

Trigonometry is used extensively in engineering and construction technology as well as many other sciences. Without trigonometry we would not be able to establish the heights of hills, mountains and buildings or the distance to stars and other planets.



The three sides of a right-angled triangle are: adjacent to the angle theta (θ), opposite the angle theta (θ) and the hypotenuse, which is opposite the right angle.

Consider this explanation to help you understand the relationships in trigonometry.

Looking at a right-angled triangle, if any of the sides increase or decrease in length, the angle θ changes accordingly. (Unknown angles are given the symbol theta, θ, from the Greek alphabet.)

There are three formulae to remember when it comes to trigonometry.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

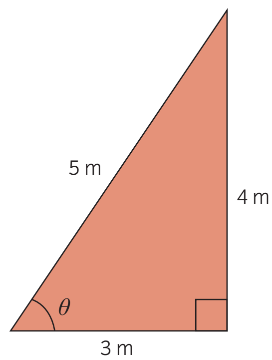
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Some people remember them by the mnemonic SOH CAH TOA.

The sine, cosine and tangent ratios for the angle are calculated using the relationships between the lengths of the sides. For example, the triangle shown here has the following dimensions:

- adjacent = 3 m
- opposite = 4 m
- hypotenuse = 5 m.



Using these values we can work out the sine, cosine and tangent values of the angle.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5} = 0.8$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5} = 0.6$$

$$\tan\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{3} = 1.333$$

Using a calculator:

- $\sin^{-1} 0.8 = 53.1°$
- $\cos^{-1} 0.6 = 53.1°$
- $\tan^{-1} 1.333 = 53.1°$

Each of the ratios determined by using the different lengths relates to the same angle. We can use these different ratios to determine any missing value from the triangle.

You need to select the right formula to do this. Choose the one that uses the information you already have and that gives you what you need to find. For example, if you knew the length of the hypotenuse and the angle, but needed to determine the length of the opposite, you would choose the formula that uses all three. This is the sine formula.

Values of sine, cosine and tangent are ratios that have been calculated for every possible angle. Many years ago, before calculators, these factors were found from books of mathematical tables. These days, all of the different possible values are programmed into your scientific calculator.

When using the **SIN**, **COS** and **TAN** functions on a calculator, pressing each button directly will provide the sine, cosine or tangent value (ratio) for the given angle. For example, **SIN** **45** will give a value of 0.7071, which means an angle of 45° has a sine value of 0.7071. To find a value of angle from a calculated or given sine, cosine or tangent, you will need to use the **SHIFT** or **2nd Function** button. You need to locate the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ symbols. They are usually accessed by applying **SHIFT** or **2nd Function** to **SIN**, **COS** and **TAN**. With this book, if you see the function $\cos^{-1}$, $\tan^{-1}$ or $\sin^{-1}$, the **SHIFT** or **2nd Function** button is required for that calcuation.

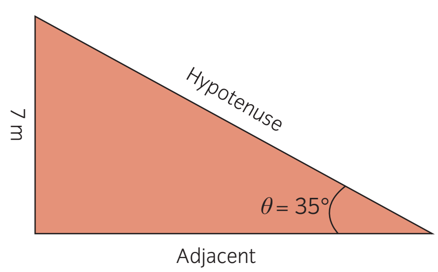
ACTIVITY

Sine, cosine and tangent are the three main trigonometric ratios. Identify three others.

KEY POINT

Calculators do not all work in the same way. Therefore, any calculator key sequence suggested is only an example of what would be required on a standard scientific (non-programmable) calculator. If the key sequence does not give the expected result on your calculator, either ask your tutor for advice or refer to the manual for your calculator. Ensure your calculator is set to DEG (for degrees), not RAD or GRAD or your results will be very different. Radians and gradians are other methods of measuring angles.

Sine



To determine the length of the hypotenuse of the triangle above, as you know the value of the angle and the length of the opposite side, use the trigonometric function sine as it includes the three known or needed ingredients.

Use the formula:

$$\sin \theta = \frac{o}{h}$$

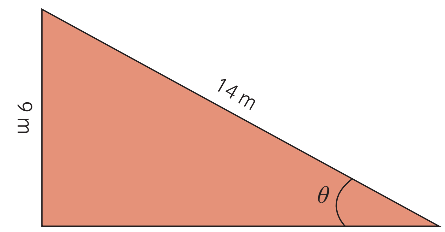
Transpose the formula:

$$h = \frac{o}{\sin \theta}$$

so:

$$h = \frac{7}{\sin 35°} = 12.2 \text{ m}$$

Example



Use the sine function to determine the value of the required angle.

Then:

$$\sin\theta = \frac{o}{h}$$

so:

$$\sin\theta = \frac{9}{14} = 0.642$$

This is not the actual value of the angle, but the sin of the angle. In order to find the angle from this, use the $\sin^{-1}$ function by pressing the **SHIFT** button or second function button, depending on your calculator. So the angle is found as:

$$\sin^{-1} 0.642 = 40°$$

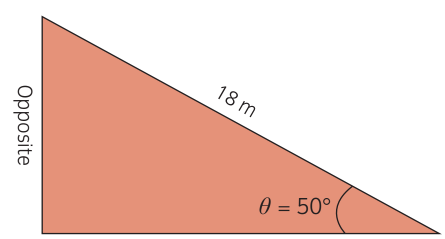
So on the calculator, press **SHIFT** **SIN** **0.642** **=**

The sequence **9** **÷** **14** **=** **sin⁻¹** **=** will give the angle as 40°. This answer is more accurate as the calculator remembers all of the values after the decimal point in the value of the sine of the angle.

KEY POINT

The button on your calculator marked **ANS** will insert the answer of the last calculation into the one being performed.

Cosine



To determine the length of the side adjacent to θ in the triangle above, use cosine as you know the size of the angle and the length of the hypotenuse, and you need to find the length of the adjacent side.

Use the formula:

$$\cos\theta = \frac{a}{h}$$

Transpose the formula:

$$a = \cos\theta \times h$$

so:

$$a = \cos 50° \times 18 = 11.57 \text{ m}$$